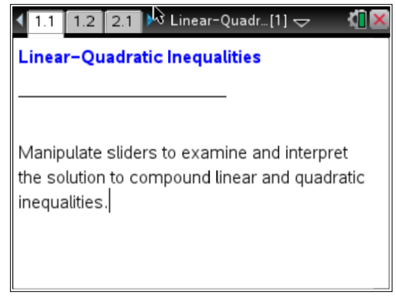

Math Objectives

- Students will be able to describe the solution to a linear-quadratic or quadratic-quadratic system of inequalities from a geometric perspective.
- Students will be able to write the solution to a linear-quadratic or quadratic-quadratic system of inequality as a compound inequality.
- Students will generate quadratic-quadratic systems of inequalities with specific types of solutions.
- Students will model with mathematics. (CCSS Mathematical Practice)

Vocabulary

- linear
- quadratic
- system of inequalities

About the Lesson

- In this lesson, students will manipulate sliders to explore the solutions of a linear-quadratic and a quadratic-quadratic system of inequalities.
- As a result, students will:
- Solve linear-quadratic and quadratic-quadratic systems of inequalities from a geometric perspective.
- Make conjectures about solutions of linear-quadratic and quadratic-quadratic systems of inequalities and verify their conjectures by manipulating sliders.
- Generate quadratic-quadratic systems of inequalities with specific types of solutions.

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- Use Teacher Software to demonstrate the activity.
- Use Class Capture to formally assess students' understanding.
- Use Live Presenter for students to share their thinking.
- Use Quick Poll for assessing students' understanding.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Linear_Quadratic_Inequalities_ Student.doc
- Linear_Quadratic_Inequalities_ Student.pdf

TI-Nspire document

- Linear_Quadratic_Inequalities.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty changing the slider, check to make sure that they have moved the cursor (arrow) until the triangles become shaded. Remind students that when they finish moving any slider, they should press esc to release.

Tech Tip: An alternate method of changing the slider values is to move the cursor near the triangles and press enter. Then, students can change the values simply by clicking up and down on the Touchpad (▲ and ▼).

Tech Tip: Tap on the arrows to change the values of the slider.

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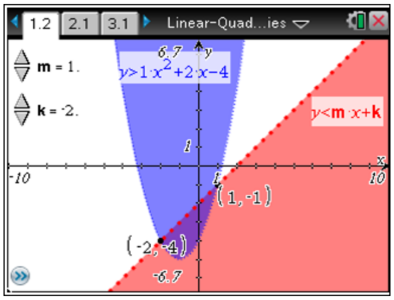

1. Select ▲ and ▼ to change the values of *m* and *k* such that *m* = 1 and *k* = –2.

   a. What do the shaded areas above the parabola and below the line represent?

   Sample answer: The shaded area above the parabola is the solution to the inequality $y > x^2 + 2x - 4$. The region below the line is the solution to the inequality $y < x - 2$.

   b. Describe the solution of the compound inequality $y > x^2 + 2x - 4$ and $y < x - 2$.

   Sample answer: The solution of the compound inequality is the region where the two shaded areas overlap. This is the area above the parabola and below the line.

   On the TI-Nspire™ CX, this is the purple region, which is the overlap of the blue shaded region above the parabola and the red shaded region below the line.

   On the TI-Nspire™, this is the dark gray region that is the overlap of the two lighter gray regions.

c. Choose a point in the solution area described in part 1b and show that it satisfies the compound inequality $x^2 + 2x - 4 < y < x - 2$.

Sample answer: The point (0, –3) lies in the solution area. By substituting $x = 0$ and $y = -3$, you get $0^2 + 2(0) - 4 < -3 < 0 - 2$, which simplifies as $-4 < -3 < -2$.

d. How would you explain to a friend what regions to shade if he or she were graphing the inequality by hand?

Sample answer: To graph $y > x^2 + 2x - 4$, first graph the parabola $y = x^2 + 2x - 4$. The *y*-values that are greater than those that lie on the parabola must lie above it. Shade the region above the parabola. To indicate that the parabola itself is not included in the solution, sketch a dotted or dashed parabola.

To graph $y < x - 2$, graph the line $y = x - 2$. The *y*-values that are less than those that lie on the line must lie below it. Shade the region below the line. To indicate that the line is not included in the solution, sketch a dotted or dashed line. Be sure the shading distinguishes this region from the previously shaded region.

The solution of the compound inequality $y > x^2 + 2x - 4$ and $y < x - 2$ is the intersection (or overlap) of the two shaded regions.

e. Select the inequality to change $y > x^2 + 2x - 4$ to $y < x^2 + 2x - 4$. Describe the solution of the compound inequality $y < x^2 + 2x - 4$ and $y < x - 2$.

Sample answer: The solution of the compound inequality is the region where the two shaded areas overlap. This is the area below the dashed parabola and below the dashed line.

On the TI-Nspire™ CX, this is the purple region, which is the overlap of the blue shaded region below the parabola and the red shaded region below the line.

On the TI-Nspire™, this is the dark gray region, which is the overlap of the shaded region below the parabola and the shaded region below the line.

TI-Nspire™ Navigator™ Opportunity: Class Capture/Live Presenter
See Note 1 at the end of this lesson.

Tech Tip: If a student clicks once on the inequality and presses [del], the whole inequality may disappear. To undo this, the student may press [ctrl] [esc] or [ctrl] [Z].

Tech Tip: To edit an equation, double-tap it, which brings up the keyboard. Use the arrows to navigate to the text you want to edit.

2. Change the inequality symbol and use the sliders to obtain the solution to compound inequality $y > x^2 + 2x - 4$ and $y < -0.5x - 0.5$.

   a. Describe the solution of $x^2 + 2x - 4 < y < -0.5x - 0.5$.

   <u>Sample answer:</u> The solution of the compound inequality is the region where the two shaded areas overlap. This is the area above the parabola and below the line.

   On the TI-Nspire™ CX, this is the purple region, which is the overlap of the blue shaded region above the parabola and the red shaded region below the line.

   On the TI-Nspire™, this is the dark gray region, which is the overlap of the shaded region above the parabola and the shaded region below the line.

   b. Use this information to solve the inequality $x^2 + 2x - 4 < -0.5x - 0.5$. Explain your reasoning.

   <u>Sample answer:</u> The *x*-values that will make the inequality true are above the parabola and below the line (the intersection of the two shaded regions). These are the *x*-values between the *x*-coordinates of the labeled intersection points, namely, $-3.5 < x < 1$.

   c. Describe the solution of $y > x^2 + 2x - 4$ and $y > -0.5x - 0.5$.

   <u>Sample answer:</u> The solution of the compound inequality is the region where the two shaded areas overlap. This is the area above the parabola and above the line.

   On the TI-Nspire™ CX, this is the purple region, which is the overlap of the blue shaded region above the parabola and the red shaded region above the line.

   On the TI-Nspire™, this is the dark gray region, which is the overlap of the shaded region above the parabola and the shaded region above the line.

Teacher Tip: Students may edit the inequality again to confirm their solutions.

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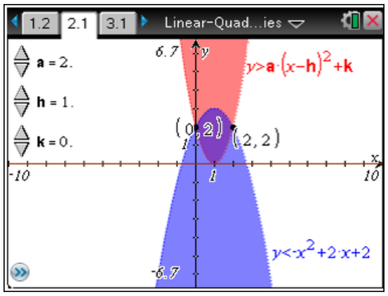

3. Use the sliders to change the values of *a, h,* and *k* such that $a = 2$, $h = 1$, and $k = 0$.

   a. Show how to obtain the coordinates of the two labeled points algebraically.

   Sample answer: The labeled points are the coordinates of the intersection of the two parabolas. To find these coordinates, set the two related equations equal to each other and solve for *x.*

$$2(x-1)^2 = -x^2 + 2x + 2$$
$$2x^2 - 4x + 2 = -x^2 + 2x + 2$$
$$3x^2 - 6x = 0$$
$$3x(x-2) = 0$$
$$x = 0 \text{ or } x = 2$$

   Substitute the *x*-values into either equation to find the corresponding *y*-values. The coordinates are (0, 2) and (2, 2).

   b. Describe the solution of $2(x-1)^2 < y < -x^2 + 2x + 2$.

   Sample answer: The solution of the compound inequality is the region where the two shaded areas overlap. This is the area above the parabola $y = 2(x-1)^2$ and below the parabola $y = -x^2 + 2x + 2$.

   On the TI-Nspire™ CX, this is the purple region, which is the overlap of the red shaded region above the parabola $y = 2(x-1)^2$ and the blue shaded region below the parabola $y = -x^2 + 2x + 2$.

   On the TI-Nspire™, this is the dark gray region, which is the overlap of the shaded region above the parabola $y = 2(x-1)^2$ and the shaded region below the parabola $y = -x^2 + 2x + 2$.

   c. What are the domain and range of the points in the solution set?

   Answer: The domain is $0 < x < 2$ and the range is $0 < y < 3$.

   d. Explain how you obtained your answer to part 3c.

Sample answer: The domain is the set of *x*-values that are inside the two parabolas (the intersection of the two shaded regions). These are the *x*-values between the *x*-coordinates of the labeled intersection points, $0 < x < 2$.

The range is the set of *y*-values that are inside the two parabolas (the intersection of the two shaded regions). To find these values, find the minimum and maximum *y*-values in the overlapping regions. These values occur at the vertices of the two parabolas. Since the parabola $y = 2(x - 1)^2$ is given in vertex form, you know that its vertex is (1, 0).

Students can calculate the vertex of the parabola $y = -x^2 + 2x + 2$ by completing the square, finding the axis of symmetry, or using the TI-Nspire™ to analyze the graph. Its vertex is (1, 3). Thus, the range is the set of *y*-values between 0 and 3, $0 < y < 3$.

TI-Nspire™ Navigator™ Opportunity: Quick Poll (Open Response)
See Note 2 at the end of this lesson.

4. If possible, give values of *a, h*, and *k* such that the solution set of the compound inequality $y < -x^2 + 2x + 2$ and $y > a(x - h)^2 + k$ is

   a. a single point

   Sample answer: The only way a single point could be the solution to the inequality would be if the parabolas shared exactly one common point. However, because these are strict inequalities (strictly greater than and strictly less than), the inequalities do not include the points that lie on the parabolas themselves. Thus, there are no values for *a, h*, and *k* that would produce a solution of a single point. (Compare and contrast this to a solution set for a system of quadratic equations.)

   b. two points

   Sample answer: For any system of two quadratic inequalities, it is impossible to have a solution set of two points. If the parabolas are tangent to each other, they will have one point in common. If the parabolas intersect in two points, there will be an overlapping region containing an infinite number of points.

   c. the empty set

   Sample answer: There are an infinite number of values for *a, h*, and *k* that would produce no solution set. For example, choose *a* = 2, *h* = 1, and *k* = 4.

5. Are your answers to question 4 the only correct answers? Explain.

   <u>Sample answer:</u> For parts 4a and 4b, there are no values for *a, h*, and *k*. So, those answers are the only correct answers. For part 4c, there are infinite possibilities for *a, h*, and *k.* By simply increasing the value of *k*, you can find an infinite number of parabolas that lie above the parabola $y = -x^2 + 2x + 2$.

6. If the inequalities in question 4 were changed to $y \le -x^2 + 2x + 2$ and $y \ge a(x - h)^2 + k$, would any of your answers change? Explain.

   <u>Sample answer:</u> If the parabolas themselves were included, the answer to part 4a would change. There are numerous possibilities. For example, the two parabolas may share a common vertex. Since the vertex of the parabola $y = -x^2 + 2x + 2$ is (1, 3), this vertex can be utilized as the vertex of the parabola $y \ge a(x - h)^2 + k$. Sample inequalities include $y \ge (x - 1)^2 + 3$, $y \ge 2(x - 1)^2 + 3$, etc.

   Thus, the point (1, 3) would be the solution set of the compound inequality $y \le -x^2 + 2x + 2$ and $y \ge 2(x - 1)^2 + 3$.

   Another possibility is for the parabolas to be tangent to each other not at the vertex. One such example is $y \le -x^2 + 2x + 2$ and $y \ge 2(x + 2)^2 - 3$.

TI-Nspire™ Navigator™ Opportunity: Quick Poll (Yes/No)
See Note 3 at the end of this lesson.

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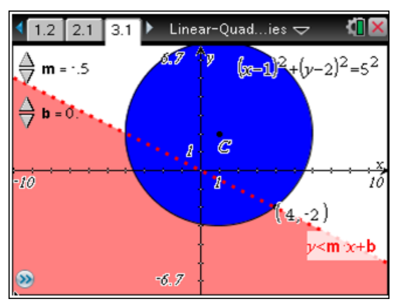

7. Use the sliders to change the values of *m* and *b* such that *m* = –0.5 and *b* = 0.
   a. Write an inequality to represent the shaded area in the interior of the circle.

      <u>Answer:</u> The circle shown has an equation of $(x - 1)^2 + (y - 2)^2 = 5^2$. The shaded area would thus represent the region $(x - 1)^2 + (y - 2)^2 < 5^2$.

b. Adjust the sliders so that the line goes through the diameter of the circle, and write its equation.

Sample answer: There are an infinite number of answers. Two possible answers are $y = -0.5x + 2.5$ and, simply, $y = 2$.

c. Is there more than one correct answer to part 7b? Why?

Sample answer: Any line that passes through the center of the circle goes through the diameter of the circle. The center of the circle is (1, 2). To write an equation for a line passing through the point (1, 2), you would have to know the coordinates of another point or know the slope of the line. In answering this question, you can substitute values of your own choosing. Thus, there are countless possibilities.

TI-Nspire™ Navigator™ Opportunity: Quick Poll (Open Response)
See Note 4 at the end of this lesson.

d. Write a compound inequality that describes

i. the area below the diameter and in the interior of the circle

Sample answer: For the equation $y = x + 1$, the area below the diameter will be $y < x + 1$. The area in the interior of the circle contains points such that $(x - 1)^2 + (y - 2)^2 < 5^2$.

Thus, the solution is $y < x + 1$ and $(x - 1)^2 + (y - 2)^2 < 5^2$.

ii. the area above the diameter and in the exterior of the circle

Sample answer: Using the equation $y = x + 1$, the area above the diameter will be $y > x + 1$. The area in the exterior of the circle contains points such that $(x - 1)^2 + (y - 2)^2 > 5^2$.

Thus, the solution is $y > x + 1$ and $(x - 1)^2 + (y - 2)^2 > 5^2$.

Teacher Tip: The answers for question 7d will vary depending on the equation students wrote for the line passing through the diameter. However, all of the inequalities will have the same inequality for the circle and the same inequality sign for the diameter. Guide students to see this for themselves.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to solve linear-quadratic and quadratic-quadratic systems of inequalities.
- How to write the solution to a linear-quadratic or quadratic-quadratic system of inequalities as a compound inequality.
- How to generate quadratic-quadratic systems of inequalities with specific types of solutions.

Assessment

Give students several linear-quadratic and quadratic-quadratic systems of inequalities, and ask them to graph their solutions by hand.

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Note 1

Question 1, Class Capture/Live Presenter: If students are working on their handhelds, you may want to use **Live Presenter** to have a student demonstrate how to utilize the sliders and change the inequality sign. Use **Class Capture** to monitor the students' progress throughout the activity.

Note 2

Question 3, Quick Poll (Open Response): Send an **Open Response Quick Poll** in which students type in their domain and range. This will give you an opportunity to see if they understand the concepts that are being taught. This will also provide you with the opportunity to have students share their thought processes in obtaining their domain and range.

Note 3

Question 6, Quick Poll (Yes/No): Send a **Yes/No Quick Poll** in which students type in their answer to question 6. (You may also want to do the same for question 5.) If students respond that their answers would change, ask them which answers would change, and why those answers would change. If none of the students respond that their answers would change, that should also promote discussion.

Note 4

Question 7, Quick Poll (Open Response): Send an **Open Response Quick Poll** in which students type in their equations for the diameter. This will enable students to see the endless possibilities for the equation and show them how easily the question could have been answered if they had simply written the equation $y = 2$ (which many of them may not have considered). It will also prepare students for the numerous possible answers for part 7d.