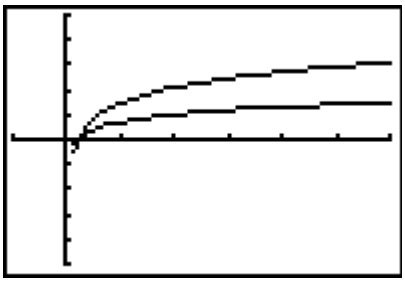
Problem 1 – Relating log functions with different bases

Execute the **DIFFBASE** program. Press PRGM and choose it from the list. Press ENTER.

Choose **SeeGraphs** from the menu. This program displays the graphs of two logarithmic functions with different bases.

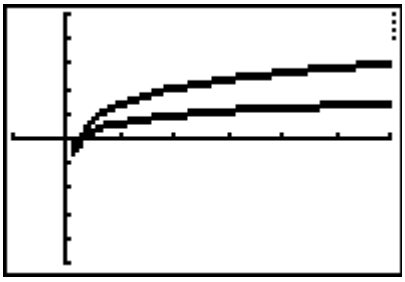
$\mathbf{Y1}(x) = \log_a x$ and $\mathbf{Y2}(x) = \log_b x$



You can determine *a* and *b* by examining points on each curve. This is a picture file, not an actual graph, so you cannot use the **Trace** feature, but you can move the cursor along the curve yourself to approximate the coordinates of points on each curve.

- What are *a* and *b*?
- What points on the graph are the best clues to the base of the logarithmic functions?

Once you think you know you *a* and *b*, run the **DIFFBASE** program again and choose **GuessBases** from the menu. Enter the values of *a* and *b* that you found.



The program graphs two logarithmic functions with bases you entered as thick lines on top of the original graph. If you choose *a* and *b* correctly, your graph will look like the one shown.

If you see more than 2 curves on your graph, try different values for *a* and *b*.

Suppose we were interested in the sum of these functions,

$$(\mathbf{Y1} + \mathbf{Y2})(x) = \log_a x + \log_b x\,.$$

How can we write this as a single logarithmic expression?

- We cannot apply the properties of logarithms unless the bases are the same.
- We need to rewrite the functions with the same base.
- We want to find a function that is equal to **Y1**, but involves logarithms base *b* instead logarithms base *a*.

We could hope that there is a constant *c* that relates the two functions, like:

$$c \cdot \mathbf{Y1}(x) = \mathbf{Y2}(x)\,.$$

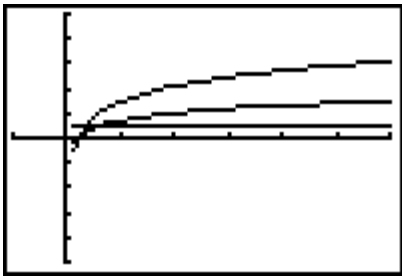
Then we would have

$$\mathbf{Y1}(x) = \frac{\mathbf{Y2}(x)}{c} = \frac{1}{c} \cdot \log_b(x)$$

which is a logarithmic function with base *b*.

We cannot be sure there is such a constant, but that doesn't have to stop us from looking for one.

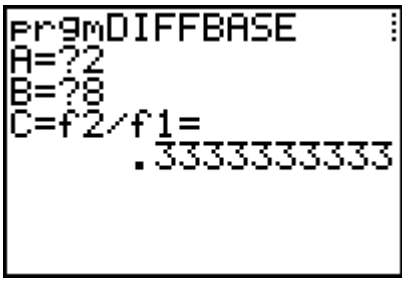
Run the **DIFFBASE** program again and choose **GraphC** from the menu. The program calculates $c = \frac{\mathbf{Y_2}(x)}{\mathbf{Y_1}(x)} = \frac{\log_b x}{\log_a x}$ and stores the result in **Y3**.



Examine the graph of **Y3** and then view the **Y3** function table. What is *c*?

Problem 2 – A closer look at *c*

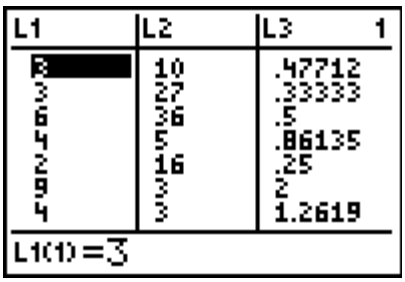
Is *c* always the same? Run the **DIFFBASE** program and choose **CalculateC** from the menu. Given *a* and *b*, the program calculates ***c*** and displays the value. Try two different values of *a* and *b*. What is *c* now?



Continue to choose **CalculateC** from the menu to experiment with different values of *a* and *b*. As you try different values, the program record the results in the **Lists**. (Values of *a* are stored in **L1**, *b* values in **L2**, and *c* values in **L3**.)

Be sure to try some powers of *a* and *b* such that one is a power of the other, like 2 and 8 or 9 and 3.

After you have tried at least 10 different values for *a* and *b*, exit the program and view the data in the lists. (Press [STAT] [ENTER].) Can you guess a formula for *c*?



Hint: Try calculating 1/*c* in **L4**.

We are convinced now that there is a constant that relates $\log_a x$ to $\log_b x$ and that the constant depends on the values of *a* and *b*. We may even have an idea how to calculate the constant *c* given *a* and *b*.

Time to use some algebra to find out for sure.

Two functions are equal if and only if their values are equal for every *x*-value in their domain. Let's pick a generic point (*x*, *y*) on the graph of **Y1**. For this (*x*, *y*), $\log_a x = y$.

If we can write *y* in terms of logarithms base *b*, we will have our function.

- Rewrite $\log_a x = y$ as an exponential expression.

- We want an expression with base *b* logarithms, so take $\log_b$ of both sides.

- Simplify using the properties of logarithms.

- Solve for *y*.

This is the relationship! You should have found that

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}.$$

You have found a formula for changing the base of a logarithm. To change a logarithm base *a* to a logarithm base *b*, simply divide the expression by $\log_a b$.

- Use this formula to find $(\mathbf{Y1} + \mathbf{Y2})(x)$ if $\mathbf{Y1}(x) = \log_3(x)$ and $\mathbf{Y2}(x) = \log_5(x)$.