Multiplicity of Zeros of Functions

Math Objectives

- Students will determine the multiplicity of zeros of a polynomial function when given its graph or its equation in factored form.
- Students will write an equation for a polynomial function when given information about its zeros and the multiplicity of the zeros.
- Students will write an equation for a polynomial function when given its graph.
- Use appropriate tools strategically (CCSS Mathematical Practice).
- Look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- degree of a polynomial
- end behavior
- multiple zeros
- multiplicity

About the Lesson

- This lesson involves students utilizing graphs and equations of five polynomial functions to determine the zeros of the functions and whether the functions cross the *x*-axis or just touch the x-axis at the zeros.
- As a result, students will:
  - Determine the degree of the polynomial functions and the effect the degree has upon the end behavior of the functions.
  - Write possible equations for a polynomial function, given information about its zeros.
  - Write the equations in factored form, given the graphs of three functions.

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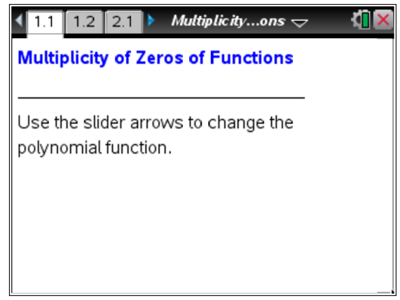
- Send the TI-Nspire document to students
- Use Class Capture or Quick Poll to examine the values of *x* for which the graph crosses the x-axis
- Use Quick Poll questions to adjust the pace of the lesson according to student understanding

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Multiplicity_of_Zeros_of_Functions_Student.pdf
- Multiplicity_of_Zeros_of_Functions_Student.doc

TI-Nspire document

- Multiplicity_of_Zeros_of_Functions.tns

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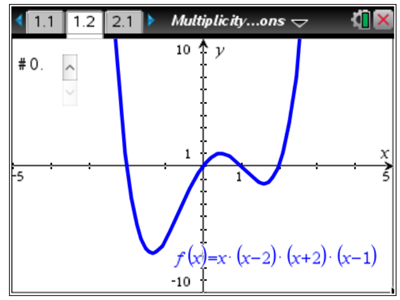
Discussion Points and Possible Answers



Move to page 1.2.

1. The initial value of the slider is **#0**.

   a. What are the zeros of the function?

   Answer: x = -2, *x* = 0, *x* = 1, *x* = 2

   b. For what value(s) of *x* does the graph of the function cross the x-axis?

   Answer: x = -2, *x* = 0, *x* = 1, *x* = 2

   c. For what value(s) of *x* does the graph of the function touch but not cross the *x*-axis?

   Answer: none

   d. What degree is the polynomial?

   Answer: 4

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

2. Use the slider to change the graph for functions **#1–#5**. For each function, answer the questions asked in Question 1. Use the table below to record your results.

   Answer:

#	Function	Zeros	Cross	Touch	Degree
1	$f(x) = (x + 1)^2(x - 2)(x - 1)$	–1, 1, 2	1,2	–1	4
2	$f(x) = (x + 1)(x - 2)^2(x - 1)$	–1, 1, 2	–1, 1	2	4
3	$f(x) = (x + 2)^2(x - 1)^2$	–2, 1	None	–2, 1	4
4	$f(x) = (x + 1)^3(x - 1)(x - 2)$	–1, 1, 2	–1, 1, 2	None	5
5	$f(x) = (x - 2)^2(x - 1)(x + 1)^2$	–1, 1, 2	1	–1, 2	5

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3. How are the zeros of a polynomial function related to the factors of a polynomial function?

Answer: The zeros of the function are the solutions when the factors are set equal to zero and solved. When the coefficient of x is 1 in the factor, the zero and the constant term in the factor have opposite signs.

> **Teacher Tip:** All the polynomial functions in this activity have a leading coefficient of 1.

4. How do the exponents in each term in the factored form of the polynomial function affect its graph?

Answer: When the exponent of the factor is odd, the graph crosses the x-axis at the corresponding zero. When the exponent of the factor is even, the graph just touches the x-axis at the corresponding zero.

5. Revisit graphs **#1**–**#5**, and observe the end behavior for the polynomial functions. What does the degree of the polynomial function tell you about its end behavior?

Sample Answers: When the degree of a polynomial function is even and the leading coefficient is positive, the arms of the graph are both up. ($x \to \infty$, $f(x) \to \infty$; $x \to -\infty$, $f(x) \to \infty$). When the degree of a polynomial function is odd and the leading coefficient is positive, then one arm is down and one arm is up. ($x \to \infty$, $f(x) \to \infty$; $x \to -\infty$, $f(x) \to -\infty$).

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

6. When a polynomial has a repeated linear factor, it has a multiple zero. Write the factored form of a polynomial function that crosses the x-axis at $x = -2$ and $x = 5$ and touches the x-axis at $x = 3$. Which of the zeros of the function must have a multiplicity greater than 1? Explain your reasoning.

Answer: $f(x) = (x + 2)(x - 5)(x - 3)^2$; $x = 3$ must have a multiplicity greater than 1 because the graph just touches the x-axis at x = 3.

> **Teacher Tip:** The other two zeros could have an odd multiplicity greater than 1 because the degree of the polynomial is not given. However, $x = 3$ is the only zero that must have an even multiplicity greater than 1.

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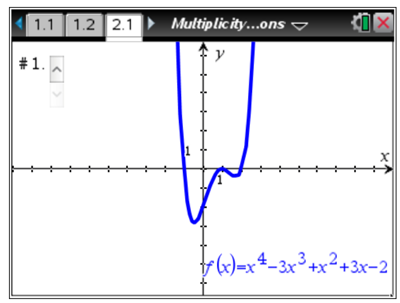
7. Write two additional polynomial functions that meet the same conditions as described in Question 6. Explain what is different from your function in Question 6, and how you determined your polynomial functions.

Sample Answers: Answers will vary. However, the exponents of the factors ($x + 2$) and ($x – 5$) must be odd because the graph crosses at these corresponding zeros. The exponent of the factor ($x – 3$) must be even because the graph just touches the x-axis at $x = 3$.

Examples: $f(x) = (x + 2)(x – 5)(x – 3)^4$
$f(x) = (x + 2)^3(x – 5)(x – 3)^2$
$f(x) = (x + 2)^3(x – 5)^5(x – 3)^2$

Teacher Tip: Students might change the a-value instead of the exponents of the factors. For example, $f(x) = 2(x + 2)(x – 5)(x – 3)^2$. Encourage the students to explore the meaning of varying exponents.

Move to page 2.1.



8. Select the slider until it reads **#1.**
   a. Write the factored form of the polynomial function graphed.

   Answer: $f(x) = (x + 1)(x – 1)^2(x – 2)$

   b. Describe how you determined the factors of the polynomial function.

   Sample Answers: The graph crosses the x-axis at $x = –1$ and $x = 2$ and touches at $x = 1$. This means there is even multiplicity greater than 1 at $x = 1$. One factor must be ($x – 1$) raised to an even power. Since the degree of the polynomial is 4, the exponents of the factors must add to 4. If ($x – 1$) takes a power of 2 for multiplicity, ($x + 1$) and ($x – 2$) can only have a power of 1.

9. Select the slider until it reads **#2**.
   a. Write the factored form of the polynomial function graphed.

   Answer: $f(x) = (x + 1)^2(x – 1)^2$

   b. Describe how you determined the degree of each of the factors of the polynomial function.

   Answer: The graph never crosses the x-axis, but it touches at $x = –1$ and $x = 1$, meaning both are multiple zeros. The degree of the polynomial is 4, so each factor has an exponent of 2.

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10. Select the slider until it reads **#3**.
    a. Write the factored form of the polynomial function graphed.

    Answer: $f(x) = (x + 2)(x + 1)(x - 1)^3$

    b. Verify your answer by expanding the polynomial and comparing to the standard form given.

    Answer: The graph of the function crosses at $x = -2$, $x = -1$, and $x = 1$. Since the degree of the polynomial is 5, one of the roots has a multiplicity of 3.

    Expansion:
    $f(x) = (x + 2)(x + 1)(x - 1)^3$
    $f(x) = (x^2 + 3x + 2)(x^3 - 3x^2 + 3x - 1)$
    $f(x) = x^5 - 4x^3 + 2x^2 + 3x - 2$

    Teacher Tip: Students might not recognize which zero should have a multiplicity of 3. If the first expansion does not match the given equation, they might have to reconsider their function. This would be a good place to discuss concavity if appropriate.

11. For what reasons would you use the factored form of a polynomial function? The standard form?

    Answer: The factored form of a polynomial function shows the x-intercepts clearly, while it is easier to find the y-intercept using the standard form.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The multiplicity of zeros of a polynomial function when given its graph or its equation in factored form.
- How to write an equation for a polynomial function when given information about its zeros and the multiplicity of the zeros.
- How to write an equation for a polynomial function when given its graph.

Extension

Move back to graph #4 on page 1.2. Compare and contrast how the graph of the function looks when it crosses the *x*-axis at $x = -1$ versus how it looks when it crosses the *x*-axis at $x = 1$ and $x = 2$. Why do you think there are differences in the graph and how does it relate to the multiplicity of each of these zeros? (Zoom in if necessary.)

Answer: Although the graph crosses the *x*-axis at all three zeros, there is a difference in the behavior of the graph at $x = -1$. At $x = -1$, the function changes concavity, which is a result of the factor $(x + 1)$ having an odd exponent greater than 1.

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Note 1

Question 2, *Live Presenter*

After students have answered all of the parts of Question 2, use Teacher Edition computer software or Live Presenter to show students the zeros of graph #1. Use the **Point On** tool and drag it to each of the three zeros. See how many students were able to identify all three. (Note: Do not use the **Graph Trace** tool because there are multiple functions graphed in this problem.)
Repeat this demonstration with graphs #2 – #5 if so desired.

Note 2

Questions 4–5, *Quick Poll*

Use several Quick Polls to test students' understanding of how exponents and degrees affect the behavior of a graph. For example:

1. A polynomial has a factor $(x - 8)^2$. What happens at the zero?
   A. Touches the *x*-axis
   B. Crosses the *x*-axis

Correct answer: A

2. A polynomial has degree 7 and has a leading coefficient of 3. The arms of the graph are:
   A. both up.
   B. both down.
   C. one up and one down.

Correct answer: C