NUMB3RS Activity: All in the Family
Episode: "Nine Wives"

Topic: Genetic Probabilities **Grade Level:** 10 - 12
Objective: To understand the higher probability of genetic problems caused by inbreeding.
Materials: TI-83 Plus/TI-84 Plus graphing calculator
Time: 15 - 20 minutes

Introduction

In "Nine Wives," Millie and Charlie examine a quilt found during an investigation. Millie shows Charlie that the quilt is actually a family tree for Abner Stone's cult. Because this cult is a small group that practices polygamy, it is quite possible that the same person can be both the paternal and maternal grandfather of a person who has unrelated grandmothers. This activity examines the genetic implications of this situation.

Discuss with Students

If your students have not studied genetics, explain that genes occur in pairs. Different versions of a gene are called alleles. During conception, each parent randomly contributes one of his or her two alleles to the child. For biology students, this presents an excellent opportunity to involve the biology department in an interdisciplinary activity. Perhaps invite a biology teacher to address the class about genetics to help students understand the increasing connection between biology and mathematics.

Some illnesses are caused by recessive alleles and will not be apparent unless the gene pair consists of two recessive alleles. Of the millions of genes in the human body, this activity considers a single pair and studies what happens when a child's mother and father both have the same father. Students calculate the probability that the child will receive the same allele from the grandfather through both the mother and the father. They then calculate the probability of inheriting two identical alleles when a child has the same grandfather on both sides of the family and compare this with the probability in the general population of inheriting two identical alleles.

Note that while some students may find it beneficial to actually key in the ALLELE program on their calculators, the program file can also be downloaded for free by going to **http://education.ti.com/exchange** and searching for "7802."

The extensions provide opportunity for interdisciplinary connections with social studies teachers to discuss the scientific reason for the taboo on incest (as opposed to the moral one that exists in most societies), as well as some problems that historians see that stem from inbreeding among royal families.

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Student Page Answers:

1.

↓***Father*** / ***Mother***→	g^1b^1	g^1b^2	g^2b^1	g^2b^2
g^1a^1	$\mathbf{g^1g^1}$, g^1b^1 a^1g^1, a^1b^1	$\mathbf{g^1g^1}$, g^1b^2 a^1g^1, a^1b^2	g^1g^2, g^1b^1 a^1g^2, a^1b^1	g^1g^2, g^1b^2 a^1g^2, a^1b^2
g^1a^2	$\mathbf{g^1g^1}$, g^1b^1 a^2g^1, a^2b^1	$\mathbf{g^1g^1}$, g^1b^2 a^2g^1, a^2b^2	g^1g^2, g^1b^1 a^2g^2, a^2b^1	g^1g^2, g^1b^2 a^2g^2, a^2b^2
g^2a^1	g^2g^1, g^2b^1 a^1g^1, a^1b^1	g^2g^1, g^2b^2 a^1g^1, a^1b^2	$\mathbf{g^2g^2}$, g^2b^1 a^1g^2, a^1b^1	$\mathbf{g^2g^2}$, g^2b^2 a^1g^2, a^1b^2
g^2a^2	g^2g^1, g^2b^1 a^2g^1, a^2b^1	g^2g^1, g^2b^2 a^2g^1, a^2b^2	$\mathbf{g^2g^2}$, g^2b^1 a^2g^2, a^2b^1	$\mathbf{g^2g^2}$, g^2b^2 a^2g^2, a^2b^2

2. *8 (in bold in the chart)* **3.** $\frac{8}{64} = \frac{1}{8} = 0.125 = 12.5\%$ **4.** *The first gene can be any of the 10. The probability that the second gene matches the first is 1/10 = 10%.* **5.** *Eight of the 64 gene pairs have probability 1 that the genes will be identical. For all other gene pairs, the probability is 1/10 that the genes will be identical. The total probability is* $\frac{8}{64} \times 1 + \frac{56}{64} \times \frac{1}{10} = \frac{17}{80} = 21.25\%$. *This probability is 9/80 = 11.25% greater than the probability in the general population of inheriting an identical pair of genes.* **6.** *Answers will vary, but for a large number of trials, results should approach 0.21.*

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Name: _____ Date: _____

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In “Nine Wives,”Millie and Charlie examine a quilt found during an investigation. Millie shows Charlie that the quilt is actually a family tree for Abner Stone’s cult. Because this cult is a small group that practices polygamy, it is quite possible that a child’s mother and father could both have the same father, even though they have different mothers. In other words, the child has two grandmothers but only one grandfather. This activity examines the genetic implications of this situation.

Genes come in pairs. Different versions of a gene are called alleles, one inherited from the father and the other from the mother. This activity considers a single pair. When a child is conceived, each parent randomly contributes one of his or her two alleles to the child. For example, if the father has genes f^1 and f^2 and the mother has m^1 and m^2, the child is equally likely to have any of the following combinations: f^1m^1, f^1m^2, f^2m^1, or f^2m^2. Since both mother and father contribute one of the two genes, there are 2 × 2, or 4, possibilities.

1. Suppose the grandfather has alleles g^1g^2, grandmother A has a^1a^2, and grandmother B has b^1b^2. In the table shown below, the possible allele combinations for the father are listed along the left. The possible allele combinations for the mother are listed along the top. If the father has allele combination g^1a^1 and the mother has allele combination g^1b^1, then the child has an equal chance of inheriting each of the four allele combinations: g^1g^1, g^1b^1, a^1g^1, or a^1b^1. Complete the table for the possible allele combinations for the child.

↓***Father / Mother***→	g^1b^1	g^1b^2	g^2b^1	g^2b^2
g^1a^1	g^1g^1, g^1b^1 a^1g^1, a^1b^1			
g^1a^2				
g^2a^1				
g^2a^2				

2. How many of the possible gene pairs in the table contain either g^1g^1 or g^2g^2?

Many of the most severe hereditary diseases are caused by recessive alleles. This means that while a person can carry and transmit the allele, he or she will not have the disease unless the same allele comes from both the mother and the father. If only one copy of the recessive allele is inherited, the dominant allele will prevail and there will be no symptoms of the disease.

3. What is the probability that a child whose paternal and maternal grandfathers are the same person will inherit the same grandfather’s allele through both the father and the mother (either g^1g^1 or g^2g^2)?

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The ultimate goal of this activity is to calculate the probability of inheriting two identical alleles when the child has the same grandfather on both sides of the family, and then compare this to the probability of inheriting two identical alleles in the general population. To simplify the calculations, assume that for a particular gene in the general population, there are only ten different possible alleles, and each is equally likely to occur.

4. What is the probability that someone in the general population will inherit two copies of the same allele?

For a child with the same grandfather on both sides of the family, we want to find the probability that the child will inherit two copies of the same allele for a particular gene. For example, if the child inherits g^1g^1, then both alleles *must* be the same. But if the child inherits g^1g^2, then they only *might* be the same allele. The allele g^1 from the grandfather could be any one of the ten possible alleles. The probability that g^1 and g^2 are the same is $\frac{1}{10}$.

5. Find the probability that this child will have two copies of the same gene. How does this compare with the probability in the general population of inheriting two copies of the same gene?

6. Design and run an experiment that randomly picks gene pairs for the grandfather and each grandmother and then randomly selects which gene is contributed to the father and mother and to the child. Use a large number of trials, and then compare the results with the theoretical probability from #5.

The following calculator program can be used to run the experiment in Question 6:

```
:0→S
:0→J
:Disp "TRIALS?"
:Input T
:While J<T
:randInt(1,10,6)→L1
:{L1(randInt(1,2)),L1(randInt(3,4)),L1(randInt(1,2)),
L1(randInt(5,6))}→ L2
:{L1(randInt(1,2)),L1(randInt(3,4))}→L3
:Disp L3
:If L3(1) = L3(2)
:S + 1→S
:J + 1→J
:End
:Disp S/T
```

The list L_1 contains the two alleles for the grandfather, the two alleles for grandmother A, and the two alleles for grandmother B. List L_2 becomes the two alleles each for the father and the mother, and list L_3 produces the two alleles for the child. This program will ask for the number of trials. It will display the ratio of how many times the child’s alleles are identical to the total number of trials. Try this several times, increasing the number of trials each time.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

This activity considers only a single allele combination. The human genome contains millions of such combination. When a particular trait is carried on a recessive allele, the trait will not be passed down unless both alleles for that trait are this recessive one. The descendent carries the allele, but does not display any characteristics of the trait.

For the Student

This is an excellent opportunity to apply mathematics to other disciplines. Throughout history, most societies have had taboos about incest. This activity shows some of the scientific basis for these taboos. But it is also true that some societies have not respected these taboos. History has many cases of royal marriages among close relatives and the resulting genetic difficulties from these unions. Ask a history teacher for examples of this and research some of these cases. Present a report to your history class on the mathematics behind these problems.

Research the laws in your state to learn about the degree to which people can be related to each other and still be permitted to marry. Calculate the probability that a common ancestor within that degree of kinship will provide a combination of identical alleles. How does it differ from the probability calculated for a common grandfather in this activity?

Additional Resources

In 1921, American geneticist Sewall Wright developed a formula to calculate an *inbreeding coefficient*. For more on this formula and its meaning, see the *NUMB3RS* activity (also for “Nine Wives”) called “A Breed Apart.” To download this activity, go to **http://education.ti.com/exchange** and search for “7803.”

For a detailed discussion of Wright’s and other inbreeding coefficients through the use of family trees of race horses, see: **http://www.highflyer.supanet.com/coefficient.htm**

Related Topic

Some argue that England’s Queen Victoria was indirectly responsible for the Russian Revolution and the rise of Communism. A mutation in one of her alleles caused many of her descendents to have hemophilia. One of these descendents was Alexis, the son of Czar Nicholas II. Although Alexis did not die of hemophilia, he needed constant attention from his parents, which caused them to neglect the affairs of state, culminating in the Russian Revolution. For the complete story of how Victoria’s hemophilia spread throughout the royal families of Europe, along with family trees, see: **http://www.sciencecases.org/hemo/hemo.asp**

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