



# Exponential Dice

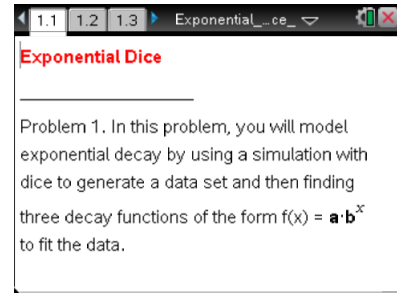
## Student Activity

Name \_\_\_\_\_

Class \_\_\_\_\_

Open the TI-Nspire document *Exponential\_Dice.tns*.

Any quantity that grows or decays at a fixed rate at regular intervals grows or decays exponentially. Many real world phenomena can be modeled by exponential functions to show how things grow or decay over time. Examples of such phenomena include the studies of the population growth of people, bacteria, and viruses; the decay of radioactive substances; the change of temperatures; and the accumulation of interest or the payment of credit.



In Problem 1, a simulation involving dice will generate a data set that can be modeled by a function in the form  $f(x) = a \cdot b^x$ . You will input a value of  $a$  and then determine three possible values of  $b$ .

In the simulation, we toss a large number of dice, remove all the dice with certain face value(s) such as 6's, 3's and 4's, etc., and then repeat these two steps until only a few dice are left. You will need to enter two values in this simulation:

- 1)  $a$  = initial number of dice (around 200 is reasonable)
- 2)  $f$  = number of face values to remove for the next toss ( $f = 1$  if you are removing one face such as the 6's only;  $f = 2$  if you are removing two faces such as the 2's and 3's, etc.)

Imagine that a stomach bug is spreading through your school and you are trying to keep track of the number of students who have not yet become ill. You can suppose that:

- Each die represents a person.
- Each toss represents a week.
- If a **6** comes up, a student becomes ill, so remove that die from the population.

**Move to page 1.2.**

Press **ctrl** **▶** and **ctrl** **◀** to navigate through the lesson.

1. Reset the simulation by changing the value of  $a$  to  $a = 0$ . Then change the values of  $a$  and  $f$  to  $a = 200$  and  $f = 1$  to simulate starting with 200 dice (students) and removing all the 6's at each trial. The right half of the page is a spreadsheet where the first two columns contain the trial number and the number of dice remaining (number of students who have not yet become ill) after that trial.
  - a. For the first function  $f_1(x) = a \cdot b^x$ , explain why  $a$  is initial the number of dice.



# Exponential Dice

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In the third column, **ratio**, the ratios between consecutive entries in Column B,  $\frac{b_2}{b_1}, \frac{b_3}{b_2}, \frac{b_4}{b_3}, \dots$  etc.,

have been calculated.

- b. Explain what each ratio represents.

The value of  $b$  for the first function is the average value of these ratios. It is calculated and stored as  $b$ .

- c. Explain why this value of  $b$  is a reasonable choice for the base of an exponential decay function to model this data.

- d. Record your first function here:  $f1(x) =$  \_\_\_\_\_ .

### Move to page 1.4.

Enter your function on this page by typing **define**  $f1(x) = a \cdot b^x$ . You can type either the variables  $a$  and  $b$  or type their numerical values.

### Move back to page 1.3.

On Page 1.3, you will see a scatterplot of your data (**trialr**, **clistr**). To view the graph of  $f1(x)$  on the scatterplot, open the entry line, move back up to  $f1(x)$ , and press enter.

- e. How well does the graph of your function fit the data?
- f. According to this model, approximately what percent of the dice are being removed during each trial?

### Move to page 1.4.

On Page 1.4, you will find the regression equation to fit the data by selecting **MENU > Statistics > Stat Calculations > Exponential Regression**. Select "**trialr**" as the  $x$ -list and "**clistr**" as the  $y$ -list, and store this equation as  $f2$ .

2. a. Record your regression function here:  $f2(x) =$  \_\_\_\_\_ .



**Move back to page 1.3.**

Go back to Page 1.3 to view the graph of the regression function on the scatterplot. If the graph of  $f2(x)$  is not displayed, open the entry line, move up to  $f2(x)$ , and press .

b. How does the graph of the exponential regression function compare to that of your first function?

Theoretically, you would expect that  $\frac{1}{6}$  of the current number of dice would be removed at every trial.

3. a. In this situation, what is the theoretical value for  $b$ ?

b. Record your third function here:  $f3(x) =$  \_\_\_\_\_ using this theoretical value of  $b$  and the initial value  $a$  you selected for the first function.

**Move to page 1.4.**

Enter the theoretical function onto this page by typing **define**  $f3(x) =$  and adding the function from part 3b after the equals sign.

**Move back to page 1.3.**

If the graph of  $f3(x)$  is not displayed on the scatterplot, open the entry line, move up to  $f3(x)$ , and press .

**Move to page 1.4.**

4. Compute and interpret the following quantities;

a.  $f1(6) - f3(6)$  and  $f2(6) - f3(6)$

b.  $f1(9) - f3(9)$  and  $f2(9) - f3(9)$

c.  $f1(12) - f3(12)$  and  $f2(12) - f3(12)$

Note: It is possible to get an error message if fewer than 12 trials were needed in the simulation. Check the spreadsheet on Page 1.2. If 18 or more trials were needed, you might want to compute these quantities when  $x = 15$  or some larger value.



5. The **half-life** of a quantity whose value decreases with time is the length of time it takes for the quantity to decay to half of its initial value. Knowing the value of the half-life of various radioactive elements is sometimes used to determine the age of fossils and other natural objects.
- a. Find the half-life of this decay model using the exponential regression function,  $f_2(x)$ .

Hint: You can use the “nsolve” command on this Calculator page.

- b. Find the half-life of this decay model using the theoretical exponential decay function,  $f_3(x)$ .
6. Suppose you ran another simulation where you removed all the 3's and 4's at each trial starting with 220 dice.
- a. Find the theoretical decay function,  $g(x)$  for this situation.

Record your answer here:  $g(x) =$  \_\_\_\_\_

- b. Find the half-life of a decreasing quantity modeled by the function  $g(x)$ .

**Move to page 2.1.**

Many things such as populations of people and animals grow at an exponential rate, In Problem 2, a simulation involving dice will generate a data set that can be modeled by a function in the form  $f(x) = a \cdot b^x$ . You will input a value of  $a$  and then determine three possible values of  $b$ .

In the simulation, we toss a small number of dice, add a die for each die with certain face value(s) such as 6's, 3's and 4's, etc. and then repeat these two steps until there are around 200 dice. You will need to enter two values in this simulation:

- 1)  $a$  = initial number of dice (3-5 is reasonable)
- 2)  $f$  = number of faces to save for the next toss ( $f = 2$  if you add a die for each of two faces such as the 3's and 4's ;  $f = 3$  if you add a die for each of three faces such as the 3's, 4's, and 5's, etc.)

Imagine that you are keeping track of the deer population in a nearby animal park. You can suppose that

- Each die represents a deer
- Each toss represents a year.
- If a **3 or 4** comes up, a deer is born, so add a die to the population.



**Move to page 2.2.**

Reset the simulation by changing the value of  $a$  to  $a = 0$ . Then change the values of  $a$  and  $f$  to  $a = 4$  and  $f = 2$  to simulate starting with 4 dice and adding a die for each of the 3's and 4's that occur.

The right half of the page is a spreadsheet where the first two columns contain the trial number and the number of dice accumulated after that trial. In the third column, **ratio**, the ratios between consecutive entries in Column B,  $\frac{b_2}{b_1}, \frac{b_3}{b_2}, \frac{b_4}{b_3}, \dots$  etc., have been calculated.

7. a. Explain what each ratio represents.

The first value of  $b$  is the average value of these ratios. It is calculated and stored as  $b$ .

- b. Explain why this value of  $b$  is a reasonable choice for the base of an exponential growth function to model this data.

- c. Record your first function here:  $f1(x) = \underline{\hspace{4cm}}$ .

**Move to page 2.4.**

Enter your function onto this page by typing **define**  $f1(x) = a \cdot b^x$ . You can type either the variables  $a$  and  $b$  or type their numerical values.

**Move back to page 2.3.**

On Page 2.3, you will see a scatterplot of your data (**trialr**, **clistr**). If the graph of  $f1(x)$  is not displayed on the scatterplot, open the entry line, move up to  $f1(x)$ , and press **enter**.

- d. How well does the graph of your function fit the data?
- e. According to this model, approximately what percent of the dice are being added during each trial?



**Move to page 2.4.**

On Page 2.4, find the regression equation to fit the data by selecting **MENU > Statistics > Stat Calculations > Exponential Regression**. Select “**trialr**” as the  $x$ -list and “**clistr**” as the  $y$ -list, and store this equation as  $f2$ .

8. a. Record your regression function here:  $f2(x) =$  \_\_\_\_\_.

**Move back to page 2.3.**

Go back to Page 2.3 to view the graph of the regression function on the scatterplot. To display the graph, open the entry line, move up to  $f2(x)$ , and press **enter**.

b. How does the graph of the exponential regression function compare to that of your first function?

Theoretically, you would expect that  $\frac{1}{3}$  of the current number of dice would be added at every trial.

9. a. In this situation, what is the theoretical value for  $b$ ?

b. Record your third function here:  $f3(x) =$  \_\_\_\_\_ using this theoretical value of  $b$  and the initial value  $a$  you selected for the first function in 7c.

**Move to page 2.4.**

Enter the theoretical function on this page by typing **define**  $f3(x) =$  and adding the function from part 9b after the equals sign

**Move back to page 2.3.**

If the graph of  $f3(x)$  is on the scatterplot, open the entry line, move up to  $f3(x)$ , and press **enter**.



Move to page 2.4.

10. Compute and interpret the quantities:

- a.  $f_1(6) - f_3(6)$  and  $f_2(6) - f_3(6)$
  
- b.  $f_1(9) - f_3(9)$  and  $f_2(9) - f_3(9)$
  
- c.  $f_1(12) - f_3(12)$  and  $f_2(12) - f_3(12)$

Note: It is possible to get an error message if fewer than 12 trials were needed in the simulation. Check the spreadsheet on Page 2.2. If 18 or more trials were needed, you might want to compute these quantities when  $x = 15$  or some larger value.

11. The **doubling time** of a quantity whose value increases over time is the length of time it takes for the quantity to double in size. It is applied to population growth, inflation, compound interest, the volume of tumors, and many other things that tend to grow over time.

- a. Find the doubling time of this growth model using the exponential regression function,  $f_2(x)$ .

Hint: You can use the “nsolve” command on this Calculator page.

- b. Find the doubling time of this growth model using the theoretical exponential growth function,  $f_3(x)$ .

12. Suppose you added a die for each of the 3's, 5's, and 6's at each trial starting with 3 dice.

- a. Find the theoretical growth function,  $g(x)$  for this situation.

Record your answer here:  $g(x) =$  \_\_\_\_\_

- b. Find the doubling time of an increasing quantity modeled by the function  $g(x)$ .