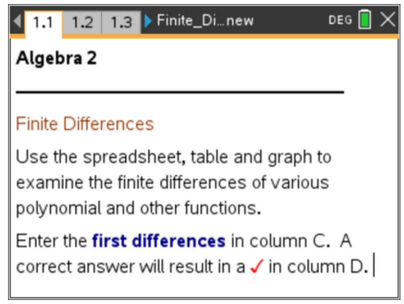
Finite Differences
Student Activity

Name ________________

Class ________________

Open the TI-Nspire document *Finite_Differences.tns.*

This activity looks at the finite differences of polynomial functions and investigates the relationship between the constant value of finite differences and the slope of a line (or the leading coefficient of a quadratic function).



Move to page 1.2.

Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

On page 1.2, there is a table of points (*xc, yc*) for a linear function. An interesting property for some functions is called the Finite Differences Method. The set of first differences is $y_2 - y_1, y_3 - y_2, y_4 - y_3, \ldots$ (the value of *y* minus the previous value of *y)* when the *x*-values increase by the same amount. In column C, enter the values of the first differences. A message will tell you if you are correct.

1. What do you notice about the set of first differences?

Move to page 1.3.

This is the graph of the linear function with the set of ordered pairs shown on page 1.2. The slope and equation are also given.

2. a. What do you notice about the set of first differences, the slope and the equation?

   b. Use the linear equation **f**(*x*) = *mx* + *b* to explain the relationship between the set of first differences and the slope. (Hint: Consider how the value of the function changes for any *x* and *x* + 1.)

Move to page 2.1.

Click the slider in the upper left corner of the screen until the differences are equal.

Move to page 2.2.

This is the graph of the function with the set of ordered pairs shown on page 2.1.

3. Is the slope related to the differences?

Move to page 3.1.

Click the slider in the upper left corner of the screen again until the differences are equal.

Move to page 3.2.

4. What is the relationship between the first set of differences (when the *x*-value increases by something other than 1) and the slope of the line? Explain.

Move to page 4.1.

Click the slider in the upper left corner of the screen until the differences are equal.

Move to page 4.2.

5. When the first set of differences is negative, what impact does that have on the graph?

6. Looking at linear data, Meredith subtracted $y_1 - y_2$ and found the constant set of first differences to be 5. Owen subtracted $y_1 - y_2$ and found the constant set of first differences to be –5. What is the slope of the line, assuming that the *x*-values are increasing by 1? Explain why the order in which the subtraction is performed is important.

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Move to page 5.1.

Click the slider in the upper left corner of the screen until the differences are equal.

7. What do you notice about the set of first differences? Second differences?

Move to page 5.2.

This is the graph of the quadratic function with the set of ordered pairs shown on page 5.1.

8. a. With an *x*-value increase of 1, what seems to be the relationship between the second differences and *a*, the leading coefficient in the equation?

   b. Tanesia made a conjecture that the rate of change for the quadratic function is a linear function. Does her conjecture seem reasonable? Why or why not?

Move to page 6.1.

Click the slider in the upper left corner of the screen until the differences are equal.

Move to page 6.2.

This is the graph of the quadratic function with the set of ordered pairs shown on page 6.1.

9. With an *x*-value increase of 2, what is the relationship between the second differences and *a*, the leading coefficient in the equation?

Move to page 7.1.

Click the slider in the upper left corner of the screen until the differences are equal.

Move to page 7.2.

This is the graph of the quadratic function with the set of ordered pairs shown on page 7.1.

10. Regardless of the *x*-value increase, what is the relationship between the second differences and *a*, the leading coefficient in the equation?

11. Revisit the three quadratics from pages 5.2, 6.2, and 7.2. Remember what you learned earlier about the relationship between the sign of the leading coefficient *a* and the direction in which the quadratic opens.
    a. Make a prediction about the relationship between the constant difference and the sign of the leading coefficient, *a*.

    b. Use the graph of the function and what you know about the rate of change of a quadratic function to explain why your prediction is reasonable.

Move to pages 8.1 and 8.2.

The points on page 8.1 are from a cubic equation.

12. Predict how many subtractions it will take until the differences are constant. Check your prediction by clicking the slider until the differences are equal.

13. Which set of finite differences would be a constant for a polynomial of n^{th} degree? Explain your reasoning.

14. Summarize your results from the investigation.
   a. For a linear function, if the first set of differences is a positive constant, the graph ______________________________.

   b. For a linear function, if the first set of differences is a negative constant, the graph ______________________________.

   c. For a quadratic function, if the second set of differences is a positive constant, the graph ______________________________.

   d. For a quadratic function, if the second set of differences is a negative constant, the graph ______________________________.

Move to page 9.1.

15. a. Suppose the terms of a sequence are given as 1, 3, 6, 10, 15, 21, …. Make a conjecture about the next three elements in the sequence. Explain the rule you are using.

   b. The function $\mathbf{f}(x)=\frac{-x^7}{504}+\frac{x^6}{18}-\frac{23x^5}{36}+\frac{35x^4}{9}-\frac{967x^3}{72}+\frac{239x^2}{9}-\frac{178x}{7}+10$ gives the values of the sequence from part 15a when the *x*-values of 1 to 6 are substituted into the equation. Input 7, 8, and 9 in the *x-value* column of the spreadsheet and complete the table below.

***x*-value**	**Function (f(*x*) from above)**	**First differences**	**Second differences**
1	1	2	1
2	3	3	1
3	6	4	1
4	10	5	1
5	15	6	
6	21		

   c. Look at the set of first and second differences for these values in the completed table. How do the results compare to your prediction in part 15a?

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Move to pages 10.1 and 10.2.

16. a. What is different about the sets of finite differences and the graph of this function, compared to the others you have looked at in this activity?

    b. Explain why the pattern of differences repeats in the way it does.