Change of Base

ID: 9469

Time required
45 minutes

Activity Overview

In this activity, students discover the change of base rule for logarithms by examining the ratio of two log functions with different bases. It begins with a review of the definition of a logarithmic function. Then students explore the hypothesis that the two functions are related by a constant. Finally, they prove the change of base rule algebraically and find the sum of the original functions.

Topic: Logarithmic Functions

- *Derive and use the equation* $\log_a x = \log_a b \cdot \log_b x$ *to covert a logarithm to another base.*

Teacher Preparation and Notes

- *Prior to beginning this activity, students should have experience applying the other properties of logarithms and solving simple*
- *This activity also offers deeper insight into the definition of a function through a discussion of what it means for two functions to be equal.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9469" in the keyword search box.***

Associated Materials

- *ChangeOfBase_Student.doc*
- *ChangeOfBase.tns*
- *ChangeOfBase_Soln.tns*

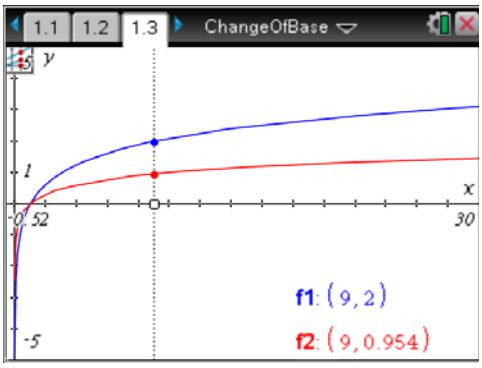
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Properties of Logarithms (TI-Nspire technology) — 9607*

Problem 1 – Relating log functions with different bases

Students are given the graphs of two logarithmic functions on page 1.3 and told on page 1.2 that they are of the form **f1**(x) = $\log_a x$ and **f2**(x) = $\log_b x$. Students should trace the graphs (**MENU > Trace > All Trace**) to find the values of a and b. Students should also realize that the most informative points on the graphs will be those at which y = 1 or some other whole number. (When y = 1, x is equal to the base of the logarithm.)

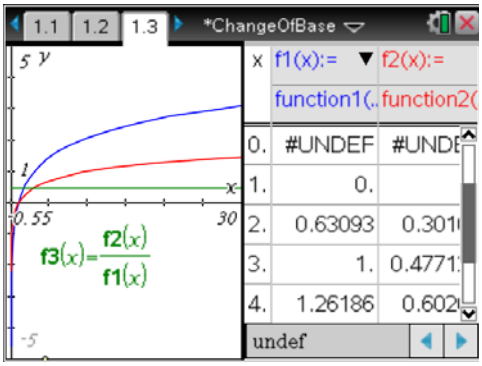


TI-Nspire Navigator Opportunity: *Live Presenter*

See Note 1 at the end of this lesson.

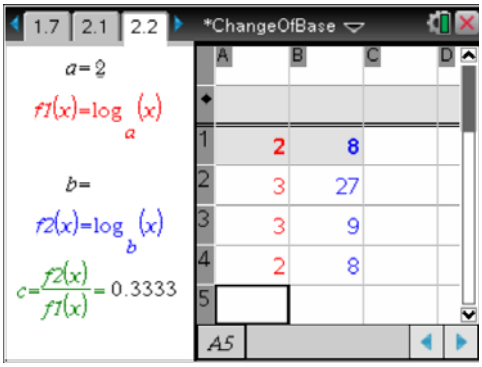
Page 1.4 presents a motivation for wanting to change the base of a logarithmic function. Page 1.6 introduces the hypothesis: suppose these two functions were related by a constant factor? That would make things much easier. Stress the fact that we do not know that such a c exists at this point, but are following a hunch.

Students should find that c = **f2**(x)/**f1**(x) and enter that expression as **f3** on page 1.3. When the graph of **f3** appears to be a straight line, that is one clue that there is indeed a constant factor c. Reviewing the values of **f3** in a function table (**MENU > View > Add Function Table**) and seeing that they are constant throughout the domain of **f1** and **f2** provides further evidence.



Problem 2 – A closer look at *c*

In Problem 2, students test different values of a and b (the bases of the logarithmic functions) to see how they affect the value of c. Students can work independently to enter values for a and b in the gray cells in the first row of page 2.2, then record a, b, and the value of c that is calculated for them in the lower rows. This creates a function table that gives students an intuitive sense of the multivariable function $c(a, b)$. Guide students to guess a rule for c based on this data. If you wish, direct them to calculate **1/*c*** in Column D as a way of giving a hint. (c = 1/($\log_a b$)).

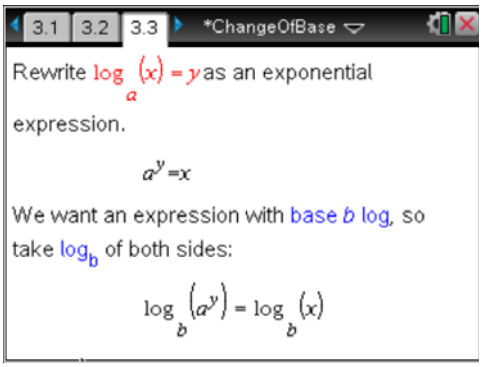


TI-Nspire Navigator Opportunity: *Live Presenter*

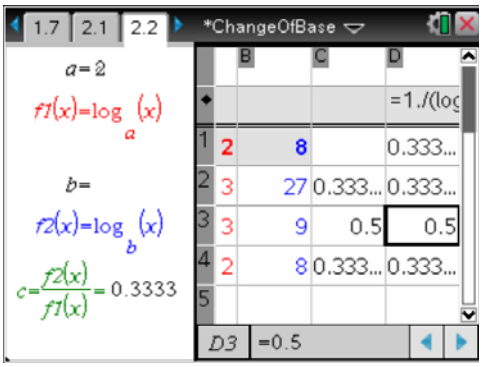
See Note 2 at the end of this lesson.

Problem 3 – Deriving the Change of Base Rule algebraically

Problem 3 steps students through the process of deriving the Change of Base rule algebraically. You may wish to have students record their work on a piece of paper instead of typing their expressions into the handheld.



On page 3.2, make sure students understand that if they can find a way to rewrite the function values at a generic point (x, y), they have rewritten the function.



On page 3.6, students are directed to check their algebra by entering the equation for c into one of the columns on page 2.2 and comparing it against the data they found there.

Finally, on page 3.7, students use the rule they derived to solve the motivating problem.

$$\log_3 x + \log_{10} x$$
$$= \frac{\log_{10} x}{\log_{10} 3} + \log_{10} x$$
$$= \frac{1}{\log_{10} 3}\left(\log_{10} x + \log_{10} 3 \cdot \log_{10} x\right)$$
$$= \frac{1}{\log_{10} 3}\left(\log_{10} x + \log_{10} x^{\log_{10} 3}\right)$$
$$= \frac{\log_{10} x^{1+\log_{10} 3}}{\log_{10} 3}$$

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 3 at the end of this lesson.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Live Presenter*
Consider using *Live Presenter* on page 1.3 to foster discussion on what the values for the bases of the logarithms.

Note 2
Problem 2, *Live Presenter*
Consider using *Live Presenter* to show students how to enter the values in the spreadsheet.

Note 3
Problem 3, *Quick Poll*
Send a *Quick Poll* asking students for their equation on page 3.4.