

## World Population

ID: 9322

Time required  
45 minutes

## Activity Overview

*In this activity, students use their handhelds to explore world population data from the years 1950–2006. They will develop various equations to model the data.*

## Topic: Exponential Functions

- Evaluate the exponential function  $f(x) = Aa^x$  for any value of  $x$ .
- Calculate the exponential line of best fit to model bivariate data and use it to predict a value of one variable corresponding to a value of the other.

## Teacher Preparation and Notes

- This activity is designed for use in a Precalculus classroom.
  - They should also have some experience with the general form of an exponential function  $f(x) = a \cdot b^x$ , be familiar with base  $e$ , know the laws of exponents, and experience functions in the form of  $f(x) = a \cdot e^x$ .
  - Before the activity you may choose to share the following information to generate interest:
    - The world population is well over the 6 billion mark with China at 1,319,175,347 people and India at 1,118,088,730 people, compared to the US at 303,665,014.
    - The population density of China is 137 people/km<sup>2</sup> and of India is 336 people/km<sup>2</sup>, compared to the US at 31 people/km<sup>2</sup>.
- Source: U.S. Census Bureau, Population Division/International Programs Center  
[www.census.gov/popest/estimates](http://www.census.gov/popest/estimates)
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
  - **To download the student and solution TI-Nspire documents (.tns files), go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “9322” in the keyword search box.**

## Associated Materials

- WorldPopulation\_Student.doc
- WorldPopulation.tns
- WorldPopulation\_Soln.tns

## Suggested Related Activities

To download any activity listed, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter the number in the keyword search box.

- Solving Exponential Equations (TI-Nspire technology) — 16058
- Solve Exponential Equations (TI-84 Plus family) — 1637
- Exponential Growth (TI-Nspire Technology) — 12003

Population growth can be modeled using various exponential functions. Students are going to model the data by using three different methods to find the best model. The data that the students are going to investigate is the midyear world population from the years 1950–2006. The years have been modified so that year 1 represents year 1950.

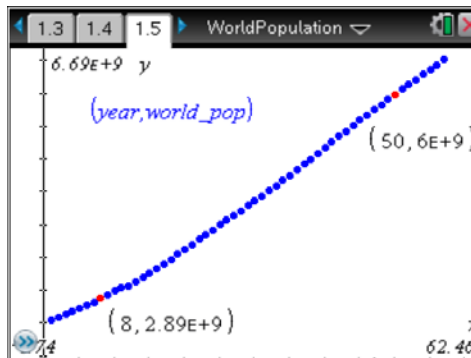
Source: U.S. Census Bureau, Population Division/International Programs Center

**Problem 1 – Find an exponential equation by hand using two points**

**Step 1:** Students begin by studying the data presented on page 1.3. They should see that an exponential or linear function would best model the data.

year	world_pop
6	2781022728
7	2833962170
8	2889768544
9	2946253650
10	2998566597

**Step 2:** On page 1.6, students will see a graph of the data. They need to find an exponential model to fit the data by using any two points. To simplify matters, two points have already been selected and are shaded differently than the others points.

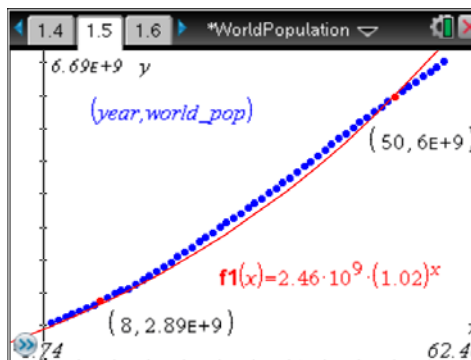


**Step 3:** On page 1.6, students will click on arrows to step through the process of finding the exponential function  $y = ab^x$  using the two selected data points.

*Substitute b into one of the exponential equations and solve.*

$$\begin{aligned} y_1 &= a \cdot b^{x_1} & 2.07612 &= b^{42} \\ y_2 &= a \cdot b^{x_2} & 42\sqrt[42]{2.07612} &= \sqrt[42]{b^{42}} \\ \frac{6 \cdot 10^9}{2.89 \cdot 10^9} &= \frac{a \cdot b^{50}}{a \cdot b^8} & 1.018 &= b \\ 6 \cdot 10^9 &= a \cdot b^{50} & 6 \cdot 10^9 &= a \cdot (1.018^{50}) \\ & & a &= 2.46 \cdot 10^9 \end{aligned}$$

**Step 4:** Students should go back to page 1.5, unhide the function entry line (**ctrl** + **G**) and enter the determined exponential function equation



**Step 5:** On page 1.7 students are asked to use their model to determine what the population will be in the year 2015? In 1890?

WorldPopulation

Go back to page 1.5, and enter the determined exponential function.  
Using your equation, what will the population be in the year 2015? In 1890?

$f(65)$	7.84406E9
$f(-60)$	8.43468E8
$f(\ )$	

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**TI-Nspire Navigator Opportunity: Quick Poll**

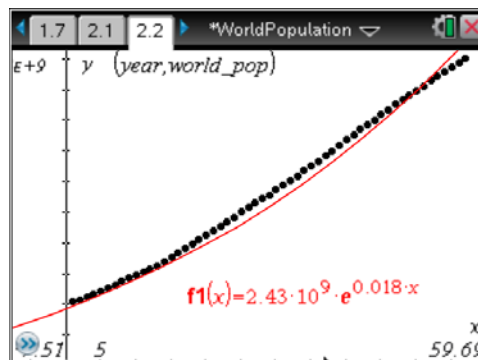
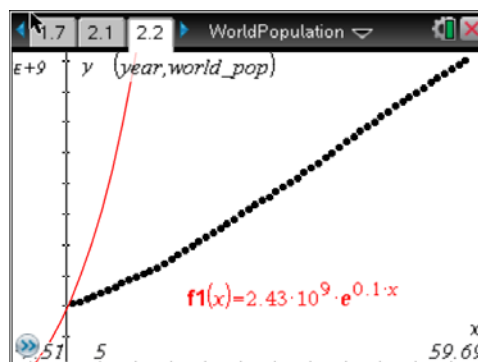
See Note 1 at the end of this lesson.

**Problem 2 – Transformation of an exponential function**

**Step 1:** An exponential function has already been graphed using the value for  $a$  that was determined in Problem 1.

The students should move their cursor to the exponent for  $e$  and click twice so they can change it to a smaller value.

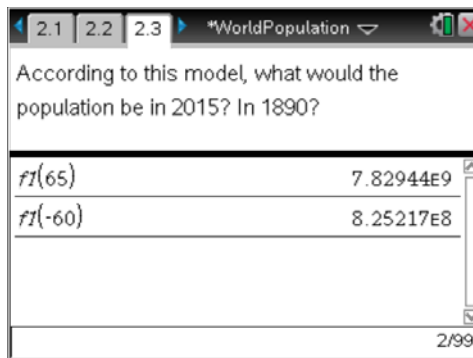
Instruct them to continue doing this until they are satisfied with the result.



**TI-Nspire Navigator Opportunity: Screen Capture**

See Note 2 at the end of this lesson.

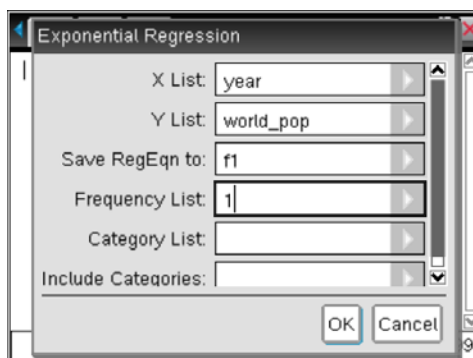
**Step 2:** On page 2.3 students are asked to use their model to determine what the population will be in the year 2015? In 1890?



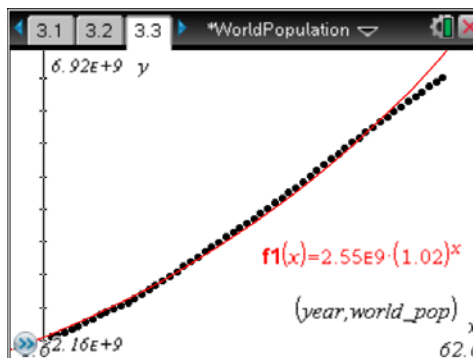
**Problem 3 – Finding the exponential regression.**

**Step 1:** Students are going to find the exponential regression of the data.

On page 3.2, students are to press **MENU > Statistics > Stat Calculations > Exponential Regression** and press **enter**. They need to select **year** for the X List, **world\_pop** for the Y List, and save the RegEqn to **f1**.



**Step 2:** Students will go to page 3.3 and up arrow to **f1(x)** in the function entry line and press **enter**. This will graph their regression equation.



**Step 3:** Student can use page 3.2 to see the exact values for **a** and **b**. To do this, they need to type **stat.** and a drop menu will appear. Select **a** and press **enter**. Then arrow up until the number is highlighted and press enter. Repeat the steps to find the value for the growth rate **b**.

"a"	0.997943
"Resid"	"(...)"
"ResidTrans"	"(...)"
stat.a	2.54657E9
2546567864.6231	2.54657E9
stat.b	1.0175
1.0174969470462	1.0175

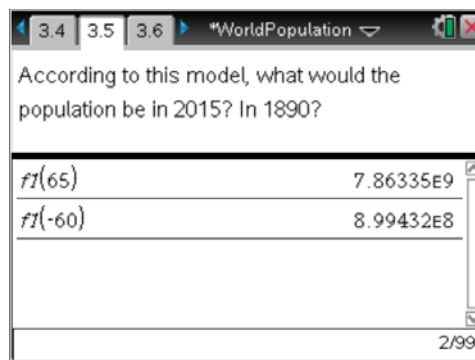
**Step 4:** On pages 3.5–3.6, students interpret the data with this model.

Page 3.5: *According to this model, what will the population be in the year 2015? In 1890?*

Page 3.6: *Which model do you feel is the best and why?*

**Step 5:** The last aspect that the students have to do is to use the graph on page 3.8 and the exponential regression equation to find the year when the world's population will reach 8.5 billion people.

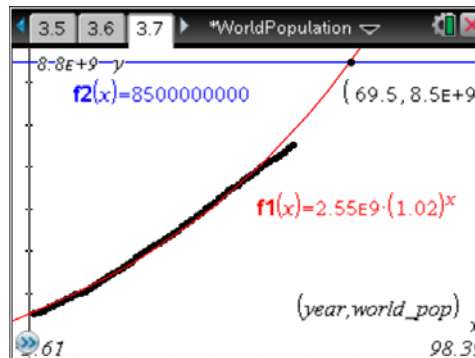
To find the year graphically, students should enter 8.5 billion in  $f2(x)$  and find the intersection point of the two equations using the **Intersection Point(s)** tool. The solution with the exponential regression model is the year 2019.



According to this model, what would the population be in 2015? In 1890?

$f1(65)$	7.86335E9
$f1(-60)$	8.99432E8

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## TI-Nspire Navigator Opportunities

### Note 1

#### Problem 1, Quick Poll

Consider using *Quick Poll* to gather student responses for the questions associated with Problem 1.

### Note 2

#### Problem 2, Screen Capture

This is an excellent opportunity to use *Screen Capture* to display all the various answers each student determines.