



### Math Objectives

- Students will develop a basic understanding of the polar coordinate system and locate points given in polar form.
- Students will convert points between polar and rectangular coordinates.
- Students will sketch graphs of polar equations.
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

### Vocabulary

- polar coordinates
- pole
- polar axis
- absolute value
- argument
- rectangular coordinates

### About the Lesson

- This lesson involves a brief introduction to the polar coordinate system.
- As a result, students will:
  - Determine the location (quadrant) of various points given in polar form.
  - Recognize cases in which a point lies on an axis.
  - Convert points between polar and rectangular form.
  - Discover that the polar coordinates for a point are not unique.
  - Sketch the graphs of polar equations.
  - Use their calculators to check graphs sketched using paper and pencil.



### TI-Nspire™ Navigator™

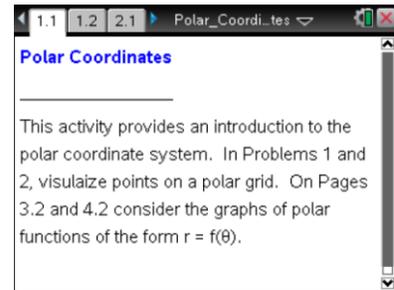
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Quick Poll to assess students' understanding.

### Activity Materials

Compatible TI Technologies :  TI-Nspire™ CX Handhelds,



TI-Nspire™ Apps for iPad®,  TI-Nspire™ Software



### Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>

### Lesson Files:

*Student Activity*  
 Polar\_Coordinates\_Student.pdf  
 Polar\_Coordinates\_Student.doc  
*TI-Nspire document*  
 Polar\_Coordinates.tns

Visit [www.mathnspired.com](http://www.mathnspired.com) for lesson updates and tech tip videos.



### Discussion Points and Possible Answers

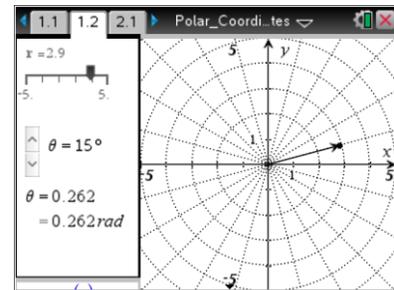


**Tech Tip:** To change slider settings, right-click in the slider box and select **Settings**. You might want to consider different values for the minimum, maximum, or step size.

The polar coordinate system is a two-dimensional coordinate system defined by a point, called the pole, and a ray from the pole, called the polar axis. In a rectangular coordinate system, the pole is usually placed at the origin, and the polar axis is represented by the positive  $x$ -axis. A point in the polar coordinate system is represented by the ordered pair  $(r, \theta)$  where  $r$  is the distance from the pole and  $\theta$  is the angle (in radians) measured counterclockwise from the polar axis.

#### Move to page 1.2.

1. On this page, the left work area contains a slider for  $r$  and a clicker for  $\theta$ . The point  $(r, \theta)$  is plotted in the right panel along with a position vector. Change the values of  $r$  and  $\theta$  as needed to answer the following questions.
  - a. Complete the following tables by finding the quadrant in which the point  $(r, \theta)$  lies.



#### Answer:

$r$	1.7	1.3	-0.6	-4.2	-3.2	3.1	-1.5	-2.5
$\theta$	$\frac{5\pi}{6}$	$-\frac{3\pi}{4}$	$-\frac{7\pi}{6}$	$\frac{3\pi}{4}$	$-\frac{4\pi}{3}$	$-\frac{13\pi}{4}$	$\frac{13\pi}{12}$	$-\frac{7\pi}{4}$
<b>Quadrant</b>	II	III	IV	IV	IV	II	I	III

$r$	0.8	2.1	2	-2.7	4	3.5	-1.4	-3
$\theta$	$\frac{19\pi}{6}$	$\frac{\pi}{4}$	$-\frac{17\pi}{12}$	$\frac{11\pi}{4}$	$\frac{7\pi}{6}$	$-\frac{\pi}{3}$	$\frac{11\pi}{3}$	$\frac{\pi}{3}$
<b>Quadrant</b>	III	I	II	IV	III	IV	II	III



(Answer to question 1a cont.)

$r$	-4	2.7	1	3.9	-5	-2	-1	1.5
$\theta$	$-\frac{\pi}{6}$	$-\frac{11\pi}{3}$	$\frac{23\pi}{12}$	$-\frac{11\pi}{6}$	$-\frac{9\pi}{4}$	$\frac{11\pi}{6}$	$-\frac{7\pi}{6}$	$\frac{9\pi}{4}$
Quadrant	II	I	IV	I	II	II	IV	I



**TI-Nspire Navigator Opportunity: *Class Capture and Quick Poll***

**See Note 1 at the end of this lesson.**

b. Describe the location of the point with the following polar coordinates:

(i)  $r > 0$  and  $\theta = 0$

(ii)  $r < 0$  and  $\theta = \frac{3\pi}{2}$

(iii)  $r < 0$  and  $\theta = \frac{\pi}{2}$

(iv)  $r > 0$  and  $\theta = -3\pi$

#### Sample Answers:

(i) The point lies on the polar axis, or the positive x-axis.

(ii) The point lies on the positive y-axis.

(iii) The point lies on the negative y-axis.

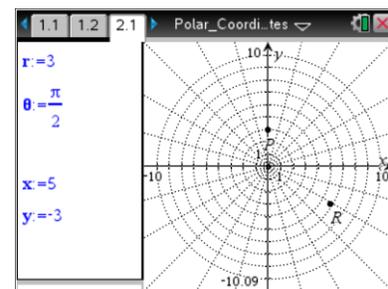
(iv) The point lies on the negative x-axis.

#### **Move to page 2.1.**

If a point has polar coordinates  $(r, \theta)$ , then the rectangular coordinates are given by  $x = r \cos \theta$  and  $y = r \sin \theta$ . Similarly, if a point has rectangular coordinates  $(x, y)$ , then the polar

coordinates are  $(r, \theta)$  such that  $r^2 = x^2 + y^2$  and  $\tan \theta = \frac{y}{x}$ ,

$x \neq 0$ .





2. Complete each of the following tables. Use Page 2.1 to enter polar coordinates and/or rectangular coordinates, to plot the points, and to check your answers. Enter coordinates in the left work area in the appropriate Math Box. The polar coordinates are represented by the point  $P$  and the rectangular coordinates are represented by the point  $R$ .
- a. For each given rectangular point in polar coordinates, find two different polar coordinates that represent the given point.

### Sample Answers:

$(r_1, \theta_1)$	$\left(2, \frac{\pi}{4}\right)$	$\left(3, \frac{7\pi}{4}\right)$	$\left(6, \frac{2\pi}{3}\right)$	$\left(1, \frac{7\pi}{6}\right)$	$\left(-2, \frac{5\pi}{4}\right)$	$\left(\frac{3}{4}, \frac{17\pi}{6}\right)$
$(r_2, \theta_2)$	$\left(2, \frac{9\pi}{4}\right)$	$\left(3, -\frac{\pi}{4}\right)$	$\left(6, \frac{8\pi}{3}\right)$	$\left(1, \frac{19\pi}{6}\right)$	$\left(2, \frac{\pi}{4}\right)$	$\left(\frac{3}{4}, \frac{5\pi}{6}\right)$
$(r_3, \theta_3)$	$\left(-2, \frac{5\pi}{4}\right)$	$\left(3, \frac{15\pi}{4}\right)$	$\left(-6, \frac{5\pi}{3}\right)$	$\left(-1, \frac{\pi}{6}\right)$	$\left(2, -\frac{7\pi}{4}\right)$	$\left(-\frac{3}{4}, -\frac{\pi}{6}\right)$

- b. For each point given in polar coordinates below, determine the rectangular coordinates.

### Answer:

$(r, \theta)$	$\left(3, \frac{7\pi}{3}\right)$	$\left(1, \frac{\pi}{6}\right)$	$\left(-2, -\frac{4\pi}{3}\right)$	$\left(\sqrt{5}, -\frac{3\pi}{2}\right)$	$\left(-8, \frac{3\pi}{4}\right)$	$\left(\frac{13}{4}, -\frac{\pi}{3}\right)$
$x$	$\frac{3}{2}$	$\frac{\sqrt{3}}{2}$	1	0	$4\sqrt{2}$	$\frac{13}{8}$
$y$	$\frac{3\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$\sqrt{5}$	$-4\sqrt{2}$	$-\frac{13\sqrt{3}}{8}$

- c. For each point given in rectangular coordinates below, determine two representations in polar coordinates.

### Sample Answers:

$(x, y)$	(3,4)	$(-\sqrt{2}, 2)$	(4,-7)	$(-\sqrt{3}, -1)$	(-5,5)	(7,24)
$r_1$	5	$\sqrt{6}$	$\sqrt{65}$	2	$5\sqrt{2}$	25
$\theta_1$	0.927	2.186	-1.052	$-\frac{5\pi}{6}$	$\frac{3\pi}{4}$	1.287
$r_2$	-5	$-\sqrt{6}$	$-\sqrt{65}$	-2	$-5\sqrt{2}$	-25
$\theta_2$	4.069	5.328	2.090	$\frac{\pi}{6}$	$\frac{7\pi}{4}$	4.429

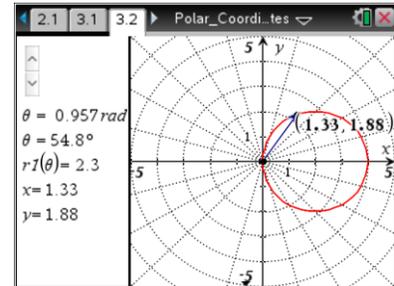


TI-Nspire Navigator Opportunity: *Class Capture*

See Note 2 at the end of this lesson.

Move to page 3.2.

- Over the next two worksheet pages, carefully sketch a complete graph of each given polar equation. Create a table of values, search for patterns, and sketch the graph on the axes provided. Check you results on the handheld and sketch the graph in the right work area of Page 3.2.

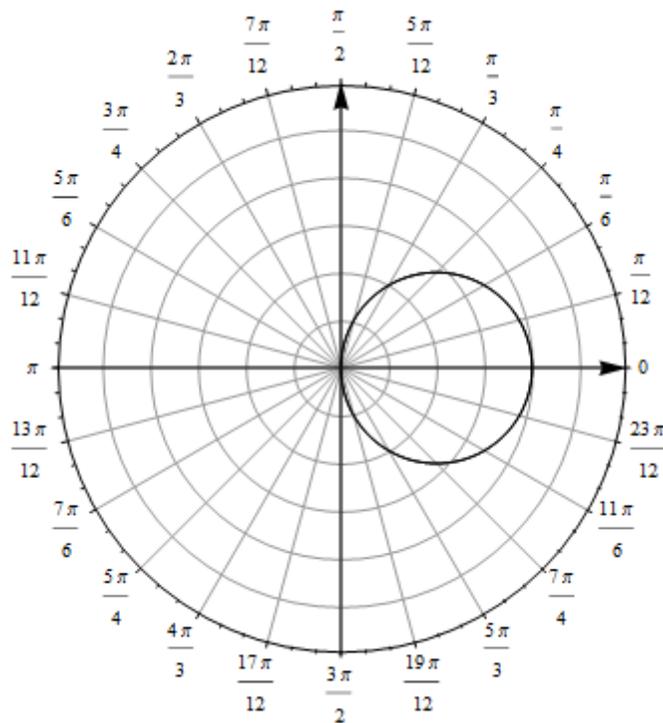


Note: Make sure the Graph Type is set to Polar. Use the clicker in the left panel to step through specific points on the curve in polar and rectangular form.

a.  $r = 4\cos\theta$ .

### Sample Answers:

$\theta$	$r$
0	4
$\frac{\pi}{6}$	$2\sqrt{3}$
$\frac{\pi}{4}$	$2\sqrt{2}$
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	-2
$\frac{3\pi}{4}$	$-2\sqrt{2}$
$\frac{5\pi}{6}$	$-2\sqrt{3}$
$\pi$	-4

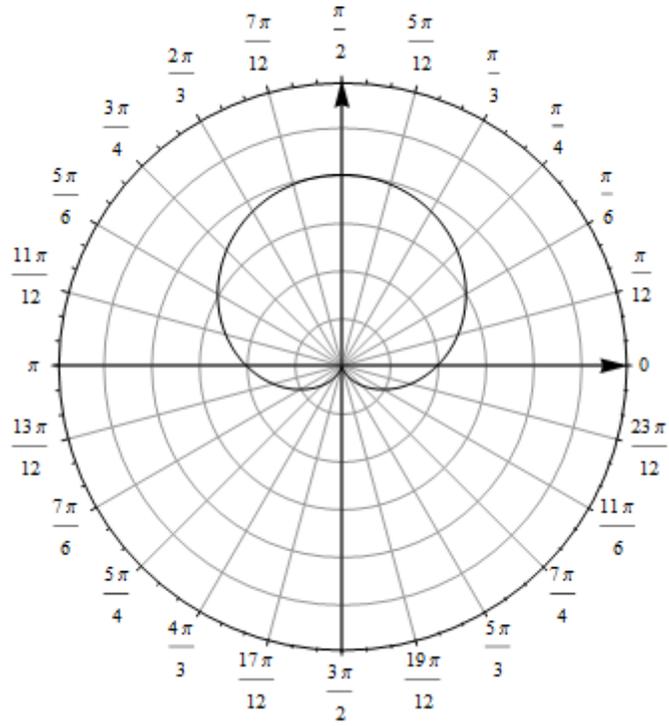




b.  $r = 2 + 2\sin \theta$ .

**Sample Answers:**

$\theta$	$r$
0	2
$\frac{\pi}{6}$	3
$\frac{\pi}{4}$	$2 + \sqrt{2}$
$\frac{\pi}{3}$	$2 + \sqrt{3}$
$\frac{\pi}{2}$	4
$\frac{2\pi}{3}$	$2 + \sqrt{3}$
$\frac{3\pi}{4}$	$2 + \sqrt{2}$
$\frac{5\pi}{6}$	3
$\pi$	2

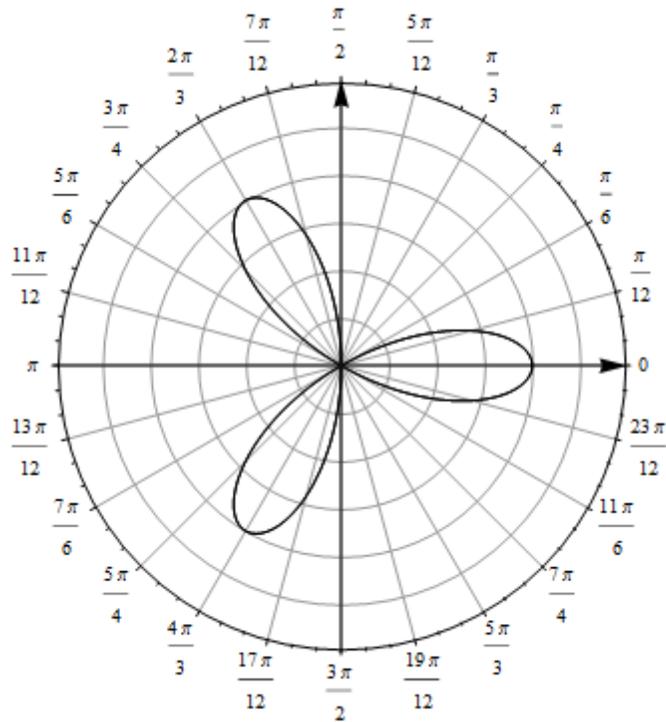




c.  $r = 4 \cos 3\theta$ .

**Sample Answers:**

$\theta$	$r$
0	4
$\frac{\pi}{6}$	0
$\frac{\pi}{4}$	$-2\sqrt{2}$
$\frac{\pi}{3}$	-4
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	4
$\frac{3\pi}{4}$	$2\sqrt{2}$
$\frac{5\pi}{6}$	0
$\pi$	-4

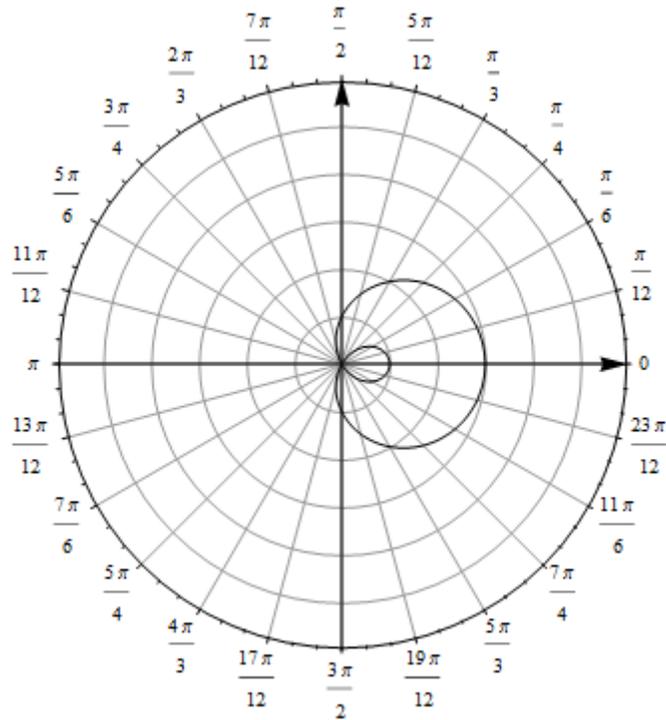




d.  $r = 1 + 2\cos\theta$ .

### Sample Answers:

$\theta$	$r$
0	3
$\frac{\pi}{6}$	$1 + \sqrt{3}$
$\frac{\pi}{4}$	$1 + \sqrt{2}$
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	0
$\frac{3\pi}{4}$	$1 - \sqrt{2}$
$\frac{5\pi}{6}$	$1 - \sqrt{3}$
$\pi$	-1



### Extensions

Here are some possible extensions to this activity:

1. Create a list of polar equations and graphs and ask students to match each equation with its corresponding graph.
2. Ask students to convert specific equations from rectangular form to polar form.
3. Ask students to sketch the graphs of other polar equations.

### Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The polar coordinate system and how to plot points given in polar form.
- How to convert points between rectangular and polar form.
- How to sketch the graph of a polar equation.



## TI-Nspire Navigator

### Note 1

#### Question 1, Class Capture and Quick Poll

Ask students to use the sliders to locate the given point. Use Class Capture to compare student responses. Conduct a Quick Poll in which students must select a quadrant: I, II, III, or IV.

### Note 2

#### Question 2, Class Capture

Ask students to plot points on Page 2.1. Take Class Captures to compare student responses.