ACTIVITY 13

How High Will It Bounce?

Math Objectives:

- Graph scatter plots
- Calculate the maximum value of a parabola
- Analyze and find an exponential regression for the rebound height

Materials:

- TI-83/TI-84 Plus Family
- Data from Activity 12

OVERVIEW

In this activity, you will use the data collected from **Activity 12: Bouncing Ball**. This time the data will be analyzed to find the relationship between the bounce number and the bounce height. When you drop a ball, its rebound height decreases from one bounce to the next. Mathematically speaking, most balls bounce in a regular pattern. You can use percentages to determine how high a ball will rebound on each bounce and make predictions about its motion. You will also be using some calculator features with which your students may not be familiar. You may want to work through the procedure together.

DATA COLLECTION

1. Have students use the data from **Activity 12: Bouncing Ball**.

2. When you exited the Vernier EasyData App™, you were told the time was stored in **L1** and the distance in **L6**. Hopefully, as suggested at the end of Activity 12, you stored these in lists named **BTIME** and **BDIST**. Set up **Plot1** as shown here. **See Figure 1.**

★ **NOTE** *For help in recalling named lists, see Appendix E.*

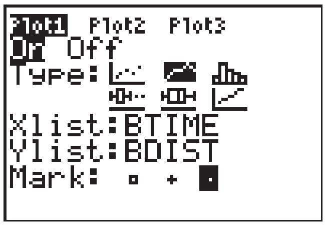


Figure 1

3. Press the ZOOM key and scroll down to **9:ZoomStat** and press ENTER. Your data points are displayed. Press TRACE and scroll right and left to see the **X**- and **Y**-values of the data points. Notice the **P1** in the upper left corner. This tells you it is tracing the points from **Plot1** and displays the lists you are using. **See Figure 2.**

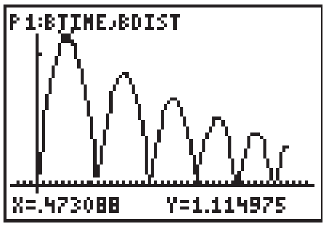


Figure 2

4. Trace to the top of the first parabola. Have the students record the **Y**-value in the chart on their worksheet as the first bounce height. Also have them store the **X**-value in the calculator under **A** and the **Y**-value in the calculator under **B**. The keystroke sequence is X,T,Θ,n STO▸ ALPHA **A** ENTER and then ALPHA **Y** STO▸ ALPHA **B**. **See Figure 3.**

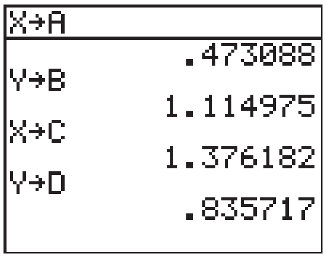


Figure 3

5. Continue scrolling to the top of each successive parabola. For each point, have the students record the **Y**-values in the chart on their worksheet and also store the **X**- and **Y**-values in successive letters **C** through **J**. Make sure students fill in the chart on their worksheet. **See Figures 3–4.**

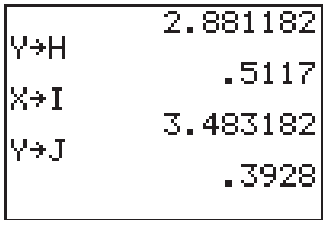


Figure 4

6. Take these **X**- and **Y**-values and put them in **L1** and **L2**. Since you stored them in the variables, you can fill in the lists by typing these variables instead of numbers. Position your cursor in the first element of **L1**. Press [ALPHA] **A. See Figure 5.** When you press [ENTER] you will see the stored value appear in the list. **See Figure 6.**

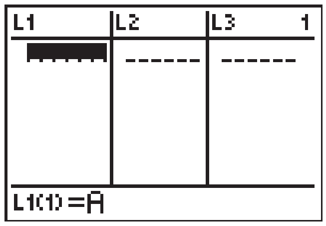


Figure 5

7. Continue filling in **L1** with the **X**-values and **L2** with the **Y**-values of the vertices parabolas. **See Figure 6.**

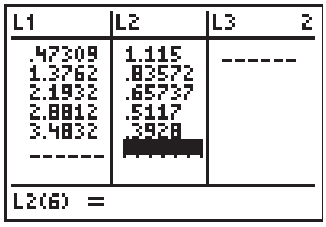


Figure 6

8. Press [2nd] [Y=] and select **2:Plot2**. Turn on **Plot2** with these lists as the **X**- and **Y**-lists as shown here. Purposely select one of the larger marks to draw attention to the vertices. Leave **Plot1** on so both the ball data plot and the plot emphasizing the vertices will be displayed together. **See Figure 7**.

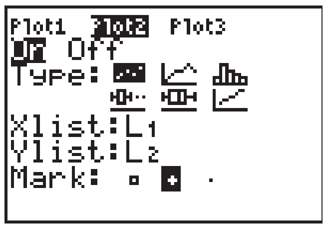


Figure 7

9. Have your students focus on the vertices. See if your students can identify an equation type that would generate the shape created if the vertices were connected. Hopefully, they will recognize it as an exponential equation, but it is likely they will think it is linear. Explore both types of equations. **See Figure 8**.

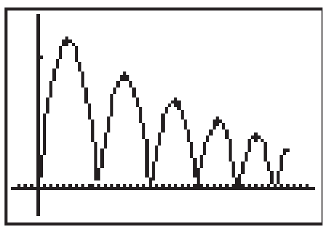


Figure 8

10. This is a great opportunity to allow the technology to **quickly** give you both the linear and the exponential equations and have them both graphed on the same axes. You can then decide which appears to be the better fit and try to identify the theory of mathematics needed to back up what seems to be visually evident.

DATA ANALYSIS

1. An easy way to get the regression equations is to use the built-in feature of the calculator. Put the linear equation in **Y1** and the exponential in **Y2**. Start with the linear equation. To generate it, press STAT, arrow over to **CALC**, and then arrow down to select **4:LinReg**, and press ENTER. On the home screen enter the arguments **L1**, **L2**, **Y1**. Press ENTER to see the equation. **See Figure 9.**

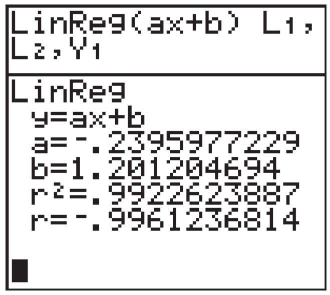


Figure 9

2. Press GRAPH to see how closely the line fits the vertices. The linear regression equation is a fairly close fit. **See Figure 10.**

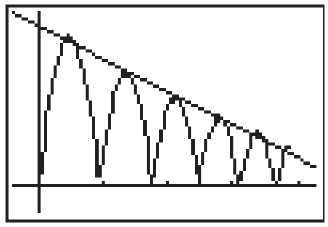


Figure 10

3. Repeat the procedure for the exponential equation and have it put into **Y2**. Press STAT, arrow over to **CALC**, and then arrow down to select **0:ExpReg**, and press ENTER. On the home screen enter the arguments **L1**, **L2**, **Y2.** Press ENTER to see the equation. **See Figure 11.**

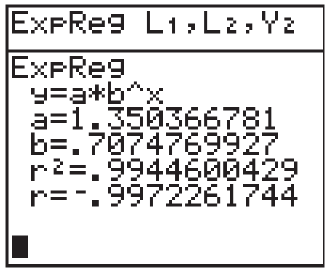


Figure 11

4. Press GRAPH to see the exponential equation graphed along with the vertices and the linear equation. **See Figure 12.** In this example, it is not easy to tell which equation is the better fit. Both correlation coefficients are very close to one. Use the calculator to help with a better visual inspection.

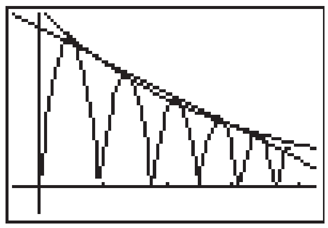


Figure 12

5. Press ZOOM. **1:ZBox** is highlighted. Press ENTER to select it. This will act like a magnifying glass and allow you to zoom in on a specific area of the graph. **See Figure 13.**

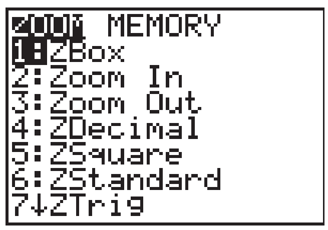


Figure 13

6. You will be taken to the graph screen and the cursor will be in the center of the screen. Imagine a box drawn around the last few vertices. Use the arrow keys to position the cursor in the upper left corner of the box and press [ENTER]. **See Figure 14.**

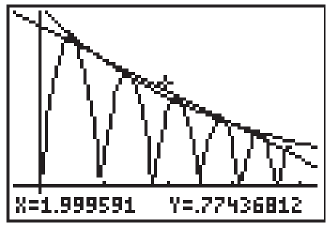


Figure 14

7. After pressing [ENTER], continue to press the arrow keys and form a box. When satisfied with its size, press [ENTER]. **See Figure 15.**

CAUTION A very common mistake is to press [ENTER] at each vertex of the box. This will generate an error message. Remind students not to press [ENTER] a second time until the box is complete.

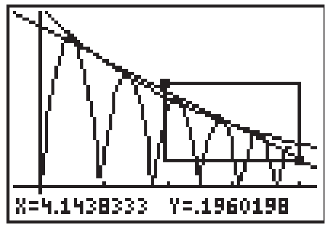


Figure 15

8. The graph will be re-drawn to include only the part outlined by the box that was drawn. You can press [TRACE] and use the up and down arrow keys to move among the plots and the equations. In the upper left corner of the screen, you will see a descriptor identifying the plot or equation where the cursor rests. The plots are designated with **P1** or **P2** and also display the lists used. **See Figure 16.**

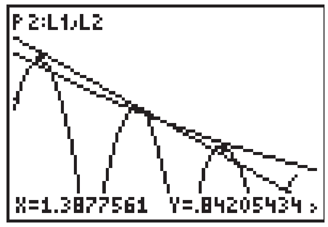


Figure 16

9. It should now be more evident to the students that the exponential equation appears to be a better fit. Guide them in identifying the mathematics that will show why this is true. **See Figure 17.**

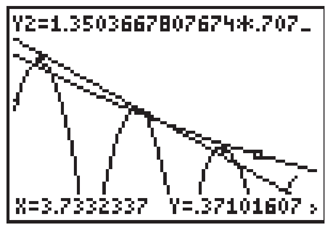


Figure 17

10. Go to the [Y=] window and position the cursor to highlight the equal sign beside **Y1**. Press [ENTER] to turn off the equal sign. This will keep the equation entered in **Y1**, but will prevent it from being graphed. It is a good habit for the students to keep all information associated with a problem until they are finished with the entire problem. **See Figure 18.**

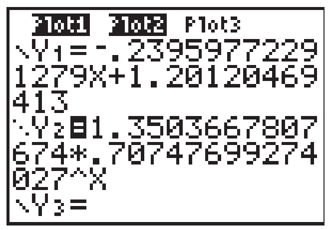


Figure 18

11. Press [GRAPH] to see only the exponential equation graphed along with the plots of the ball data and the vertices. **See Figure 19.**

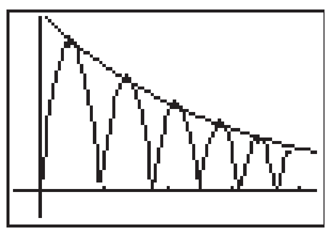


Figure 19

12. Return to the [Y=] screen and encourage students to examine the format of the equation. **See Figure 20.** Tell them to round the numbers in their head to make the equation appear simpler. Point out that the **a** value ($y = ab^x$) is the initial height. Demonstrate to the students that an exponential is no more than repeated multiplication, e.g., $a(.71)^0 \rightarrow a(.71), \rightarrow a(.71)(.71), \rightarrow a(.71)(.71)(.71)$

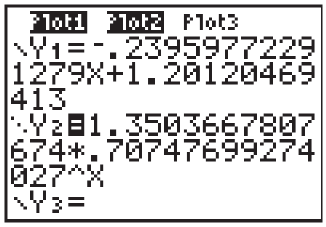


Figure 20

13. Have a discussion with your class about exponentials and set up a chart to show your students how an exponential pattern is established. For example, a particular ball has a bounce ratio of 75% of the bounce height. We saw above that the standard form of the regression equation is $\mathbf{Y = a\, b^x}$.

Initial drop height	**$a\, b^0 = a$**
Vertex of bounce 1 =	**$a\, b^1 = a\,(.75)$**
Vertex of bounce 2 =	**(Vertex of bounce 1) times (.75) = $a\,(.75)\,(.75) = a\,(.75)^2$**
Vertex of bounce 3 =	**(Vertex of bounce 2) times (.75) = $a\,(.75)\,(.75)\,(.75) = a\,(.75)^3$**
Vertex of bounce 4 =	**(Vertex of bounce 3) times (.75) = $a\,(.75)\,(.75)\,(.75)\,(.75) = a\,(.75)^4$**
Vertex of bounce *n* =	**(Vertex of bounce *n-1*) times (.75) = $a\,(.75)^{n-1}\,(.75) = a\,(.75)^n$**

EXPLORATIONS

1. This next exploration will identify the common ratio of the ball bounce for your data. When students traced the graph up to the vertices, they stored the values. Go back to that stored data and use it to fill in the chart with the bounce ratios and rebound percentages. Since the values are stored in variables, simply type in the variables. Start by dividing the **Y**-value of the second vertex by the **Y**-value of the first vertex. Make sure your students understand why you only need the **Y**-values. You can store the answer in the first element of list 3 as shown in the screenshot. The keystroke sequence is [ALPHA] **D** / [ALPHA] **B** [STO▸] [2nd] [3] [(] [1] [)]. The **L3 (1)** means the data will be stored in the first element of **L3. See Figure 21.**

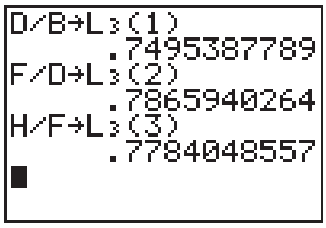


Figure 21

2. Continue dividing every vertex's **Y**-value by the previous vertex's **Y**-value and saving them in successive elements of **L3**. Press [STAT] [ENTER] to see **L3** filled in with these values. **See Figure 22.**

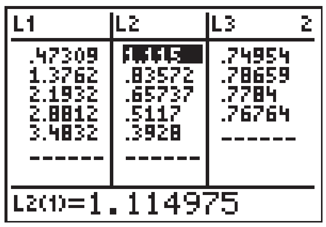


Figure 22

3. The average of this list would be a reasonable candidate for the rebound percent for this particular ball. To find this average, press [2nd] [LIST] and arrow over to **MATH**. Arrow down to select **3:mean(** and press [ENTER].
See Figure 23.

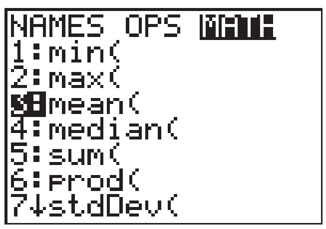


Figure 23

4. You will be taken to the home screen where you need to enter [2nd] [3] [)] and press [ENTER] to have this command executed. The mean of **L3** represents the approximate rebound percentage for this ball. In this example, the percentage is 77%. The height of each successive vertex of a bounce is approximately 77% of the previous bounce's height. Press [STO▶] [ALPHA] **M**. You will see **Ans →M**, which means it is using the last answer from the line above and storing it in the variable **M**. Press [ENTER] to execute the command and save this for easy retrieval. **See Figure 24.**

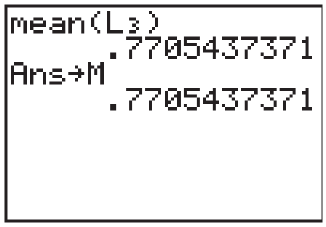


Figure 24

5. In order to find your own exponential equation, you need to know the initial drop height. You did not write it down, but you should be able to get a close approximation for it. If the height of the first bounce (already labeled as **B**), is 77% of the initial drop height (call it **Z**), then **B** = **.77Z**. We can use this to solve for **Z**. **Z** = **B**/.77. Let the calculator do this computation for you. Press [ALPHA] **B** / [2nd] [(-)] to access [ANS] . . . or [ALPHA] **B** / [ALPHA] **M**. Press [ENTER] to perform the calculation. **See Figure 25.**

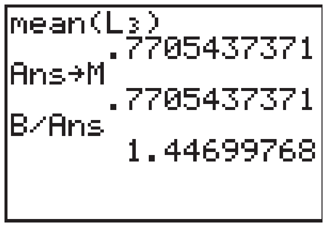


Figure 25

6. To store this answer in **Z** for easy retrieval, press [STO▶] [ALPHA] **Z** [ENTER]. **See Figure 26.**

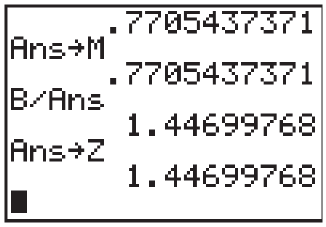


Figure 26

7. Go to the [Y=] window and use these values to substitute into the basic form of the exponential **y = ab**^{x}, using **Z** for **a** and **M** for **b**. Enter it in **Y3**. Leave **Y2** on also. Change the graph style of **Y3** to the ball tracker. **See Figure 27.**

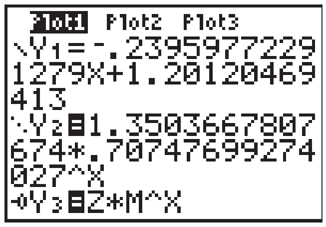


Figure 27

8. Press [GRAPH] to see how close of a fit you have. It is not a close fit. Give the students some time to see if they can figure out why this exponential equation does not fit the data or match the regression equation the calculator found. **See Figure 28.**

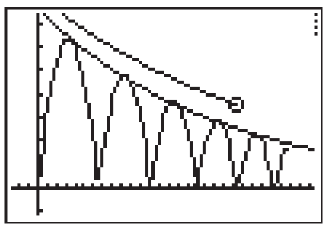


Figure 28

9. The rebound ratio is with respect to the number of the bounce. Our **X**-axis is time instead of the bounce number. Create a new list in **L4** with the numbers 1 through 5, representing the bounce number. **See Figure 29.**

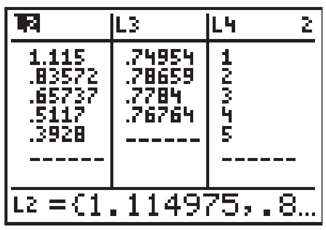


Figure 29

10. Setup a new scatter plot with the **Xlist** as **L4** and the **Ylist** as **L2**. These are the **Y**-values of the vertices that stand for the heights of the bounces. Choose one of the bigger **Marks**. Leave the other plots and the **Y2** and **Y3** all turned on. **See Figure 30.**

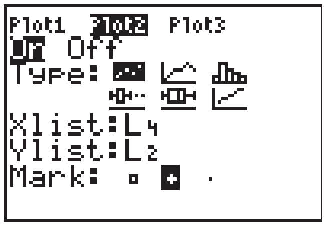


Figure 30

11. Press GRAPH to see your regression equation go right through these new points. **See Figure 31.**

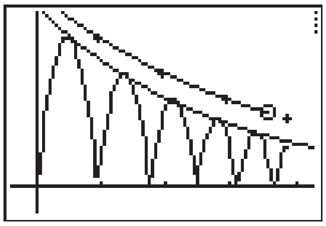


Figure 31

12. The idea of having the heights graphed vs. time or vs. the number of bounces might be a little difficult for some students to understand. To help, try tracing to the first vertex from **Plot1** and notice its **Y**-value. **See Figure 32.**

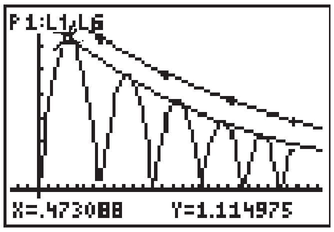


Figure 32

13. Press the down arrow key to put the cursor on **P2**. Trace to the first data point on **Plot2** and notice that it has the same **Y**-value. **See Figure 33.**

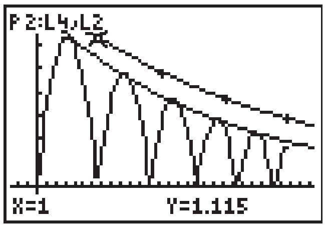


Figure 33

14. It may also be helpful to replace the variables in **Y3** with their values rounded off to the nearest hundredth. This will help the students to recognize both the initial drop height and the rebound percentage. **See Figure 34.**

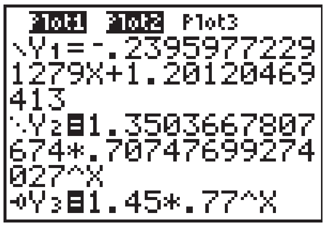


Figure 34

TEACHER NOTES

1. Notice in steps 12 and 13 how the name of the plot displayed in the upper left corner of the screen blocks parts of the graph. You can shift this to the right side of the screen with just the **P1** or **P2** being displayed. To do this, press 2nd ZOOM to access the [FORMAT] menu. Press the down arrow key until the cursor is on the last line and then press the right arrow key to highlight the **ExprOff** option. Press ENTER to select it. **See Figure 35.**

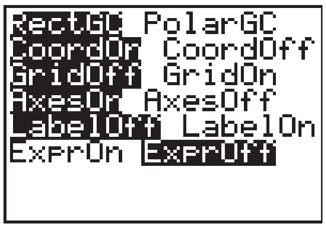


Figure 35

2. Press GRAPH and then TRACE. You will see **P1** in the upper right corner where it will not be in the way of viewing your graph. **See Figure 36.**

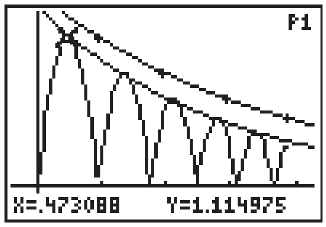


Figure 36

3. Press the down arrow key to trace along **P2. See Figure 37.**

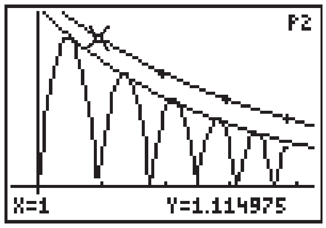


Figure 37

4. Another solution for the problem of the expression display in the upper left corner of the screen is to lengthen the range of the **Y**-values instead of turning off the expression display. Press WINDOW to view the current settings. Remember the EasyData App was measuring the height of the ball in meters. The **Ymax** in the window set by the calculator is 1.3 meters for this example. **See Figure 38.**

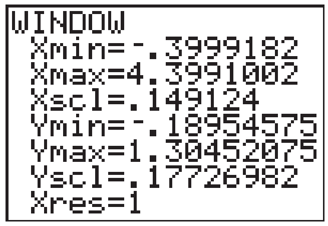


Figure 38

5. Position the cursor beside **Ymax** and type in 1.5. As soon as you start to type the new entry, the old value will be erased. **See Figure 39.**

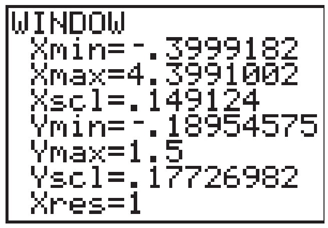


Figure 39

6. Press GRAPH and then TRACE. You will see **P1** in the upper left corner. Even though the expression is displayed, showing you the names of the lists in the Stat Plot, it is not in the way of viewing your graph. **See Figure 40.**

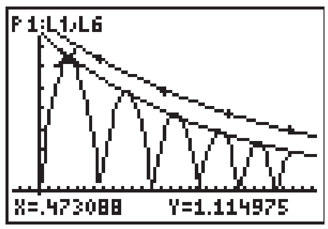


Figure 40

WORKSHEET ANSWERS

There is no separate answer sheet for this activity since the answers will all vary. The description of the activity should provide you with any information you need.

ACTIVITY 13

Name: ______________________________

How High Will It Bounce?

Math Objectives:

- Graph scatter plots
- Calculate the maximum value of a parabola
- Analyze and find an exponential regression for the rebound height

Materials:

- TI-83/TI-84 Plus Family
- Data from Activity 12

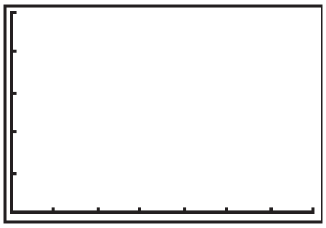
OVERVIEW

In this activity, you will use the data collected from **Activity 12: Bouncing Ball**. This time the data will be analyzed to find the relationship between the bounce number and the bounce height. Your teacher will outline the procedure for you.

1. Create a scatter plot in **Plot1** with the data saved from Activity 12. Use the **BTIME** and **BDIST** for the **Xlist** and **Ylist**, respectively. Use the **9:ZoomStat** feature to set your window. Trace to each vertex and fill in the chart on the worksheet with the coordinates of the vertices. Follow your teacher's instruction as to how to save those values in your calculator in **L1** and **L2.**

Bounce #	Bounce vertex Xlist	Bounce height Ylist
1		
2		
3		
4		
5		
6		

2. Create a scatter plot in **Plot2** of the data (**L1**, **L2**). Sketch both **Plot1** and **Plot2.** Use one of the bigger **Marks** from the setup for **Plot2**.



3. Determine a linear model that closely fits the data in the (**L1**, **L2**) **Plot2**. Write that equation here. ______________________

4. Enter this equation in **Y1** and add it to your sketch to see how well it fits the data.

5. Determine an exponential model that closely fits the data in the (**L1**, **L2**) **Plot2**. Write that equation here.

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6. Enter this equation in **Y2** and add it to your sketch to see how well it fits the data.

7. Which equation appears to be the better fit? ______________________

8. Explain the mathematics that would verify this. ______________________

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9. Fill in this chart with the bounce heights and the rebound ratios.

	Bounce Ratio	Rebound %
$\frac{\text{Bounce 2}}{\text{Bounce 1}}$ =		
$\frac{\text{Bounce 3}}{\text{Bounce 2}}$ =		
$\frac{\text{Bounce 4}}{\text{Bounce 3}}$ =		
$\frac{\text{Bounce 5}}{\text{Bounce 4}}$ =		

10. Store the rebound ratios in **L5**. Find the average of **L5** and store it in **M**. Write that average here. ______________________

11. Each time the ball rebounded, its new height was approximately ________ percent of its previous height.

12. Using this percentage, work backwards and find the initial drop height. Write it here and store it in **Z** in your calculator ____________

13. Using this drop height and the rebound percentage, find your own exponential regression of the form **$y = ab^x$** and enter it here and into **Y3**. You can enter it in the calculator as **Y = Z ∗ M ^ X.** ______________________

14. Using either the trace or the table feature, determine how high the ball would bounce on its next bounce.

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15. Describe how you found the answer to #14. ______________________________________

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