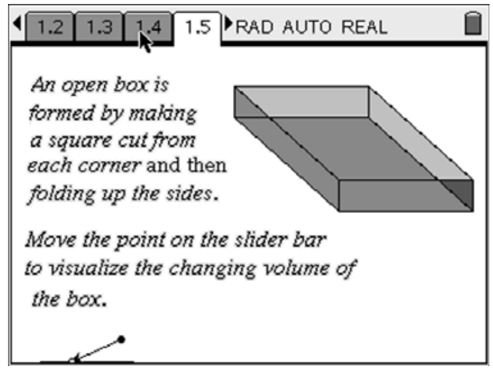
Part I: The Open Box_An Exploration of Maximum Volume *Hands on Exploration*

An open box can be created by making a square cut from each corner of a flat sheet and then folding up the corners of the box. On page 1.2, grab the open circle on the slider and visualize the box created as the corners are folded up.

Question: If the sheet is 24 x 18 cm, what is the size of the cut that will maximize the volume of the box? Discuss with your partner. On pg. 1.4 you are asked to make a prediction about the height of the square that will produce the maximum volume. Write your Prediction here:____________________



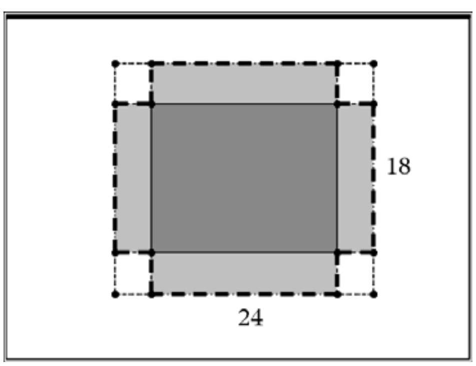
You will be modeling the open box volume by cutting and folding a box. Take the sheet of paper you have been given and cut off the white border so that just the graph paper remains. It should be 18 cm squares wide and 24 cm squares long.

1st: Cut a square from each corner of a piece of cm grid paper and then fold up and tape the sides together to form an open box depending on what group you are in:

Group A: 1x1 Group E: 5x5
Group B: 2x2 Group F: 6x6
Group C: 3x3 Group G: 7x7
Group D: 4x4 Group H: 8x8

Group ____: Names: ____

Cut a square from each corner of the paper that is _____ squares by ______squares.



2nd: Fill the "box" with starbursts and calculate the "starburst volume" of the box. Record the length of the side of your square (height of box), the dimensions of the base, and compute the number of Starburst candies your box holds, the "starburst volume", on this worksheet.

height of box:_________.

length of base: ________

width of base: _________

Starburst Volume: ___________

3rd: How do you find the volume of a rectangular prism? ________________________________pg. 1.8

Calculate the actual volume of your box in terms of cubic centimeters using the volume formula. Record your answer here.

Actual Volume: ____________

4th: Finally, record the height of the box, the "starburst volume" and the actual volume onto the table at the right and on the board. Collect and record all class values onto the table at the right.

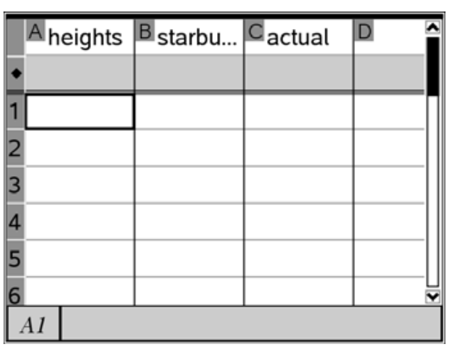
height:________

"starburst volume: _______

actual volume in cm^(3): ______ pg. 1.9

Length of square	*Volume of box in Starbursts*	*Actual Volume of the box*
0 cm		
1 cm		
2 cm		
3 cm		
4 cm		
5 cm		
6 cm		
7 cm		
8 cm		
9 cm		
x cm		

5th: Enter each group's data for the size of the cut, which is the height of the box, the "starburst volume", and the actual volume on a list and spreadsheet document. (pg. 1.11).



After all the data has been collected, explore the relationship between the starburst volume and the actual volume. Graph the data on a data and statistics application.
What type of relationship exists? Is it constant? Linear? Quadratic? Cubic? How can you tell?

____

Find the function that models the relationship between the two volumes using the Ti-Npsire and record this function here. Find the least square regression equation for this set of data.

f(x) = ____, where x is the starburst volume and f(x) is the actual volume.

What is the meaning of the coefficients and constants in you function? ____________________________

Next, graph the data that represents the height and "starburst volume" of the box. (pg.1.15) Use the graph to locate the height of the box that produces the maximum "starburst volume" of the box. What type of function is represented by the data?_________________________
Review how you found the number of Starburst that would fit in your box.

Now let the size of the cut be x-cm. Imagine that you have cut out the squares and folded the box up. Represent the volume in terms of x, where x is the height of your box:

1) The number of starbursts that fit along the length of the box._________________
2) The number of Starbursts that fit along the width of the box._________________
3) The number of Starbursts that along the height of the box. _________________

The Volume of Starbursts, in terms of x is ______________________________________

Find the function that represents the Starburst volume using the regression tools that models the data.

Record this "starburst volume" function:f(x) = ____________________________________, where x is the height of the box and f(x) is the starburst volume.

What is the meaning of the constant in your function? _________________________________

What is the maximum volume of the box in terms of starbursts? ________________

What is the size of the cut that produces the "starburst" maximum volume?______________(pg. 1.16)

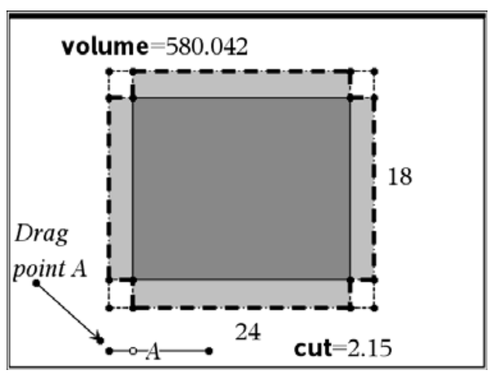
What is the actual maximum volume of the box from your data?____________________

What is the size of the cut that produces the actual maximum volume?__________________

Is it the same cut as the maximum starburst volume? _______________

Why or why not?_______________________________ (pg. 1.17)

Part II: Exploring Maximum Volume with the Ti-Nspire



Go to pg.1.19 where the volume problem is modeled on the Ti-Nspire. Drag the open circle A to create the "cut" and find the volume of the box. Observe the volume as you drag A.

Can you find the maximum volume? Data will be captured on the following page. Graph this data on a Data & Statistics page. Click on the axis to add the variables: height and Volume. Which one is the independent variable and which one is the dependent variable?
____

Discuss with your partner what type of polynomial function the data suggests, linear, quadratic, cubic, etc.

____

Find the algebraic function of the volume problem using the geometric formula and plot this function over your data. Does the data seem to be a good fit?

F(x) = ____, where x is the height of the box and f(x) is the actual volume of the box.

Use the features of the Ti-Nspire to find the maximum volume of your function by plotting the algebraic function on a graph & geometry page.

Find the maximum volume for the function. Go to menu; (menu) 6: Points and Lines; 2: Point on. Place the point on your function, hit escape, (esc) then click and drag the point on the curve to find the maximum point. When the word "maximum" appears click once to set value on the screen.

What is the size of the cut that will produce the maximum volume? ___________________

What is the maximum volume? __________________

What is the integral length that gives the maximum volume? _______________

What is this volume? _________________

Did the size of the cut produced by the starburst maximum volume agree with the actual volume? ______

What is the Domain of your function?_____________________

What is the Range of your function? ______________________

Another way to fit a function to data is to use the least squares regression feature on the handheld. Recall that the equation for a cubic function is $y = ax^3 + bx^2 + cx + d$. Using your TI-Nspire handheld, find the cubic regression for the set of data already listed in your spreadsheet. Go to pg. 1.20. Go to cell C1. Press menu; 4: Statistics; 1: Stat Calculations; 7: Cubic Regression. The next screen that appears will ask you to name the x-list and y-list. Click on x List: select "cut", tab to the y List, click then select "volume". Tab to next cell and save RegEqn to f2. Press enter to exit and the regression equation will now appear in your spreadsheet.

Write the regression equation here: ____
(round decimals off to the thousandths place)

Check with your partner and another group to verify the regression equation. Compare the regression equation with the algebraic equation. What is the same? What is different? Do you think one is a better fit than the other? When would one of the functions be more desirable than another?

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Lastly, how close was your prediction for the maximum volume at the beginning of this worksheet.
____
What generalizations can be made about the maximum volume of a box produced from this activity?

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