Math Objectives

- Students will understand the role of the values of *a* and *n* in the equation $r = a\sin(n\theta)$.
- Students will be able to predict the number of petals and their length by examining the polar equation.
- Student will understand the relationship between the equation of a rose curve and the equation of a sinusoidal function.

Vocabulary

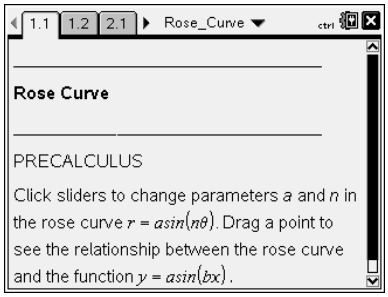
- amplitude
- frequency
- rose curve
- sinusoidal function

About the Lesson

- Students will click on sliders to observe the effect of changing the values of *a* and *n* in the equation $r = a\sin(n\theta)$.
- Students will generalize the roles of *a* and *n* in the equation.
- Students will grab a point and drag it along a sinusoidal function. As the point is dragged, the corresponding polar equation will be formed.
- Students will compare the equations of the function and the rose curve and make generalizations about the relationship between the two equations.
- Students will write equations of rose curves when given information about the petals of the curve.

TI-Nspire Navigator™

- Use Quick Poll to assess students' understanding.
- Use Live Presenter to share students' figures.
- Collect student documents and analyze the results.
- Utilize Class Analysis to display students' answers.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Utilize a slider
- Grab and drag a point
- Check a Self-Check Learn Check question

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- In the *Graphs & Geometry* application, you can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity
Rose_Curve_Student.PDF
Rose_Curve_Student.DOC

TI-Nspire document
Rose_Curve.tns

Visit www.ti-mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

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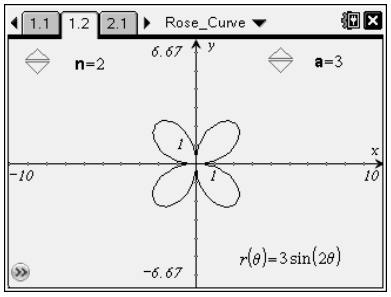


1. A polar curve with an equation in the form $r = a\sin(n\theta)$ is called a rose curve. Why do you think this is so?

   Answer: The graph of the polar equation $r = a\sin(n\theta)$ forms a curve that looks like a flower.

2. Press ▲ and ▼ to change the value of n in the equation $r = a\sin(n\theta)$.
   a. What effect does the value of n have on the graph of the curve?

      Answer: The value of n determines the number of petals on the rose curve.

   b. Explain what happens to the curve when you increase the value of n and when you decrease it.

      Answer: At first, it appears that as we increase the value of n, we increase the number of petals and as we decrease the value of n, we decrease the number of petals. However, when we look carefully, we see that there is a little more involved. When n is an even integer, there are $2n$ petals. When n is an odd integer, there are n petals.

      Teacher Tip: Students may not be able to determine that the number of petals is dependent upon whether n is an odd or even integer. Hopefully, the next two questions will help guide them to this discovery.

3. How many petals does the curve have when $n = 3$? When $n = 4$? Predict the number of petals when $n = 9$ and when $n = 10$.

   Answer: When $n = 3$, there are 3 petals. When $n = 4$, there are 8 petals. When $n = 9$, there would be 9 petals. When $n = 10$, there would be 20 petals.

   Teacher Tip: Most students quickly see the relationship between the value of n and the number of petals in the rose curve. If students are having difficulty, you may want to suggest that they make a table with columns for values of n and the number of petals formed. They can then utilize the TI-Nspire file to obtain the information to fill in their table.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

4. Write a rule to predict the number of petals of a rose curve.

   Answer: When n is an even integer, there are $2n$ petals. When n is an odd integer, there are n petals.

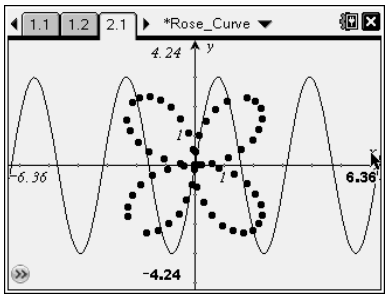
5. Press ▲ and ▼ to change the value of a in the equation $r = a\sin(n\theta)$.
   Explain the effect that the value of a has upon the graph. Be sure to include a description of what happens to the curve when you increase the value of a and when you decrease it.

   Answer: The value of a determines the length of each of the petals. As we increase the value of a, the length of the petals increases. As we decrease the value of a, the length of the petals decreases.

6. Explain how your knowledge of sinusoidal functions can help you understand the effect the value of a has upon the graph of a rose curve.

   Answer: In the sinusoidal function $y = a\sin(bx)$, a is the amplitude of the function. It determines the height of the sine curve. In the rose curve $r = a\sin(n\theta)$, a determines the distance a point on the graph is from the pole (origin).

Move to page 2.1.



7. Grab point P (located at the origin) and slowly drag it along the function $y = 3\sin(2x)$, $0 \le x \le 2\pi$. As you drag point P, you will see the polar curve $r = 3\sin(2\theta)$, $0 \le \theta \le 2\pi$ also being sketched.

   a. Explain the effect of the coefficient 2 on the graph of each of the two curves.

   Answer: The coefficient 2 determines the frequency of the sinusoidal function. We see two complete sine curves in the interval $[0, 2\pi]$. Each curve has an upper and lower portion. Thus, there are four arches in the graph of our sinusoidal function. There are four petals in the rose curve.

   Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☜) getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text*. Then press ctrl [ ] to grab the point and close the hand (✊).

   Tech Tip: Be sure that students are dragging point P **very** slowly. Otherwise, the rose curve will not be drawn properly.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

b. What part of the rectangular graph of the function corresponds to the first quadrant petal in polar coordinates? Generalize this idea for all four petals of the curve.

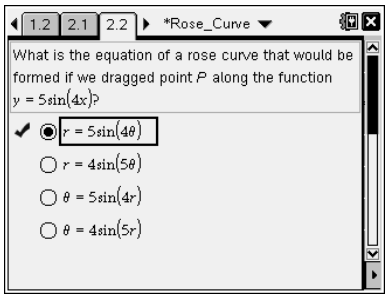
Answer: The part of the rectangular coordinate graph of the function between 0 and $\frac{\pi}{2}$ corresponds to the first quadrant petal. Each arch of the sinusoidal function corresponds to one petal of the rose curve.

c. What effect does the coefficient 3 have upon the graphs of each of the two curves?

Answer: The coefficient of 3 in the sinusoidal function is the amplitude of the function. The amplitude is the distance between the midline of the function and its maximum or minimum point.

The 3 in the equation of the rose curve is the length of each petal. It corresponds to the maximum distance from the center of the rose curve to it furthest point.

Move to page 2.2 to answer Question 8.



8. What is the equation of a rose curve that would be formed if we dragged point *P* along the function $y = 5\sin(4x)$?

Answer: The rose curve $r = 5\sin(4\theta)$ would be formed by point *P* along the function $y = 5\sin(4x)$.

Teacher Tip: This question may be answered on the worksheet or on page 2.2 of the TI-Nspire document. It is a Self-Check question, enabling students to check if their answers are correct. If you have Connect-to-Class software or a TI-Nspire Navigator system, you may want to collect students' responses to analyze them.

TI-Nspire Navigator Opportunity: *Screen Capture and/or Live Presenter*
See Note 3 at the end of this lesson.

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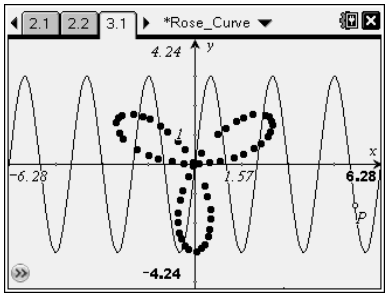


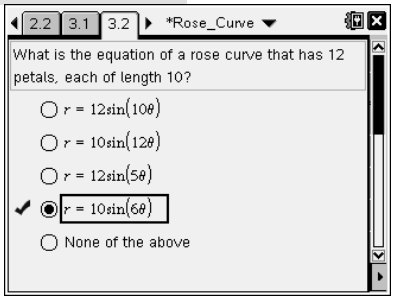
9. Drag point *P* along the function $y = 3\sin(3x)$, $0 \le x \le 2\pi$. As you drag *P*, the polar curve $r = 3\sin(3\theta)$, $0 \le \theta \le 2\pi$ will be sketched. Explain why the polar curve $r = 3\sin(3\theta)$ has only three petals, while the function $y = 3\sin(3x)$ has six arches.

Answer: In the interval $0 \le \theta \le 2\pi$, there are actually six petals sketched in the rose curve. However, only three of the petals are unique. The three petals of the rose curve are sketched for values of θ in the interval $0 \le \theta \le \pi$. As θ continues in the interval $\pi \le \theta \le 2\pi$, the three petals are re-sketched.

Teacher Tip: Have students pay careful attention to what is being sketched as they drag point *P*. If you have a Teacher Edition of the TI-Nspire software, you may want to display this page for students to see what is happening.

Teacher Tip: Ask students to generalize why the number of petals is *2n* when *n* is even and *n* when *n* is odd.

Move to page 3.2.



10. What is the equation of a rose curve that has 12 petals, each of length 10?

Answer: Since the length of each curve is 10, $a = 10$. To sketch out 12 petals, we would need an *n* value of 6. (When *n* is an even number, the number of petals sketched is 2*n*.)

The correct equation is $r = 10\sin(6\theta)$.

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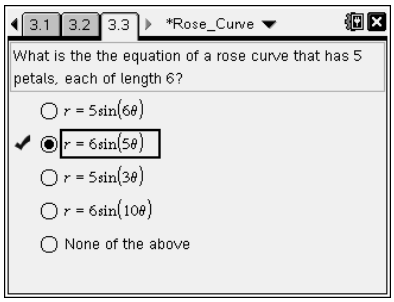

11. What is the equation of a rose curve that has 5 petals, each of length 6?

Answer: Since the length of each curve is 6, $a = 6$. To sketch out 5 petals, we would need an *n* value of 5. (When *n* is an odd number, the number of petals sketched is *n*.) The correct equation is $r = 6\sin(5\theta)$.

Teacher Tip: These questions may be answered on the worksheet or on pages 3.2 and 3.3 of the TI-Nspire document. They are Self-Check questions, enabling students to check if their answers are correct. If you have Connect-to-Class software or a TI-Nspire Navigator system, you may want to collect students' responses to analyze them.

TI-Nspire Navigator Opportunity: ***Screen Capture and/or Live Presenter***
See Note 3 at the end of this lesson.

12. Explain the similarities and differences you would expect if we replaced the sine graphs on pages 2.1 and 3.1 with cosine graphs.

Teacher Tip: This might make a good extension question. You can develop this question into a large part of your lesson, or omit it entirely.

Answer: The cosine functions would be shifts of the sine functions.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The role of the values of a and n in the equation $r = a\sin(n\theta)$.
- How to predict the number of petals and their length by examining the polar equation.
- How the equation of a rose curve is related to the equation of a sinusoidal function.

Extension

Ask students to determine what happens to the rose curve when n is a non-positive integer.

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Note 1

Question 3, *Quick Poll*: You may want to send a Quick Poll to determine whether or not students understand that the number of petals in a rose curve is determined by whether *n* is an odd or an even integer. You could send out four individual Open Response Quick Polls and discuss the responses after each poll.

Note 2

Question 7, *Live Presenter*: Some students may have difficulty dragging point *P* to form the rose curve. Having a student as the Live Presenter will enable other students to see how the curve is formed. It validates the process of forming a curve.

Note 3

Questions 8, 10, 11, *Learning Check and Class Analysis*: These questions were created as Self-Check questions, enabling them to be utilized by students whether or not they have access to TI-Nspire Navigator. You might want to let students know that you will be collecting their answers at some point. If you choose to use students' scores for assessment, you may want to change the questions from Self-Check to Exam questions.