



### Math Objectives

Students will be able to:

- Students will determine the domain of rational functions.
- Students will use algebraic concepts to determine the vertical asymptotes of a rational function.
- Students will determine the removable discontinuities of a rational function.
- Students will describe the graph of a rational function given the equation.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

### Vocabulary

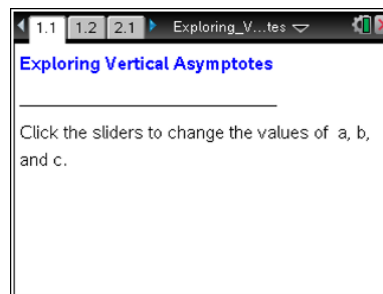
- rational function
- domain
- asymptote
- factors
- zeros
- removable discontinuity

### About the Lesson

- This lesson involves observing how changing the values in a rational function affects the continuity of the graph of the function.
- As a result, students will:
  - Manipulate the factors of the numerator and denominator to observe the effects of each value.
  - Explain how the values in a rational function determine the vertical asymptotes.
  - Identify the conditions that must be met for a rational function to have removable discontinuity.

### TI-Nspire™ Navigator™ System



- Use Document Collection/Quick Poll/Screen Capture to assess student understanding during the lesson.
- Use Screen Capture to examine patterns that emerge.
- Use Quick Poll to assess understanding.
- Use Live Presenter to increase student understanding.



### TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use a slider

### Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- In the *Graphs & Geometry* application, you can hide the function entry line by pressing  .

### Lesson Materials:

#### Student Activity

- Exploring\_Vertical\_Asymptotes\_Student.pdf
- Exploring\_Vertical\_Asymptotes\_Student.doc

#### TI-Nspire document

- Exploring\_Vertical\_Asymptotes.tns

Visit [www.mathnspired.com](http://www.mathnspired.com) for lesson updates and tech tip videos.



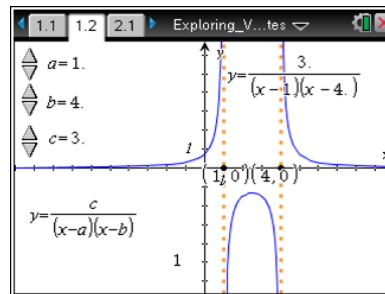
### Discussion Points and Possible Answers

**Tech Tip:** Students can self-check their answers to the included assessment questions by selecting **MENU > Self Check** and then pressing  .

Move to page 1.2.

1. Use ▲ and ▼ to change the value of  $a$ . Describe how the graph changes.

**Sample answers:** One of the vertical asymptotes moves. The other vertical asymptote stays the same. When  $a = b$ , there is only one vertical asymptote. The forms of the curves bounded by the asymptotes dilate but do not reflect.



2. Use ▲ and ▼ to change the value of  $b$ . Describe how the graph changes.

**Sample answers:** The other vertical asymptote moves, while the first vertical asymptote stays the same. When  $a = b$ , there is only one vertical asymptote. The forms of the curves bounded by the asymptotes dilate but do not reflect.

3. What do the values of  $a$  and  $b$  represent in the function?

**Answer:**  $a$  and  $b$  are the zeros of the denominator or where the function is undefined.

4. What are the equations of the vertical asymptotes?

**Answer:**  $x = a$  and  $x = b$ . When  $a = b$ , there is only one asymptote,  $x = a = b$ .

**TI-Nspire Navigator Opportunity: *Live Presenter/Screen Capture***  
See Note 1 below.



5. State the domain of the function in terms of  $a$ ,  $b$ , and  $c$ .

**Answer:**  $(-\infty, a) \cup (a, b) \cup (b, \infty)$  when  $a < b$ ,  
 $(-\infty, b) \cup (b, a) \cup (a, \infty)$  when  $b < a$ ,  
 or  $(-\infty, a) \cup (a, \infty)$  when  $a = b$

6. Use ▲ and ▼ to change the value of  $c$ . How does changing  $c$  affect the domain?

**Answer:** Changing  $c$  only dilates the curves bounded by the asymptotes. It does not move the asymptotes. Therefore, the domain is not affected.

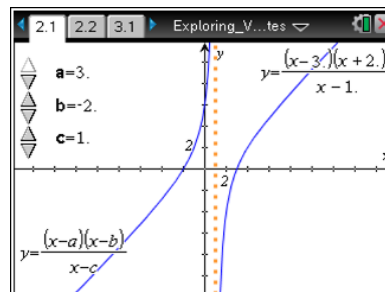
7. Describe how you could find the vertical asymptotes for any rational function with a constant numerator.

**Answer:** Factor the denominator. Solve the denominator to find the zeros of the denominator.

**Teacher Tip:** Removable discontinuity will be addressed in the next section. Students could discuss the need for the “with a constant in the numerator” qualifier.

Move to page 2.1.

8. Use ▲ and ▼ to set  $a = 2$  and  $b = -1$ , and then change the value of  $c$ . For which values of  $c$  are there no asymptotes? Explain why there are no asymptotes for these values of  $c$ .



**Answer:** When  $c = b = -1$  and  $c = a = 2$ , there is no asymptote. The graph looks like a line with a “hole” at  $x = c = a$  or  $x = c = b$ .

When  $a = c$ , the factor  $(x - a)$  in the numerator reduces with the factor  $(x - c)$  in the denominator, leaving the graph  $y = x - b$ . When  $b = c$ , the factor  $(x - b)$  in the numerator reduces with the factor  $(x - c)$  in the denominator, leaving the graph  $y = x - a$ .

**TI-Nspire Navigator Opportunity: Screen Capture**

See Note 2 below.



9. The “hole” in the graph is called a removable discontinuity. Explain why the hole exists and how you might remove it by modifying the function definition.

**Sample answer:** The domain of the function is  $(-\infty, c) \cup (c, \infty)$ . The value of  $c$  will always be undefined regardless of the values of  $a$  and  $b$ . When the value of  $c$  does not equal either  $a$  or  $b$ , there is an asymptote at  $x = c$ . However, when  $c$  equals either  $a$  or  $b$ , there is only a hole. If that hole was removed, the graph would be continuous.

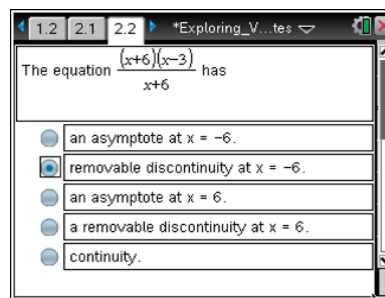
Move to page 2.2.

10. Answer the question on Page 2.2.

Describe the graph of the function

$$y = \frac{(x+6)(x-3)}{x+6}$$

**Answer:** See picture to the right.



**TI-Nspire Navigator Opportunity: Quick Poll**

See Note 3 below.

Move to page 3.1.

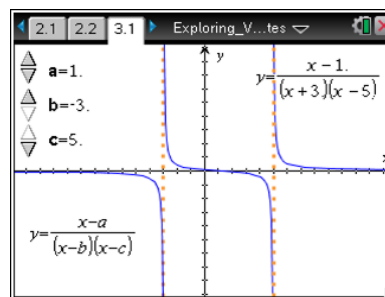
11. Use ▲ and ▼ to set  $b = -1$  and  $c = 4$ . Then use ▲ and ▼ to change the value of  $a$ .
- Describe how the graph changes as the value of  $a$  changes.

**Sample answer:** The graph bounded by the two asymptotes  $x = -1$  and  $x = 4$  dilates, but the asymptotes do not move. For certain values of  $a$ , there is only one asymptote.

- What is the domain of the function in terms of  $a$ ,  $b$ , and  $c$ ?

**Answer:** The domain is  $(-\infty, b) \cup (b, c) \cup (c, \infty)$  when  $b < c$ .

**Teacher Tip:** It is important to note that the domain would be  $(-\infty, c) \cup (c, b) \cup (b, \infty)$  when  $c < b$ , or  $(-\infty, b) \cup (c, \infty)$  when  $b = c$ .





- c. For which values of  $a$  is there only one asymptote? Describe the graph at these values.

**Answer:** When  $a = b = -1$  or  $a = c = 4$ , there is only one asymptote. The graph looks like a translation of an inverse variation. There is also a hole or removable discontinuity.

- d. Explain algebraically why the graph looks as it does at these points.

**Answer:** When  $a = b$ , the factor  $(x - a)$  in the numerator reduces with the factor  $(x - b)$  in the denominator, leaving the graph  $y = \frac{1}{x - c}$ . When  $a = c$ , the factor  $(x - a)$  in the numerator

reduces with the factor  $(x - c)$  in the denominator, leaving the graph  $y = \frac{1}{x - b}$ . The domain of

the function remains the same regardless of the factors that reduce. When  $a$  equals either  $b$  or  $c$ , there is a removable discontinuity instead of an asymptote.

12. Describe how the domain would change if you changed the values of  $b$  and  $c$ .

**Answer:** The domain is  $(-\infty, b) \cup (b, c) \cup (c, \infty)$  when  $b < c$ ,  
 $(-\infty, c) \cup (c, b) \cup (b, \infty)$  when  $c < b$ ,  
 or  $(-\infty, b) \cup (c, \infty)$  when  $b = c$ .

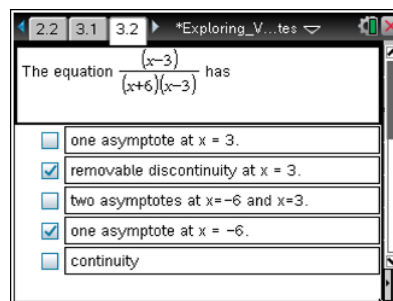
**Teacher Tip:** This is a good time to revisit Question 7. Students could discuss how to find the vertical asymptotes for *any* rational function.

**Move to page 3.2.**

13. Answer the question on Page 3.2.

Describe the graph of the function  $y = \frac{x - 3}{(x + 6)(x - 3)}$ .

**Answer:** See screenshot to the right.



**TI-Nspire Navigator Opportunity: Quick Poll**  
 See Note 4 below.



### Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- How to determine the vertical asymptotes of a rational function.
- What conditions must be true for a rational function to have removable discontinuity.

### Assessment

Embedded questions on Pages 2.2 and 3.2.

### TI-Nspire Navigator

#### Note 1

##### Question 4, Name of Feature: Live Presenter/Screen Capture

Choose a student to become the Live Presenter. Have the student demonstrate and explain how he or she found both values that resulted in no asymptotes. Discuss the domain for each scenario. This will help students answer Question 5.

Turn off Live Presenter. Turn on the Auto-Refresh feature (30-second intervals might be reasonable).

TI-Nspire Navigator can be used to monitor student progress. You might want to leave Screen Capture projected (without student names). This will enable both you and the students to monitor their progress. Alternatively, you might want to turn off the projector so the student screens are visible only to you.

As you circulate around the classroom, frequently check Screen Capture to identify students who need help. Use this information to decide whether to provide individual assistance or bring the class together to address what seems to be a common misunderstanding.

After the class discussion, students should begin Question 5.

#### Note 2

##### Question 8, Name of Feature: Screen Capture

After most students have found what conditions result in the function having no asymptotes, you might ask students to change the values of  $a$ ,  $b$ , and  $c$ . Use Screen Capture to show the results of each student.

Ask students to comment on any patterns they may see. This might help students answer Question 8b. After the class discussion, students should begin Question 8b.



### Note 3

#### **Question 10: Name of Feature: Quick Poll (*Multiple Choice*)**

Instead of having students self-check, change the teacher settings for this question to exam and send Page 2.2 as a Quick Poll. You can save the results and show a Class Analysis to discuss possible misunderstandings students may have.

If students had difficulty, you can review the concepts covered in Questions 1–9 above. It might be useful to pair students up, assigning students of different abilities to the same group.

After the class discussion, students should go to Page 3.1 and begin Question 11.

### Note 4

#### **Question 13: Name of Feature: Quick Poll (*Multiple Choice*)**

Instead of having students self-check, change the teacher settings for this question to exam and send Page 3.2 as a Quick Poll. You can save the results and show a Class Analysis to discuss possible misunderstandings students may have.

### Note 5

You might want to delete students' files from their handhelds. If students share handhelds, you will have to resend the documents for your next class.



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