Rational Roots of Polynomial Functions

Lisa Blank, Lyme Central School, Chaumont, NY

Time Required
40 minutes

Activity Overview

In this activity, students apply the Rational Root Theorem in determining the rational roots of 4 polynomial functions. Results of the application of the theorem are compared to results obtained graphically to identify the presence of irrational roots.

Topic: Rational Root Theorem

- *Rational and Irrational Roots or Zeros*
- *Conjugate Pairs*

Teacher Preparation and Notes

- *Load the Ratrootthm.tns file onto student handhelds.*
- ctrl ▸ *moves students to the next page and* ctrl tab *will enable movement between regions on a split screen page.*
- *Problems 1, 2, and 3 may be done in class and problem 4 could either be done in class or assigned as homework. Questions may be answered on the handheld or associated worksheet.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "12222" in the quick search box.***

Associated Materials

- *Ratrootthm_worksheet_TI-Nspire.doc*
- *Ratrootthm.tns*
- *Ratrootthm _Soln.tns*

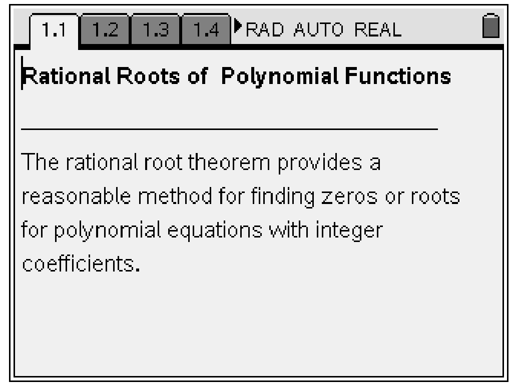
Suggested Related Activity

To download any activity listed, go to education.ti.com/exchange and enter numbers or key words:

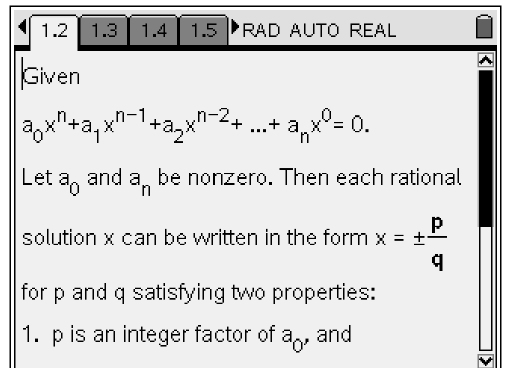
- *Watch Your P's and Q's (TI-Nspire technology)*

Problems 1 & 2 – Introduction & Practice

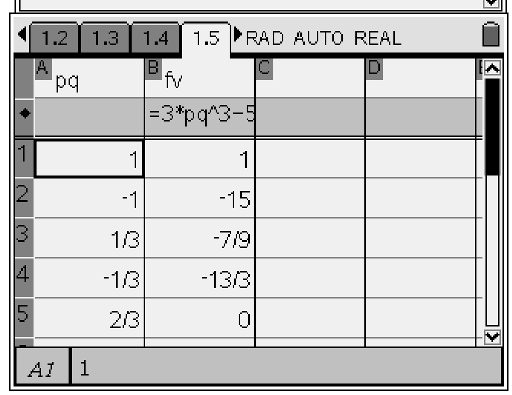
In this activity, students are introduced to the Rational Root Theorem as a means of obtaining the rational roots of a polynomial function.



The Rational Root Theorem is described and the possible rational roots ($\pm\frac{p}{q}$) are determined for a cubic equation.



Students enter the possible values for $\frac{p}{q}$ into column A of a spreadsheet and then evaluate the given function at each $\frac{p}{q}$ value in column B.



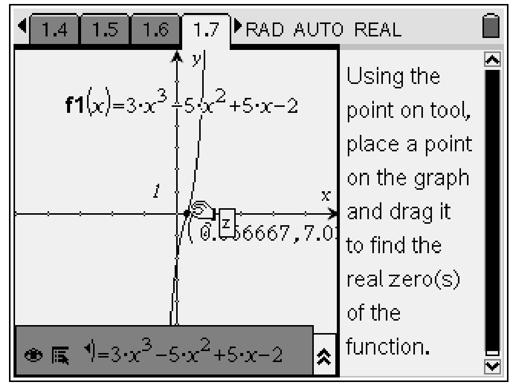
Note that the label for column A must be typed into the polynomial equation in place of the variable x. This is done in the grey shaded box near the top of column B.

Ask students what the value of zero implies for the $\frac{p}{q}$ *value of 2/3.*

Students explore the graph of the function to identify zeros.

Discuss how the graph compares with the spreadsheet.

What types of zeros might a graph show that the spreadsheet will not show?



Problem 3 – Beyond Rational

Problems 3 and 4 involve quartic (fourth degree) polynomials.

The polynomial in problem 3 has 2 rational and 2 irrational roots. Students use the Rational Root Theorem to obtain the rational roots.

Students will first identify the rational roots by applying the theorem. Next, they will explore the function graph.

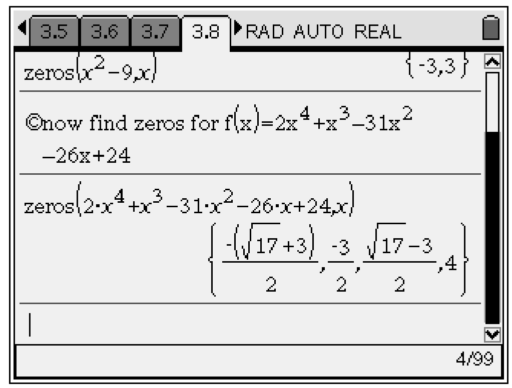
Students should notice that the function graph crosses the x-axis 4 times. Given that they have only two rational roots, the remaining roots must be irrational.

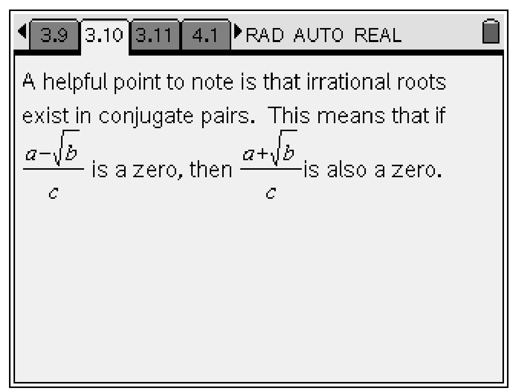
CAS Nspire users may use a calculator page to determine all real zeros of the function. Non CAS users will need to use algebraic methods including polynomial division or synthetic division and factoring, use of the quadratic formula, or completing the square to achieve the same results.

It is noted that irrational roots exist in conjugate pairs.

Ask students how this knowledge of irrational roots existing in conjugate pairs helps them in identifying all real roots of polynomial functions.

Ask students if this concept of conjugate pairs extends to complex roots of polynomial functions.





Problem 4 – Retained Impressions

This problem provides additional practice with using the Rational Root Theorem in identifying rational roots of a quartic function.