Going Back To Your Roots

ID: 11759

Time Required
40 minutes

Activity Overview

In this activity, students apply the Fundamental Theorem of Algebra in determining the complex roots of polynomial functions. The theorem is applied both algebraically and graphically, utilizing features of TI-Nspire CAS technology to enhance student understanding.

Topic: Polynomial Functions

- *Fundamental Theorem of Algebra*
- *Complex roots*
- *Multiplicity*

Teacher Preparation and Notes

- ctrl + ► *moves students to the next page and* ctrl + tab *will enable movement between regions on a split screen page.*
- *Problems 1 through 3 are to be done in class. The extension can be used for further exploration of the topic. Additional practice is provided on the associated worksheet for guided practice or homework. Questions for problems in the student TI-Nspire document (.tns) may be answered on the handheld or associated worksheet.*
- *This activity is intended to be used with CAS technology; however the activity can be completed without CAS by factoring and finding roots by hand.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11759" in the quick search box.***

Associated Materials

- *PrecalcWeek16_BackToRoots_worksheet_TINspire.doc*
- *PrecalcWeek16_BackToRoots.tns*
- *PrecalcWeek16_BackToRoots_Soln.tns*

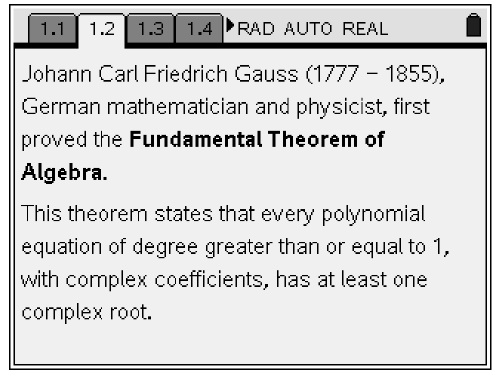
Suggested Related Activity

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Discriminating Against the Zero (TI-Nspire technology) — 11521*

Before beginning the activity, it is important that students understand the definition of a complex number and how to determine the degree of a polynomial.

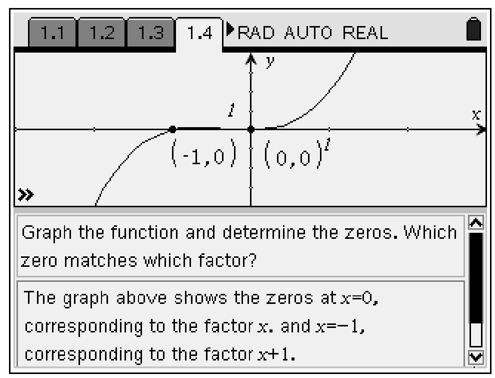
Problem 1 – The Fundamental Theorem of Algebra



In this activity, students are introduced to the Fundamental Theorem of Algebra and apply it in obtaining all complex roots of polynomial functions. After introduction of the theorem, explain to students that all real numbers can be written as complex numbers. For example, the number 3 can be written as 3 + 0*i*. Therefore, every real zero is also a complex zero.

Students will consider the polynomial $f(x) = x^3 + x^2$. To factor this polynomial, they will need to first factor out the greatest common factor of x^3 and x^2, which is x^2. Further simplification to linear factors yields.

$$\begin{aligned} f(x) &= x^2(x+1) \\ &= x \cdot x \cdot (x+1) \end{aligned}$$

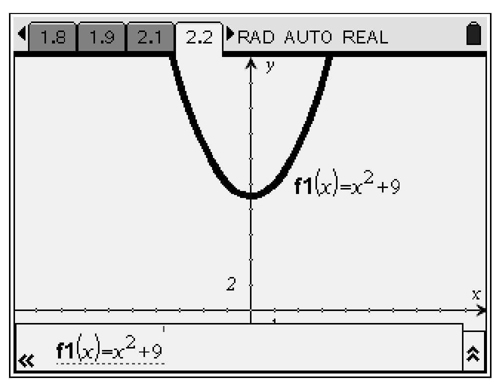


Then students are to graph the functions and use the **Intersection Point(s)** and **Coordinates and Equations** tools to determine the roots. Page 1.5 introduces the multiplicity of each root. Make sure that students understand that multiplicity is the number of times a factor shows up in a polynomial.

Students should notice that the multiplicities of all roots or zeros add up to the degree of the polynomial. The degree is the highest power on the variable in the expanded form of the polynomial.

Pages 1.7 to 1.9 test students understanding with different functions.

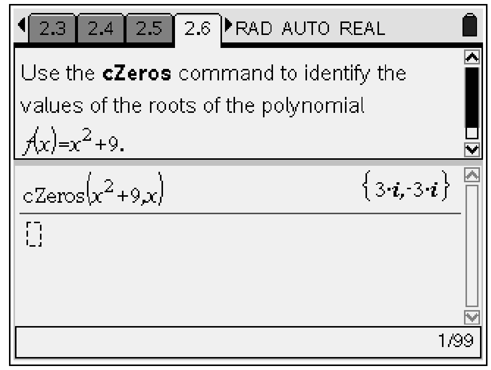
Problem 2 – Beyond Real



In this section, students explore the graph of a polynomial with no real roots. They are to graph the function $f(x) = x^2 + 9$ and then use the graph to determine how many complex roots and what type of roots it has.

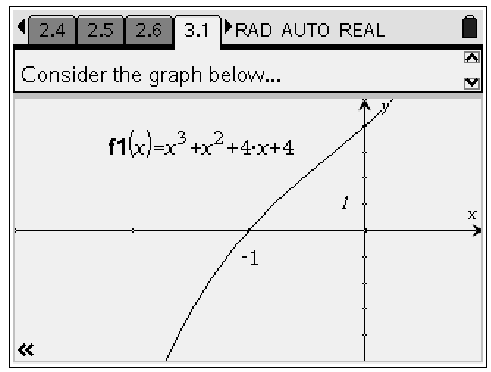
Students need to understand the difference between a complex number and an imaginary number. An imaginary number is a complex number, $a + bi$, where $a = 0$, $b \neq 0$, and $I = \sqrt{-1}$.

With the use of CAS technology, students are to use the **cZeros** command (**MENU > Algebra > Complex > Zeros**) to identify all complex roots for the polynomial. If CAS technology is not used, students will need to use other methods such as completing the square or the quadratic formula.



Problem 3 – The Mixed Case

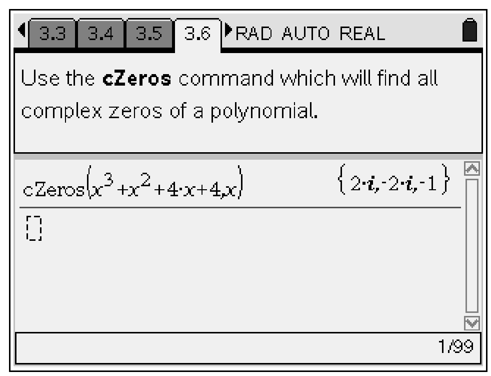
Students are provided with a polynomial that has both real and imaginary roots. The graph illustrates the real root and other methods must be employed to find the remaining roots.



CAS TI-Nspire users may use the **Factor**, **cFactor**, and **cZeros** commands to determine the roots. Non-CAS users will need to apply a combination of such methods as synthetic division, polynomial long division, completing the square, and the quadratic formula.

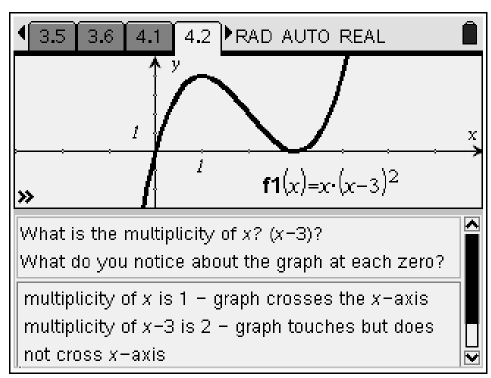
Students not using the CAS Nspire will need additional time for completing the algebraic work involved in factoring and determining zeros.

Confirm with students that the sum of the multiplicities of the roots equals the degree of the polynomial.



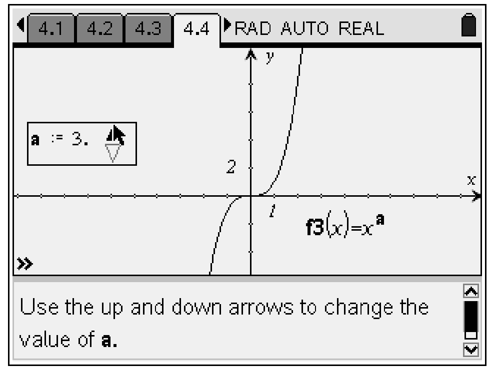
Extension – Even and Odd Multiplicity

In this problem, students explore the effect of even and odd multiplicities of roots by first looking at two specific graphs already in factored form. They need to compare the graph at the *x*-value of the root and the multiplicity of the root.



On page 4.4, students are to use the up and down arrows to change the value of ***a***. For the function $f(x) = x^a$, the zero is at $x = 0$ for all values of a.

As part of further exploration, students can change the function to have other roots, such as $f(x) = (x - 2)^a$. To do this, students need to double click on the function on the screen and edit it or they can press ctrl + G to reveal the function entry line and edit it.



Application & Practice Answers:

Polynomial	Factor(s)	Roots	Multiplicities
$f(x) = x^4 - 9x^3 + 27x^2 - 31x + 12$	$x - 4$	4	1
	$x - 3$	3	1
	$x - 1$	1	2
$f(x) = x^3 - 7x^2 + 11x - 5$	$x - 5$	5	1
	$x - 1$	1	2
$f(x) = x^5 + 9x^4 + 31x^3 + 63x^2 + 108x + 108$	$x - 2i$	$2i$	1
	$x + 2i$	$-2i$	1
	$x + 3$	-3	3