NUMB3RS Activity: Cycloid I
Episode: "The Mole"

Topic: Geometry and Algebra **Grade Level:** 10 - 12
Objective: Draw a cycloid
Time: 20 - 30 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator

Introduction

In "The Mole," Charlie helps the FBI analyze a hit-and-run accident involving a woman and a car. To better understand the situation, Charlie examines the mechanics of walking. Charlie states that "when you walk, it's really a series of little circles rotating inside a larger circle. The heel orbiting backwards, then forward past the knee is a small circle within the larger circle of walking." The objective then becomes to determine the path of the heel not only as it moves in the circle, but as the person walks forward.

Discuss with Students

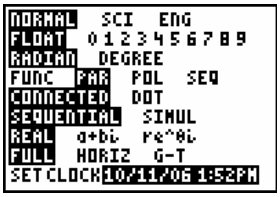
Students may not have seen sine and cosine functions nor parametric equations. The goal of the activity is to have them predict the behavior of a physical action, by constructing a physical model of the action and observing how that model is expressed in algebraic terms.

In the activity, students will examine the movement of a point on the wheel of a bicycle as the bicycle travels in a straight line. The path swept out by the point on the wheel is called a cycloid, which is a combination of translation (the bicycle moving forward) and rotation (the wheel turning). If needed, review with the students the concepts of translation and rotation. Students can often visualize these actions separately, but the cycloid results from a combination of both.

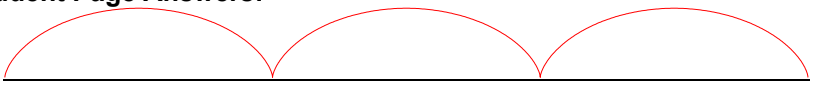
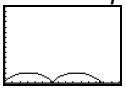
Students begin by predicting the path the point would follow. Take the opportunity to have students compare predictions and discuss what happens to the point as the bicycle moves forward. Many students will initially think that the point must go around in a circle, discuss with them how moving forward effects this "circular" movement.

For the physical construction of the cycloid, make sure students are careful that the coin does not slip. Discuss with them what would happen if the tires on the bicycle slipped.

When graphing the parametric equation, the calculators must be in parametric mode as well as radian mode. To do this, press MODE and select **Par.**



Student Page Answers:

1.  **2.** *Answers will vary.*
3. *when the point is at the bottom of the circle* **4.** *when the point is at the top of the circle*
5.  **6.** *Answers will vary.* **7.** *The height of the curve is 2 units.* **8.** *0, 2π, 4π* **9.** *2π*
10. *It is the circumference of the circle.*

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Name: _____ Date: _____

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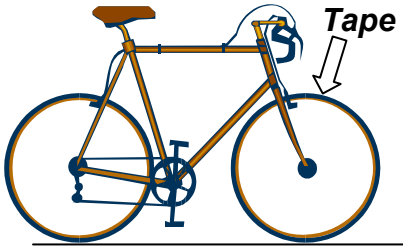
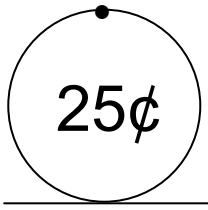
Imagine that Charlie attaches a piece of reflective tape to the tire of Larry's bicycle. As Larry rides his bicycle, Charlie plots the path of the tape.

1. By sketching below, make a prediction of the tape's path as Larry rides his bicycle.



The path that the tape follows is called a cycloid, and is a combination of translation (the bike moving forward) and rotation (the wheel turning). You will now create you own cycloid by using a coin, poker chip, or any flat, circular object. Place a book or a ruler on the line below and mark on the edge of the circular shape the point you want to plot. For a quarter use the ***e*** above Washington's head. Roll the shape along the book or ruler being careful not to let it slip. Mark the location of ten points that you can connect to form the path.



2. How does the path you drew compare to your prediction from Larry's bicycle?

3. Is there an instant where the point is not moving forward?

4. Where along the curve would the point be moving the fastest?

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We can generate a cycloid by using the parametric equation

$$x = a(t - \sin t)$$
$$y = a(1 - \cos t)$$

where *a* is the radius of the circle and *t* is time.

For this example we can let *a* = 1.

Enter the equation into your graphing calculator with the window settings shown below:

```
Tmin = Ø        Xmin = Ø        Ymin = Ø
Tmax = 4π       Xmax = 15       Ymax = 15
Tstep = π/12    Xscl = 1        Yscl = 1
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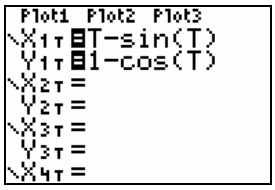
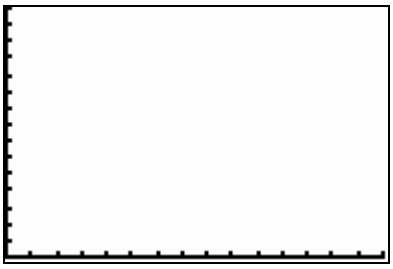


5. Sketch the graph generated from the equation.



6. How does this graph compare to the one you generated by hand?

7. What is the height of the curve?

8. The bottom is a called a cusp. Where does the cusp occur?

9. What is the distance between the Cusps?

10. How does this distance relate to the circle?

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

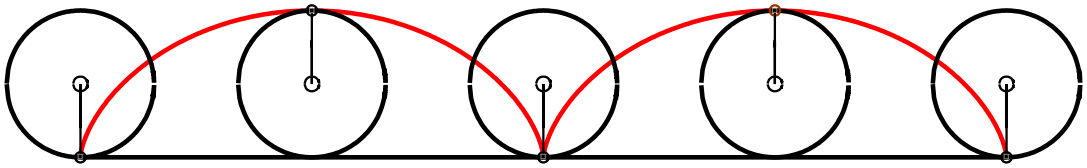
Extensions

Introduction

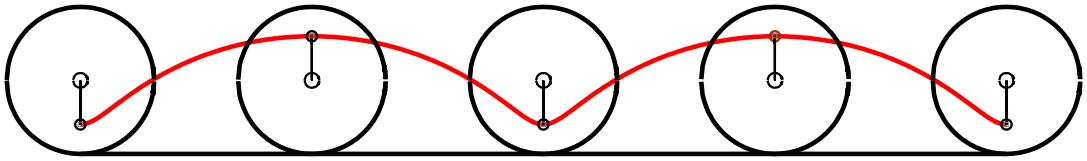
The cycloid is one of a family of curves called the Trochoid Curves, which can be thought of as the trace of a point fixed on a circle that rolls along a line. The general parametric equation of the Trochoid Curves is

$$x = aT - b\sin T$$
$$y = a - b\cos T$$

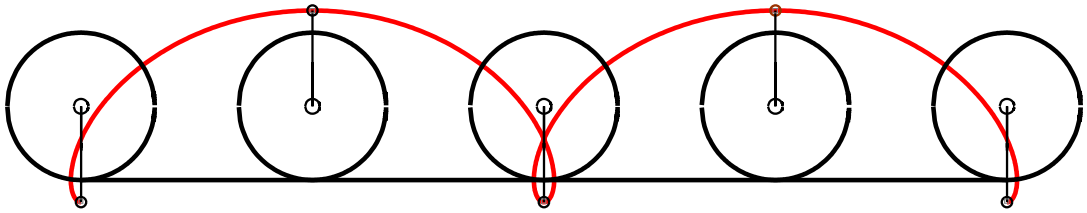
where a is the radius of the circle, b is the radius of the point traced, and T is time.



In the activity you examined a cycloid where $a = b$ so the point was fixed on the circle.



If the point is located inside the circle where $a > b$, then you create a curtate cycloid.



If the point is located outside the circle where $a < b$, then you create a prolate cycloid.

Try different values of a and b on your calculator to explore this family of curves. Sketch your different curves and record the values of a and b you used.

Additional Resources

- The applet on this site has an adjustable radius and graphs cycloids. **http://www.univie.ac.at/future.media/moe/galerie/geom3/geom3.html**
- The cycloid is sometimes called the Helen of Geometers because the number of arguments that have centered on it among mathematicians. This Web site gives an account of many of these disputes: **http://www.mathpages.com/rr/s8-03/8-03.htm**.
- Why can't human beings walk as fast as they run? Explore the math behind both at **http://plus.maths.org/issue13/features/walking**.

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