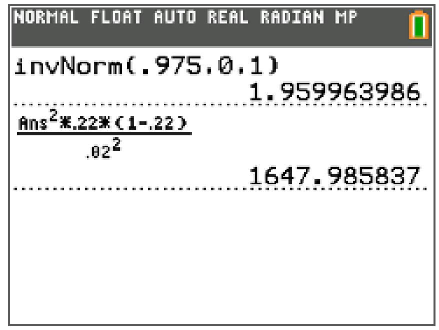
Estimating a Population Proportion



About the Lesson

In this activity, students find the confidence interval for a population proportion by first finding the critical value and the margin of error. They confirm their answers by using the **1-PropZTest** command. They find confidence intervals for real-life scenarios and use those intervals to make a judgment about a claim. Finally, they use two formulas for finding the required sample size for a survey, given a confidence interval and margin of error. As a result, students will:

- Estimate the proportion in a population.
- Calculate the confidence interval for an estimate of *p*.

Vocabulary

- confidence interval
- margins of error

Teacher Preparation and Notes

- Students should already be familiar with the concepts of margins of error and confidence intervals. They should also be familiar with the binomial distribution and its requirements.
- Using a confidence interval to make a decision, as done in Problem 2, is a precursor to hypothesis testing.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

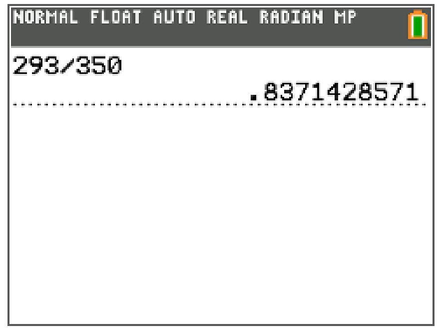
- Estimating_a_Population_Proportion_Student.doc
- Estimating_a_Population_Proportion_Student.pdf

Problem 1 – Margin of Error and a Confidence Interval

Introduce the concept of using $\hat{p}$ to estimate *p* and how it relates to a binomial distribution. Provided certain conditions are met, the normal distribution can be used to approximate the binomial distribution.

1. Find $\hat{p}$.

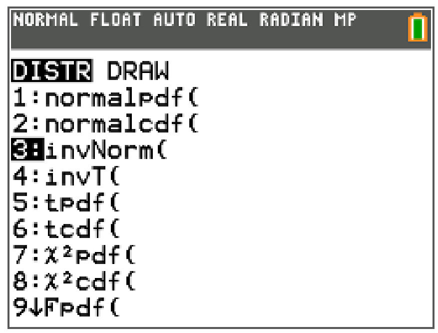
<u>Answer:</u> The sample proportion is about 0.837.



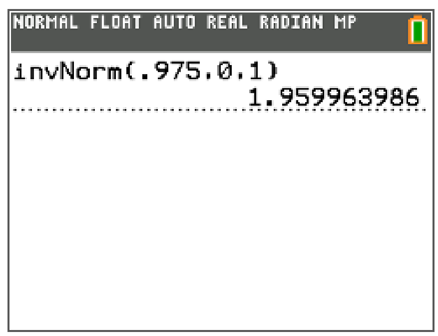
Formulas for the margin of error and confidence interval are given on the worksheet. Note that some books will use $\hat{q}$ in place of $1-\hat{p}$.

The confidence interval for *p* could also be written as $\hat{p} \pm E$ or $(\hat{p}-E, \hat{p}+E)$.

Students are to find the critical value, the margin of error, and the intervals at both the 95% and 99% level.



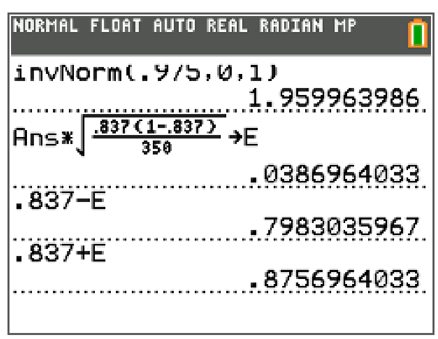
Critical values are found by using **invNorm** ([2nd] [DISTR]) to find the area to the left of that value. For a 95% interval, 5% is in both tails or 2.5% is in one tail, so 97.5% is to the left of the positive critical value.



When finding the margin of error, students can round the sample proportion to 0.837.

The margin of error is about 3.9%. Students must subtract and add this to the sample proportion: 83.7% or 0.837.

Note: For the second part of the formula (second line shown in the screenshot), students are to press [2nd] [(-)] [2nd] [x²] [alpha] [y=] and select **n/d**. Then enter **.837(1–.837)** and press [enter] to move to the bottom of the fraction. Enter **350** and press [▶] [▶] to move out from under the square root. Then press [sto→] [alpha] [E] [enter] to finish.



2. Find a 95% confidence interval. State your findings.

Answer: (0.798, 0.876); Sample answer: The percentage of voters that support the bill is about 87.3%, with a margin of error of plus or minus 3.9%. I am 95% confident that the true percentage of voters that support the bill is between 79.8% and 87.6%.

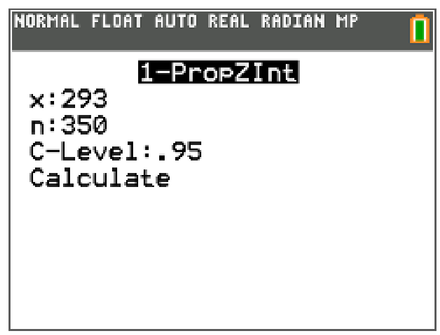
When repeating the process for the 99% interval, students will use an area of 0.995 to the left of the critical value. This margin of error is about 5.1%. Students should explain why it makes sense that this margin of error is larger than the margin of error at the 95% level.

Students should state their findings in complete sentences. For example, *The percentage of voters that support the bill is about 83.7%, with a margin of error of plus or minus 3.9%*. Or, *We are 95% confident that the true percentage of voters that support the bill is between 79.8% and 87.6%.*

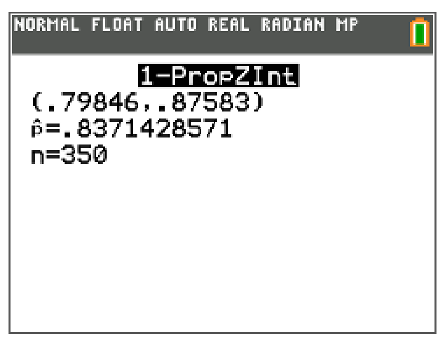
3. Find a 99% confidence interval. State your findings.

Answer: (0.786, 0.888); Sample answer: The percentage of voters that support the bill is about 87.3%, with a margin of error of plus or minus 5.1%. I am 99% confident that the true percentage of voters that support the bill is between 78.6% and 88.8%.

The calculator allows this confidence interval to be found without having to first find the critical value and margin of error. Students can check their work by pressing stat, move to the **TESTS** menu and choose **1-PropZInt**. Then, they enter *x*, *n*, and the confidence level (C Level).



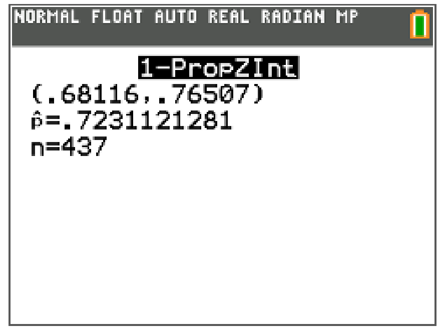
The display shows lower and upper bounds of the interval (answers are slightly different than what was found before because $\hat{p}$ is rounded), $\hat{p}$, and the number of observations.



Problem 2 – Practice Problems

Students are to read the scenario posed on page 2 of the worksheet and then find the 95% confidence interval. You can have students find the CI using the formula or the **1-PropZInt** command.

They should discuss their answer to the question about the reporter's claim. Students should be wary of the reporter's claim because 80% is not within the confidence interval.



4. Find a 95% confidence interval for the true proportion of teens that go to the mall at least once per week.

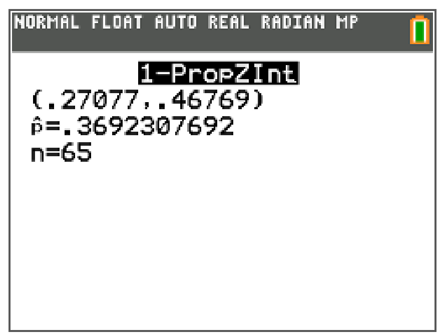
Answer: (68.1%, 76.5%)

5. What do you think of the reporter's claim?

Answer: The claim appears to be incorrect because 80% is not in the confidence interval.

Students are to read the scenario posed on the worksheet and then find the 90% confidence interval.

They can discuss their answer to the question about the principle's claim. Students should say that the principal's claim appears to be correct, or that there is no reason to tell the principal that he or she is incorrect, because the claimed percentage is within the confidence interval.



6. Find a 90% confidence interval for the true proportion of students who support the switch.

Answer: (27.1%, 46.8%)

7. What do you think of the principal's claim?

Answer: There is no reason to dispute the claim because 33% is in the confidence interval.

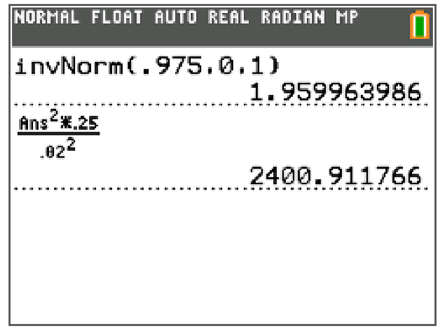
Students should look at and discuss the differences in the margins of error for the two problems and tell why the second is so much larger than the first (consider sample size).

Problem 3 – Sample Size

Introduce the formulas on page 3 of the student worksheet as the formulas used for estimating the sample size that must be taken to estimate a population proportion with a given margin of error.

Students are to use the formula when an estimate of $\hat{p}$ is not known to answer Question 8.

The answer of 2400.91 should be rounded up to 2401. Ask students why the sample size should *always* be rounded *up*. (The larger the sample size, the smaller the margin of error is. A maximum margin of error is given.)

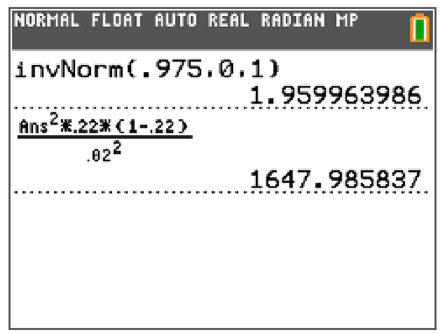


8. How many citizens must be surveyed?

<u>Answer:</u> 2401

Students are to use the formula when an estimate of $\hat{p}$ is known to answer Question 9. (An estimate might be known due to previous studies or expert opinion.)

Discuss why the sample size needed for this situation is less than the sample size when an estimate was not given (an estimate for success is now given, now $\hat{p}(1-\hat{p})$ is less than 0.25).



Tech Tip: Students find the z-value multiple times during the activity. They can use the up arrow key (▲) to highlight a previous entry and then press [enter] to place it on the current entry line and edit it as necessary.

Problem 4 – Extension

10. Complete the table.

Answers:

$\hat{p}$	$1-\hat{p}$	$\hat{p}(1-\hat{p})$
0.1	**0.9**	**0.09**
0.2	**0.8**	**0.16**
0.3	**0.7**	**0.21**
0.4	**0.6**	**0.24**
0.5	**0.5**	**0.25**
0.6	**0.4**	**0.24**
0.7	**0.3**	**0.21**
0.8	**0.2**	**0.16**
0.9	**0.1**	**0.09**

Students should explain how the two formulas for sample size are related and why 0.25 replaces $\hat{p}(1-\hat{p})$.

11. Use the values in the table to explain the derivation of the formula used when no estimate of $\hat{p}$ is given.

Answer: The maximum value of the product is 0.25. It replaces $\hat{p}(1-\hat{p})$ in $n=\frac{\left(z_{\alpha/2}\right)^2\cdot\hat{p}(1-\hat{p})}{E^2}$. It gives the greatest value of *n* that could be needed. It is always safer to survey more people than needed, rather than not enough. (Students may think then that the first formula should always be used. Remind them of the practical limitations of time and money.)

