Cycloid Curves **Name:** _____

Date: _____ **Per:** _____

In this exploration you will examine the cycloid curve. All page references refer to the TI-Nspire document ***Cycloid****.*

1. (Page 1.2) Consider a point on a circle, of radius 1, rolling on a plane. (Press the play button to view the animation.) What path does the point follow? Draw the path in the space below.

2. (Pg 1.3) Select **Geometry Trace**, from the **Trace** menu, click on the point at the bottom of the circle and press play to view the path of the point on the circle. Revise your drawing of the path, if needed. (If you would like to see the path traced another time, select the **Trace** menu, then **Erase Geometry Trace** and follow the previous instructions again.)

   The curve that follows the path is called the cycloid curve. This curve is defined parametrically as follows:

$$\begin{cases} x = r(t - \sin t) \\ y = r(1 - \cos t)' \end{cases}$$

   where r is the radius of the rotating circle and t is the parameter.

3. (Pg 1.4) This page shows the cycloid curve given by...

$$\begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}, \text{ for } 0 \le t \le 4\pi.$$

   Manipulate the open point *t* to answer the following questions (Note that the *x* and *y* coordinates of the point on the circle are shown at the bottom of the page.):

   a. At what value(s) of t does the curve reach its maximum height? What is the maximum height? How does your answer relate to the circle?

b. At what value of t does the circle complete its first rotation?

c. How far horizontally has the point moved after one rotation? How does your answer relate to the circle?

4. (Pg 1.5) On this page the general form of the parametric equation of the cycloid is given. Change the values of r to determine the following:

   a. The maximum height of the curve, in terms of r.

   b. The horizontal change in the curve over one period, in terms of *r*.

5. Algebraically determine the values of *t* for which the cycloid curve intersects the *x*-axis. Does the value of *r* affect your answer?

6. What is the maximum height of the cycloid curve given by parametric equation:

$$\begin{cases} x = 3(t - \sin t) \\ y = 3(1 - \cos t) \end{cases}?$$

7. What is the horizontal change in the curve over one period for the curve in the previous problem?

8. Insert a **Graphs and Geometry** page and graph one period of the curve, to verify your answer. (Remember to graph a parametric equation, you must press **Menu** and change the **Graph Type** to **Parametric**.) Revise your equation, if necessary.

9. Write a parametric equation of a cycloid curve that reaches a maximum height of 4.

10. Insert a **Graphs and Geometry** page and graph one period of the curve, to verify your answer. Revise your equation, if necessary.

11. Write a parametric equation of a cycloid curve that that has a horizontal change of 6π over one period.

12. Insert a **Graphs and Geometry** page and graph one period of the curve, to verify your answer. Revise your equation, if necessary.

13. What if the circle was rolling under the x-axis instead of on top of it? What would the parametric equation for such a curve be if the maximum height of the curve is 2?

14. Insert a **Graphs and Geometry** page and graph one period of the curve, to verify your answer. Revise your equation, if necessary.

15. Go online and investigate the terms *brachistochronous* and *tautochronous* and how they relate to the cycloid curve. Write a paragraph summarizing your findings. Be prepared to share your findings with the class.