NUMB3RS Activity: Meltdown
Episode: "Harvest"

Topic: Direct variation with the square **Grade Level:** 9 - 12
Objective: Students will investigate the relationship between the radius and area of circles that change size over time
Time: about 20 minutes
Materials: calculator

Introduction

In "Harvest," Don and David discover a secret operating room in the basement of an old motel, which is being used to perform illegal kidney transplants. They find blood-soaked sheets and a pile of ice melting on a sheet of plastic in a corner. When Charlie sees the FBI's pictures, he notices that the size of the puddle formed by the melting ice depends on the time the picture was taken. He and Amita discuss how this information can be used to determine when the ice first started to melt. This will tell them when their suspects last used the operating room.

Discuss with Students

This activity uses a small number of modeling assumptions to reduce the problem to a simpler one. It is important to discuss with students how these modeling assumptions are used when working with complex problems. If a model uses too many variables, it will be difficult to work with mathematically; on the other hand, if too many assumptions are made, the model can be too simple to be of any value.

In this activity, we use the following assumptions about melting ice: it is on a level surface, the room is large enough that the air temperature remains constant, and the puddle is circular with constant thickness. With these assumptions in place, the ice turns to water at a constant rate, and the area that the puddle covers varies directly with the square of its radius. Soliciting these assumptions from the students could drive an interesting discussion, but do so before distributing the Student Page, as they are listed there.

Here is a possible "mini-lab" you can use to introduce the topic. For each pair (or group) of students, it requires an eyedropper, water, a level desktop, and a ruler. Place one drop of water on the desktop and measure its diameter. Then find the radius of the puddle and compute the area that the puddle covers. Record the data as the ordered pair (radius, area). Do the same after adding a second drop, third drop, and so on. The method for determining that the area is directly proportional to the square of the radius of the puddle is completely at the teacher's discretion. As an extension, you could also have students record the number of drops used to make the puddle and explore how the area varies with the number of drops.

Student Page Answers: **1.** *The area of the puddle increases at a constant rate.* **2.** 25π *cm*^{2}, 100π *cm*^{2}, 225π *cm*^{2}, 400π *cm*^{2} **3.** *Area increases by* 75π, 200π, 375π*; the next question is the generalization.* **4.** *Because* $(n + r)^2 = n^2 + 2nr + r^2$*, the difference is* $n^2 + 2nr$*, which checks with the answers in #3.* **5.** *First picture: 2,826 cm*^{2}*, second picture: 3,215.36 cm*^{2}*, a difference of 389.36 cm*^{2}*/hr or 6.49 cm*^{2}*/min* **6.** *The puddle in the first picture covers an area of 2,826 cm*^{2}*, and has been increasing at the rate of 6.49 cm*^{2}*/min. So, the ice started melting 2,826 ÷ 6.49 ≈ 435.4 minutes earlier. This is about 7.26 hours, or about 7 hours and 15 minutes. So, the ice started melting at around 1:30 A.M., a perfect time for an illegal kidney transplant.*

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Name: ________________________________ Date: ________________

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In this activity, we will assume that the ice is on a level surface, that it melts into a circular puddle of constant thickness, and that the room’s temperature remains constant.

1. If the ice melts at a constant rate, what does that tell us about the rate at which the area of the puddle increases?

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2. Use the formula $A = \pi r^2$ to complete the following table for the area of a growing puddle. Leave your answers in terms of π.

Puddle Number	1	2	3	4
Radius	5 cm	10 cm	15 cm	20 cm
Area				

3. How does the area of the puddle increase when the radius increases from its original size by 5 cm, 10 cm, and 15 cm? Can you generalize the change in area for an increase of *n* cm?

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4. Algebraically, how much larger is $(n + r)^2$ than r^2? Compare this to your answers to #3.

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Suppose Charlie has two pictures of the melting ice; the first one was taken at 8:45 A.M. and the second one was taken at 9:45 A.M. In the first picture, he determines the radius of the puddle to be 30 cm. In the second picture, it has grown to 32 cm.

5. What is the area in square centimeters (cm^2) that the puddle covered in each picture? What is the corresponding rate of increase in the area (cm^2/min)? (Use 3.14 for π.)

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6. When did the ice start to melt? (Hint: use the rate of increase in the area to find how long it took the puddle to grow to a radius of 30 cm.)

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

The model in this activity used a few assumptions. At a real crime scene, the puddle is certain to be irregularly shaped and the floor would be neither smooth nor level. This raises several questions about the mathematics of true reality, as well as the physical chemistry involved. Charlie and Amita have quite a discussion about this, and many of the basic principles can be studied in high school mathematics and science.

For the Student

- In the episode, Charlie also refers to **surface tension**. On a smooth surface, the surface tension of water causes it to produce thicker puddles, like the way water “beads up” on a freshly waxed car. Suppose that in the previous activity, the thickness of the puddle has to be considered. Explain how changing the thickness of the puddle changes the relationship between the melting rate and the rate at which the area of the puddle changes.

- Part of Charlie and Amita’s puddle discussion is about properties of water on different surfaces. Their discussion involves **adhesion** (molecules’ attraction to surfaces) and **cohesion** (molecules’ attraction to each other). You can experiment with this yourself. Place a drop of water on a piece of aluminum foil, glass, and wax paper. Examine the “thickness” of the drop in each case. Think about how this relates to the concepts of adhesion and cohesion.

Additional Resources

For more about the properties of water, visit:
http://www.uni.edu/~iowawet/H2OProperties.html

For applications of these properties, along with some possible ideas for science or research projects, visit:
http://hyperphysics.phy-astr.gsu.edu/hbase/surten.html

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