To Simplify or Not to Simplify…

Overview – Activity ID: 8942

Students will discuss fractions and what it means to simplify them using prime factorization. Students will then use the calculator to simplify fractions manually as well as investigate both the prime factorization and greatest common factors of the numerator and denominator in various fractions.

Math Concepts

- Number sense
- Finding common factors
- Prime factorization
- Division

Materials

- TI-34 MultiView™

Activity

Discuss and review the concept of prime factorization with students.

Write an example of a prime number on your paper. Now, write a non-prime number (called a composite number).

What makes your first number prime and the second composite?

Anticipate a variety of responses. The best response would be that a prime number has no factors or divisors other than itself and 1, while a composite number has at least one additional factor or divisor.

Show the class an example of a composite number, and discuss how, regardless of the composite number chosen, it can be broken down into the product of prime numbers.

Let's consider the number 100. One hundred is a composite number because it has factors. Let's find the prime factorization. Here's one way:

$$\begin{array}{c} 100 \\ 4 \cdot 25 \\ 2 \cdot 2 \quad 5 \cdot 5 \end{array}$$

The prime factorization of 100 is 2·2·5·5.

Now, let's look at a second number. Find the prime factorization of 35:

$$\begin{array}{c} 35 \\ 5 \cdot 7 \end{array}$$

Simplifying a fraction containing two composite numbers can be done by dividing both the numerator and denominator by a common factor. Given the fraction $\frac{35}{100}$*, what is a factor both numbers have in common?*

Looking at the prime factorization of both numbers, the students should be able to see that 5 is the only common factor or common divisor.

Dividing the numerator and denominator by a factor of 5 will give us $\frac{35}{100} \div \frac{5}{5} = \frac{7}{20}$*, which is in simplified form.*

Now show a fraction that is more challenging to simplify.

Let's consider the fraction $\frac{81}{54}$*.*

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Writing the numerator as a product of primes gives this:

$$\begin{array}{c} 81 \\ 9 \cdot 9 \\ 3 \cdot 3 \quad 3 \cdot 3 \end{array}$$

And doing the same with the denominator gives this:

$$\begin{array}{c} 54 \\ 6 \cdot 9 \\ 2 \cdot 3 \quad 3 \cdot 3 \end{array}$$

Now, if we were to attempt to simplify the fraction $\frac{81}{54}$*, what would you suggest we do first? What factors or divisors do both have in common?*

Students might suggest dividing both the numerator and denominator by 3 first, since they see a 3 in the prime factorization of both. Then, encourage them to see that the fraction can be simplified by a factor of 3 two additional times because there are three factors of the number 3 in total.

The simplest form of this fraction is $\frac{3}{2}$*. Simplifying by the prime factors one at a time can be time consuming. We can also use the TI-34 MultiView to help with this process.*

First, go over the calculator settings with the class. The calculator needs to be set to Manual Simplification in order to allow students to simplify by the smallest common factor or divisor each time. **MANSIMP** is the default, but it is still necessary to check the settings.

Follow these steps:

1. Press θ ∃ ∃ ∃ ∃.
2. Screen should show this:

3. Press ∀ <, then ∃ ∀ <
   again to set the calculator to Manual Simplification and **n/d**.,
4. Press % σ to return to the home screen.

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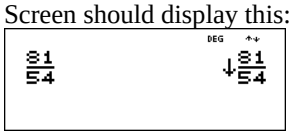
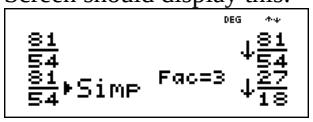
Let's again work with the fraction $\frac{81}{54}$, this time using the calculator's capabilities to help us simplify.

Follow these steps:

1. Press 8 1 Π 5 4 ∀ <.
2. Screen should display this:



3. Press } < to simplify the fraction by the least common factor.
4. Screen should display this:



This tells us the fraction can again be simplified.

5. Repeat } < until there is no longer an arrow next to the fraction.

Now discuss how to find the greatest common factor or greatest common divisor of both 81 and 54.

What factors did 81 and 54 have in common? In this case, we already saw they had 3, 3, and 3 in common because we did the prime factorization by hand. What is 3 ·3 ·3? (Answer: 27) Can the numbers 81 and 54 both be divided by 27? (Answer: Yes)

The product of the common prime factors is the greatest common factor (GCF) or the greatest common divisor (GCD). Show the students how they can find this number with the calculator's help.

Even without doing the prime factorization by hand, we can easily find the GCD of 81 and 54. Each time we press the SIMP key on the calculator, we can see the prime factor. Here, we can see that the calculator simplified by 3, then 3 again, then 3 one last time. This equates to a GCD of 27. What does that mean?

Review the concept that both the numerator and denominator can be simplified by a factor of 27 right away to get a simplified fraction of $\frac{3}{2}$.

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Directions: Complete the table. Use your TI-34 MultiView™ to help find the prime factorization of each number, then circle all common factors. Problems with  should be calculated using mental math rather than the calculator.

	Fraction	Prime factorization of each number	GCD	Fraction $\div \frac{\text{GCD}}{\text{GCD}}$	Simplified fraction
1. 	$\frac{18}{24}$				
2.	$\frac{70}{112}$	$70 = \textcircled{2} \cdot 5 \cdot \textcircled{7}$ $112 = \textcircled{2} \cdot 2 \cdot 2 \cdot 2 \cdot \textcircled{7}$	14	$\frac{70}{112} \div \frac{14}{14}$	$\frac{5}{8}$
3.	$\frac{128}{640}$				
4.	$\frac{343}{735}$				
5.	$\frac{121}{165}$				
6.	$\frac{242}{528}$				
7.	$\frac{169}{400}$				
8.	$\frac{480}{512}$				
9.	$\frac{405}{729}$				
10.	$\frac{236}{944}$				
11.	$\frac{180}{432}$				
12. 	$\frac{210}{39}$				
13.	$\frac{340}{255}$				

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14. Explain, in your own words, what the calculator is doing to simplify these fractions.

15. Explain how you found the greatest common divisor in each of the problems.

16. Find the greatest common divisor of 432 and 828 by hand. Show all your work.

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Answer Key

Directions: Complete the table. Use your TI-34 MultiView™ to help find the prime factorization of each number, then circle all common factors. Problems with 💡 should be calculated using mental math rather than the calculator.

	Fraction	Prime factorization of each number	GCD	Fraction $\div \frac{\text{GCD}}{\text{GCD}}$	Simplified fraction
1. 💡	$\frac{18}{24}$	$18 = 2\cdot3\cdot3$ $24 = 2\cdot2\cdot2\cdot3$	6	$\frac{18}{24} \div \frac{6}{6}$	$\frac{3}{4}$
2.	$\frac{70}{112}$	$70 = \textcircled{2}\cdot 5\ \textcircled{7}$ $112 = \textcircled{2}\cdot 2\cdot 2\cdot 2\ \textcircled{7}$	14	$\frac{70}{112} \div \frac{14}{14}$	$\frac{5}{8}$
3.	$\frac{128}{640}$	$128 = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$ $640 = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot5$	128	$\frac{128}{640} \div \frac{128}{128}$	$\frac{1}{5}$
4.	$\frac{343}{735}$	$343 = 7\cdot7\cdot7$ $735 = 3\cdot5\cdot7\cdot7$	49	$\frac{343}{735} \div \frac{49}{49}$	$\frac{7}{15}$
5.	$\frac{121}{165}$	$121 = 11\cdot11$ $165 = 3\cdot5\cdot11$	11	$\frac{121}{165} \div \frac{11}{11}$	$\frac{11}{15}$
6.	$\frac{242}{528}$	$242 = 2\cdot11\cdot11$ $528 = 2\cdot2\cdot2\cdot2\cdot3\cdot11$	22	$\frac{242}{528} \div \frac{22}{22}$	$\frac{11}{24}$
7.	$\frac{169}{400}$	$169 = 13\cdot13$ $400 = 2\cdot2\cdot2\cdot2\cdot5\cdot5$	None	N/A	$\frac{169}{400}$
8.	$\frac{480}{512}$	$480 = 2\cdot2\cdot2\cdot2\cdot2\cdot3\cdot5$ $512 = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$	32	$\frac{480}{512} \div \frac{32}{32}$	$\frac{15}{16}$
9.	$\frac{405}{729}$	$405 = 3\cdot3\cdot3\cdot3\cdot5$ $729 = 3\cdot3\cdot3\cdot3\cdot3\cdot3$	81	$\frac{405}{729} \div \frac{81}{81}$	$\frac{5}{9}$
10.	$\frac{236}{944}$	$236 = 2\cdot2\cdot59$ $944 = 2\cdot2\cdot2\cdot2\cdot59$	236	$\frac{236}{944} \div \frac{236}{236}$	$\frac{1}{4}$
11.	$\frac{180}{432}$	$180 = 2\cdot2\cdot3\cdot3\cdot5$ $432 = 2\cdot2\cdot2\cdot2\cdot3\cdot3\cdot3$	36	$\frac{180}{432} \div \frac{36}{36}$	$\frac{5}{12}$
12. 💡	$\frac{210}{39}$	$210 = 2\cdot3\cdot5\cdot7$ $39 = 3\cdot13$	3	$\frac{210}{39} \div \frac{3}{3}$	$\frac{70}{13}$
13.	$\frac{340}{255}$	$340 = 2\cdot2\cdot5\cdot17$ $255 = 3\cdot5\cdot17$	85	$\frac{340}{255} \div \frac{85}{85}$	$\frac{4}{3}$

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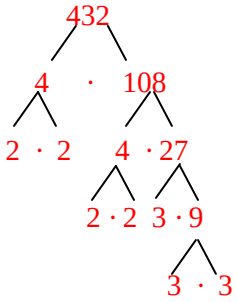
14. Explain, in your own words, what the calculator is doing to simplify these fractions.

The calculator is simplifying the fraction by the smallest common divisor each time.

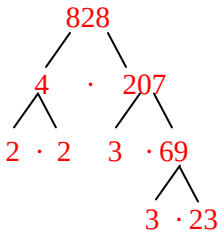
15. Explain how you found the greatest common divisor in each of the problems.

The GCD is found by multiplying the common prime numbers together.

16. Find the greatest common divisor of 432 and 828 by hand. Show all your work.



$432 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$



$828 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 23$

$GCD = 2 \cdot 2 \cdot 3 \cdot 3 = 36$