

## Xtreme Calculus: Part 2

ID: 11488

Time Required  
40–45 minutes

## Activity Overview

*In this activity, students will explore distance-time and velocity-time graphs to deepen their understanding of the application of the derivative. Students will investigate kinematic relationships—graphically, numerically, and algebraically—using animations, and with self-check multiple choice questions. A calculus lab using a CBR2 motion detector is also included to dynamically demonstrate the relationship between displacement, velocity, and acceleration.*

## Topic: Derivative Applied to Motion

- Interpret the derivative as a rate of change in various applied contexts
- Average velocity, instantaneous velocity, speed, acceleration

## Teacher Preparation and Notes

- Students can write their responses directly into the TI-Nspire handheld. On self-check questions, students can then press  and select **Check Answer** (or  + ).
- For the Extension Calculus Motion Lab, students will need a racquet ball or some other smooth ball, an incline plane, and a CBR 2 or Go! Motion sensor. If these materials are not available, then Problem 8 provides data for this problem.
- The activity is designed to be a student-centered discovery and instruction of differential calculus as it relates to motion. Teaching the basics before beginning the activity is optional. After completing the activity, students should be more successful with AP questions like 2002AB3, 2003AB2, 2004AB3, 2004formB AB3&BC3, 2005B AB3, 2005AB5&BC5, mc1998BC90, mc1998AB24, and mc 2003BC84.
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- **To download the student TI-Nspire document (.tns file) and student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “11488” in the quick search box.**

## Associated Materials

- XtremeCalculusPart2\_Student.doc
- XtremeCalculusPart2.tns
- Calculator-Based Ranger 2™ (CBR 2™) or Go!® Motion (optional)

## Suggested Related Activities

- Time Derivatives (TI-Nspire CAS technology) — 9537
- Graphical Derivatives (TI-Nspire CAS technology) — 8499
- Parachuting (TI-89 Titanium) — 3021
- Move My Way (TI-84 Plus) — 4324

**Part 1 – Extreme Cyclist**

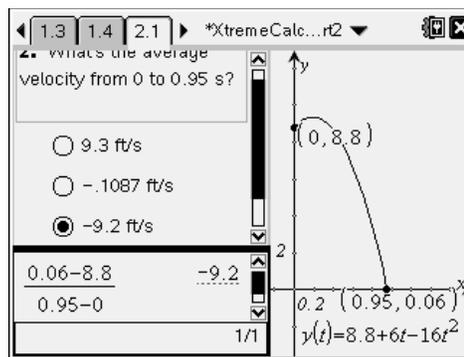
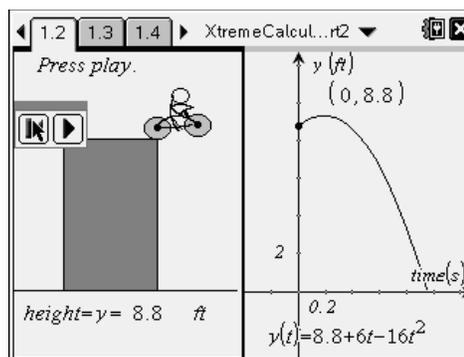
On page 1.2, students are to press play and watch the animation. Students should think about the slope of the tangent line to the position graph.

The tangent line on page 1.4 can be moved by grabbing the point of tangency. A chord can be drawn to depict the average velocity.

Discussion question

- How does average velocity differ from instantaneous velocity?

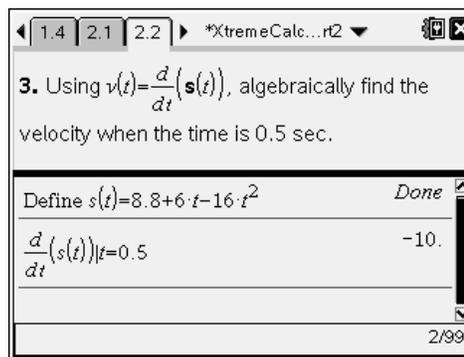
On page 2.1, the average velocity is the slope between  $t = 0s$  and  $0.95s$ . Students should show their work in the calculator area provided in the bottom-left of the page. Point out that the students should realize that the slope is obviously negative.



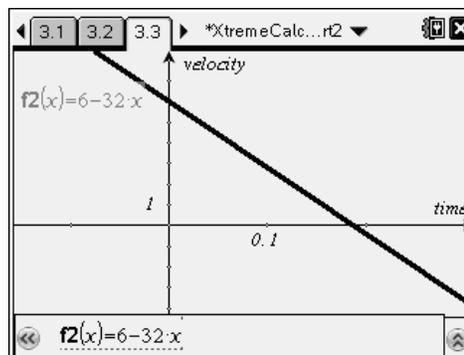
**TI-Nspire Navigator Opportunity: Screen Capture or Live Presenter**  
**See Note 1 at the end of this lesson.**

Explain that the velocity function is the derivative of the position function. Students will calculate the velocity at  $t = 0.5$  algebraically on page 2.2. Students can use the **Derivative** command (if CAS technology is available) or **nDeriv** command to aid them with this task.

On page 2.3, students will graphically verify their answer on page 2.2 by constructing a tangent line and finding the slope. To construct a tangent line, select **MENU > Points & Lines > Tangent** and click on the point on the graph. To find the slope of the line, select **MENU > Measurement > Slope** and click on the tangent line. This slope should be  $v(0.5) = -10$ .



On page 3.1, students are to find the velocity function and graph it in **f2(x)** on page 3.2. (Note: this is a function of  $x$ , so the  $t$  will need to be replaced with an  $x$ .) The slope of this graph is  $-32$ . On page 3.3, even though the velocity is zero at the peak, students should notice that a parabolic position-time graph indicates a constant acceleration.



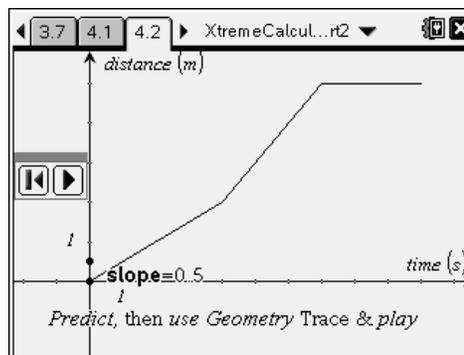
**Student Solutions**

1. The cyclist has an initial positive velocity. The slope of the tangent of the position-time graph is positive. The height  $y$  is increasing. Up is the positive  $y$  direction.
2.  $(0.06 - 8.8)/(0.95 - 0) = -9.2$  ft/s
3.  $s'(0.5) = v(0.5) = -10$  ft/s
4.  $v(t) = 6 - 32t$
5. Acceleration is constant  $-32$  ft/s<sup>2</sup>.
6. Velocity was always decreasing, since the acceleration was negative. The last multiple-choice answer in the .tns file (“because the graph is going down”) is not the best answer (i) because it doesn’t use calculus, and (ii) it isn’t clear to what graph it is referring.
7. Sometimes. Zero is not a positive or negative number. Speed is always  $\geq 0$ .
8. When the velocity and acceleration are both in the same direction, the speed is increasing.

**TI-Nspire Navigator Opportunity: Quick Poll**  
 See Note 2 at the end of this lesson.

**Part 2 – Predict the Graph & Geometry Trace**

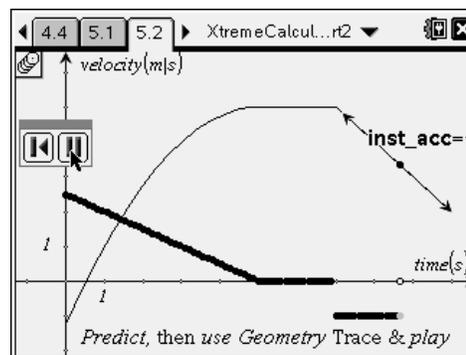
This part of the activity begins with Problem 4. Students will examine a distance-time graph on page 4.2 and consider what the corresponding velocity-time graph looks like. It is recommended for students to sketch the distance-time graph and directly below that, with the same time scale, to draw the velocity-time graph.



To view the velocity-time graph, students will set up a **Geometry Trace** and press play. As the tangent line travels along the position graph, the velocity-time graph will be traced out. To use **Geometry Trace** to display the velocity-time graph, students will select **MENU > Trace > Geometry Trace**. Next, they will click on the point not on the graph (it will display “point to click” when the cursor is hovering over the point) and then press play.

As students analyze the motion of the object, a common mistake is “graph as picture errors.” This is when a student considers a graph to be like a photograph of a situation rather than an abstract mathematical representation. Warn students to not use this thought when answering Question 10.

Students will repeat this process for another position-time graph on page 5.2.



**TI-Nspire Navigator Opportunity: Screen Capture**

See Note 3 at the end of this lesson.

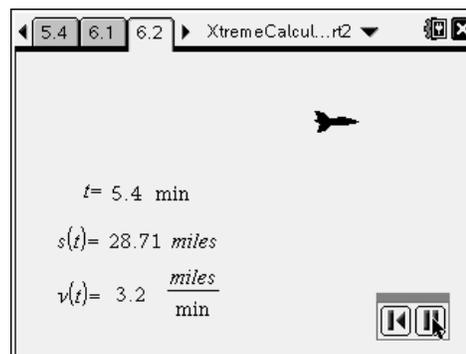
**Student Solutions**

9. Student answers will vary. Students should have sketched their prediction of the velocity-time graph.
10. The solution is that the graph depicts an object that starts at the origin moving at a constant velocity; it suddenly moves at a higher constant speed and then abruptly stops. These jumps in the velocity time graph make this *extreme motion*.
11. When  $t = 5$  seconds, the acceleration exists because the left limit of the rate of change of velocity equals the right limit as  $t$  approaches 5.
12. When  $t = 7$ , the acceleration does not exist because the limit from the left does not equal the right limit of the slope.

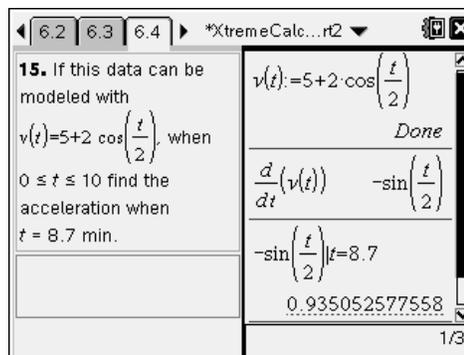
**Part 3 – Extreme Airplane**

Students will examine the flight of an airplane on page 6.2. Students should predict how the position-time graph and the velocity-graph look before moving on to the next page. The time and velocity data are given on page 6.3. This is used to determine the approximate acceleration using a difference quotient.

This data can also be used when the MVT is studied. Similar free responses to questions on calculus exams address the Mean Value Theorem in the context of data.



On page 6.4, students are given the velocity function of the airplane. Students should find the acceleration when  $t = 8.7$  min algebraically.

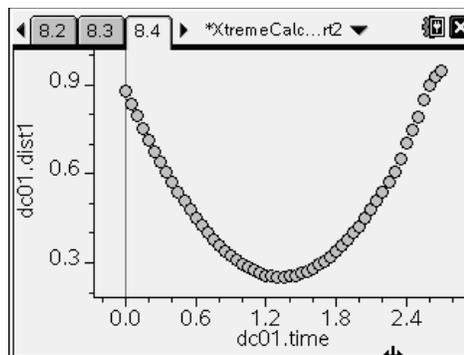


**Student Solutions**

- 13. Student answers will vary. Students should have sketched their predictions of the position-time graph and the velocity-time graph.
- 14.  $a = -0.6$  miles/min<sup>2</sup>
- 15.  $a(8.7) = 0.935$  miles/min<sup>2</sup>
- 16. The plane is speeding up because both the velocity and acceleration are positive.

**Extension – Calculus Motion Lab with CBR 2**

Students will set up the lab experiment as shown on page 7.1. Students should practice rolling the ball so that it goes up the approximately 1 meter (3 feet) ramp and get close to the CBR 2, but doesn't hit it. When the CBR 2 is plugged in, it automatically begins working. Choose to display the data in a *Graphs & Geometry* application because this graph labels the axes clearly and includes units.



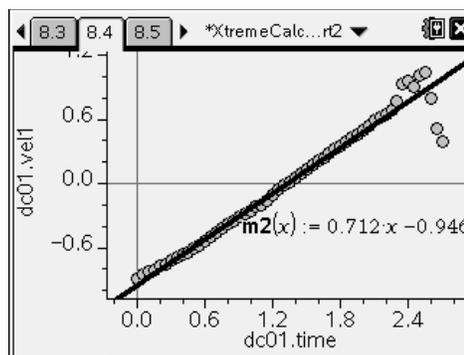
Before pressing **play**, the students should sketch their prediction of the position-time graph.

On the handhelds, students will press **(enter)** to begin and stop collecting data.

Page 7.4 explains how to display the data in a *Data & Statistics* page (**MENU > Experiment > Display Data In > New Data & Statistics**). This will enable the student to easily change the y-axis to velocity and see the dynamic change. Again, students should predict what the graph will look like before changing the axis.

Problem 8 is provided in case a CBR 2 is unavailable or for those students that have trouble collecting the data. If motion sensors and other materials are available, then Problem 8 can be disregarded.

Answers will vary for the slope of the velocity-time graph. The slope of the velocity-time graph represents the acceleration and is approximately  $0.7 \text{ m/s}^2$ .



**TI-Nspire Navigator Opportunities****Note 1****Part 1, *Screen Capture or Live Presenter***

These problems would be good places to do screen captures to verify students are able to run the animation and enter the equations correctly. You may also choose to have a student or group of students lead students through the lesson by projecting their results.

**Note 2****Part 1, *Quick Poll***

Throughout the lesson, you may choose to use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.

**Note 3****Part 2, *Screen Capture***

You may choose to use Screen Capture to verify that students are able to use the ***Geometry Trace*** application and run the animation.