

TI-89 Activities for PreCalculus and AP Calculus

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The statistical, graphical, and symbolic capabilities of the TI-89 make it a powerful tool for investigating and developing mathematical concepts. In this session, we will explore some of the main features of the TI-89 and work on some activities designed to help develop students' understanding of various concepts and procedures in pre-calculus and AP calculus.

TI-89 Preliminaries

- The ON key is in the lower left corner.
- The blue **ENTER** key is in the lower right corner. Other blue keys include the:
 - ♦ F1-F5 keys, just below the screen, which are used to access menus displayed at the top of the screen,
 - ♦ four **ARROW** keys, at the top right, which are used to scroll around the screen,
 - ♦ **ESC** key, at the top middle just to the left of the cursor keys.
 - ♦ **APPS** (applications) key, just below the **ESC** key, which accesses a menu of different TI-89 applications for the user to choose.





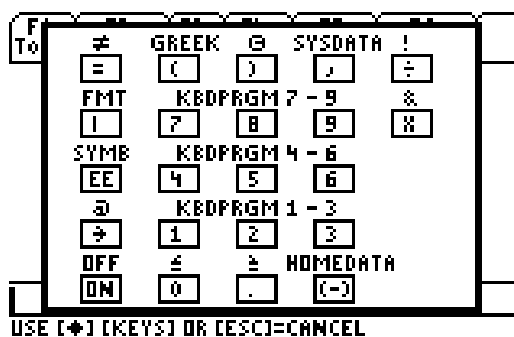
APPS Menu for older TI-89 code




APPS Menu for newer TI-89 code

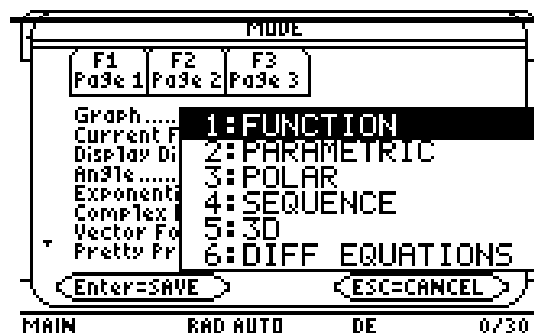
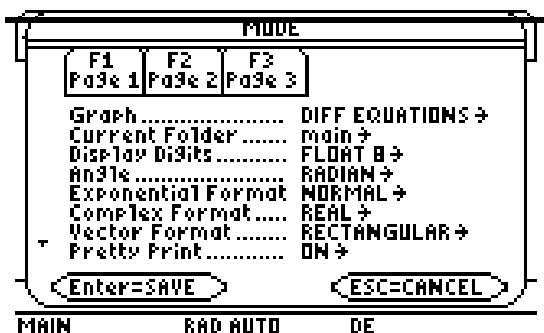
- The yellow **2nd** key is used to access the yellow “functions” of each key.
- The purple **alpha** key is used to access alphabetic characters shown in purple above certain keys. Pressing the **alpha** key twice locks the calculator in “alphabetic” mode.

- The  (green diamond) key is used to access the green “functions” of each key. Due to space limitations, the green functions of certain keys are not displayed next to the keys. Pressing the  key and then the EE key will display the green “functions” for these keys.

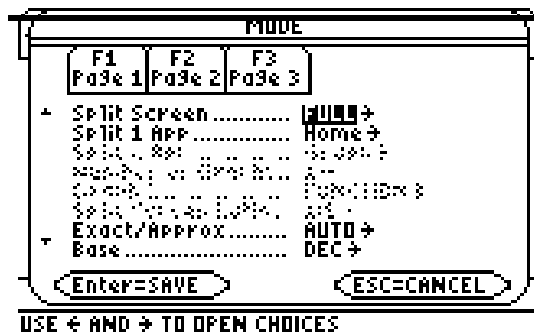
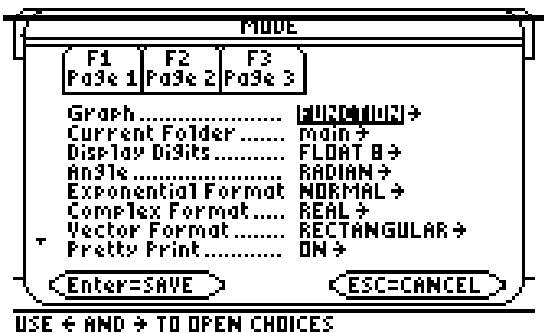


- Contrast is adjusted by holding down the  key and pressing either the + key to darken or the - key to lighten.

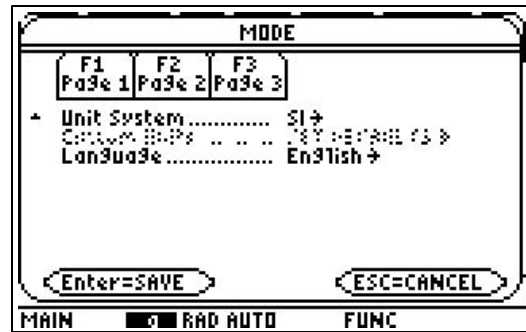
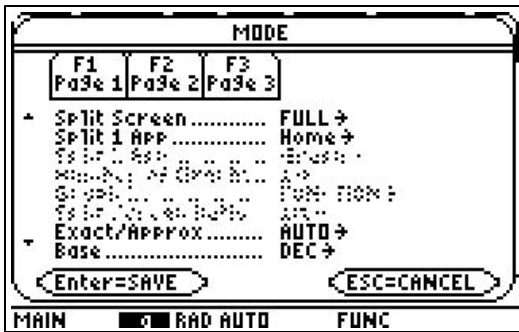
Setting Modes: The MODE key is below the alpha key in the upper left. Pressing this key will access the different settings possible on the TI-89. To change any of the MODE settings, first scroll to that setting, then press the right arrow key. This will open a submenu that shows the different possibilities for that particular setting.



To select a new setting, either press the number of that setting, or scroll to that setting and press ENTER. The submenu will disappear and the new setting should appear on the MODE screen.

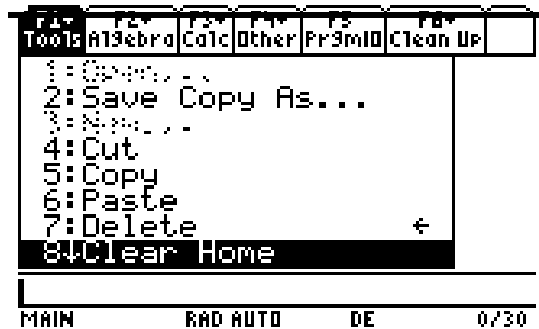


The TI-89 has three different mode screens that can be accessed by pressing the F1, F2 or F3 keys. Once all the settings are to your liking, press ENTER to save them. If you don't, all the settings will revert back to where they were before you accessed the MODE screen. Press HOME to get to the home screen, if you aren't there already.



Exact vs. Approximate: One of the mode settings, Exact/Approx, tells the calculator how it should do calculations. In **Exact** mode, the TI-89 will give exact answers only or will give no answer when an exact answer is not possible (though an answer may exist). In **Approximate** mode, the TI-89 will give decimal answers (like the TI-83). In **Auto** mode, the TI-89 will give an exact answer if at all possible, but will revert to decimal answers when it cannot (assuming an answer is possible). Note that in **Auto** mode, you can force an answer to be in decimal form by inserting a decimal point after an integer in the calculation. You can also get a decimal approximation of an answer by pressing \blacklozenge then ENTER instead of just ENTER to evaluate.

Clearing the History Area: If the history area of the Home screen has previous work you wish to clear, press F1 (“Tools” menu) followed by 8:Clear Home.



Executing and Editing Commands: New commands first appear at the bottom of the TI-89 screen in the “entry line” (the area of the screen just above the word MAIN). Pressing ENTER places the command and answer in the “history area”. By default the TI-89 is in insert mode with a vertical cursor bar. New text is inserted to the left of the cursor on the entry line. Errors are deleted using the backspace key (the back arrow key, to the left of the CLEAR key). Pressing the CLEAR key (once or twice) to delete the entire line to start over.

Editing Previous Entries: Instead of using 2nd ENTER multiple times to recall previous commands, use the up arrow multiple times to enter the “history area” of the home screen. With the desired previous entry or answer highlighted, press ENTER to copy it to the edit line.

Catalog: The CATALOG key is below the APPS key. Catalog is an alphabetized list of reserved words and symbols. Many commands not easily located through the TI-89 menus can be found quickly here. 2nd up arrow or 2nd down arrow can be used to page up or down through CATALOG. The syntax of the selected command appears at the very bottom of the TI-89 screen. For example, when the command lcm(is highlighted, the syntax $EXPR1, EXPR2$ appears at the bottom of the screen.

Custom Menu: The TI-89 has a custom menu (2nd HOME) which contains many commonly used words so you don't have to type them in. The custom menu will replace the regular menu bar at the top of the screen. To return to the regular menu bar, just press CUSTOM again.

Activity: Exploring Capabilities

Let's see how the TI-89 handles a variety of calculations.

- | | | |
|--------------------------------------|--|---------------------------------------|
| 1. $100!$ | 2. $\frac{\sqrt{5} + \sqrt{2}}{2\sqrt{2} - \sqrt{5}}$ | 3. $\sqrt{6}(\sqrt{-18} + 2\sqrt{3})$ |
| 4. $\frac{2-i}{4+3i}$ | 5. $e^{-\infty}$ | 6. $\log(0)$ |
| 7. $\cos\left(\frac{\pi}{12}\right)$ | 8. $\cos^{-1}\left(\frac{\sqrt{2} + \sqrt{6}}{4}\right)$ | 9. $\tan\left(\frac{\pi}{12}\right)$ |
| 10. $\sin(\cos^{-1} x)$ | 11. $\sin(\sec^{-1} x)$ | 12. $\tan(\csc^{-1} x)$ |

Activity: Solving Systems of Equations

The TI-89 can solve systems of equations (even non-linear systems) directly using. However, we can use the symbolic manipulation capabilities of the TI-89 to solve systems of equations in a manner that can develop, or assess, students' understanding of the processes involved. Let's look ways the TI-89 can be used to solve the system of equations $3x + 2y = 11$ and $2x - 5y = 13$.



```

■ solve(3·x+2·y=11 and 2
  x=81/19 and y=-17/19
...=11 and 2x-5y=13, {x,y})
MAIN          RAD AUTO    DE          1/30
  
```

Note: A solve(command for solving a system of equations can be found by pressing 2nd HOME to access the CUSTOM menu, then pressing F2 (Solve) followed by 4: Solve(and ,{x,y}). The two equations you wish to solve can then be typed in the appropriate spots in this command.

Elimination Method

Begin by multiplying each equation by appropriately chosen values and then adding the equations to "eliminate" x. Next, solve the resulting equation for y.



```

■ (3·x+2·y=11)·2+(2·x-5
  19·y=-17
...y=11)*2+(2x-5y=13)*-3
MAIN          RAD AUTO    DE          1/30
  
```



```

■ (3·x+2·y=11)·2+(2·x-5
  19·y=-17
■ solve(19·y=-17,y)
  y=-17/19
solve(19*y=-17,y)
MAIN          RAD AUTO    DE          2/30
  
```

Once the value for y is obtained, it can be substituted into one of the original equations using the $|$ key. Finally, the resulting equation can be solved for x. The values obtained for x and y can then be substituted into the other equation to verify that they are a solution to the system.

F1*	F2*	F3*	F4*	F5	F6*
Tools	Algebra	Calc	Other	Fr3Mid	Clean Up
■ solve(19·y = -17, y)					
$y = -17/19$					
■ 3·x + 2·y = 11 y = -17/19					
$3·x - 34/19 = 11$					
■ solve(3·x - 34/19 = 11, x)					
$x = 81/19$					
...=13 y=-17/19 and x=81/19					
MAIN	RAD AUTO	DE	4/30		

F1*	F2*	F3*	F4*	F5	F6*
Tools	Algebra	Calc	Other	Fr3Mid	Clean Up
■ 3·x + 2·y = 11 y = -17/19					
$3·x - 34/19 = 11$					
■ solve(3·x - 34/19 = 11, x)					
$x = 81/19$					
■ 2·x - 5·y = 13 y = -17/19					
true					
...=13 y=-17/19 and x=81/19					
MAIN	RAD AUTO	DE	5/30		

Substitution Method

Begin by solving one of the equation for y in terms of x. Then substitute this expression for y into the other equation, using the $|$ key obtaining a linear equation in x. Solve this equation for x.

Once the value for x is obtained, it can be substituted into the equation for y found in the first step. These values for x and y can then be substituted into one of the original equation to verify that they represent a solution to the system.

F1*	F2*	F3*	F4*	F5	F6*
Tools	Algebra	Calc	Other	Fr3Mid	Clean Up
■ solve(3·x + 2·y = 11, y)					
$y = \frac{-(3·x - 11)}{2}$					
2x-5y=13 y=-(3*x-11)/2					
MAIN	RAD AUTO	DE	1/30		

F1*	F2*	F3*	F4*	F5	F6*
Tools	Algebra	Calc	Other	Fr3Mid	Clean Up
$y = \frac{-(3·x - 11)}{2}$					
■ 2·x - 5·y = 13 y = $\frac{-(3·x - 11)}{2}$					
$\frac{19·x}{2} - 55/2 = 13$					
2x-5y=13 y=-(3*x-11)/2					
MAIN	RAD AUTO	DE	2/30		

F1*	F2*	F3*	F4*	F5	F6*
Tools	Algebra	Calc	Other	Fr3Mid	Clean Up
■ solve($\frac{19·x}{2} - 55/2 = 13, x$)					
$x = 81/19$					
■ y = $\frac{-(3·x - 11)}{2}$ x = 81/19					
$y = -17/19$					
y=-(3*x-11)/2 x=81/19					
MAIN	RAD AUTO	DE	4/30		

F1*	F2*	F3*	F4*	F5	F6*
Tools	Algebra	Calc	Other	Fr3Mid	Clean Up
$x = 81/19$					
■ y = $\frac{-(3·x - 11)}{2}$ x = 81/19					
$y = -17/19$					
■ 3·x + 2·y = 11 x = 81/19					
true					
...=11 x=81/19 and y=-17/19					
MAIN	RAD AUTO	FUNC	5/30		

Solving systems of equations in this manner places the focus on the equation-solving process itself and not on the algebraic manipulations necessary to complete the process. This makes it possible to assess, or develop, students' understanding of the process without the algebra "getting in the way" of students' success or failure at solving systems of equations.

Matrix Methods

The coefficients from the system of equations can be entered into the TI-89 as a coefficient matrix $\begin{bmatrix} 3 & 2 \\ 2 & -5 \end{bmatrix}$ and

solution matrix $\begin{bmatrix} 11 \\ 13 \end{bmatrix}$ and stored as variables.

F1▼	F2▼	F3▼	F4▼	F5	F6▼	
Tools	Algebra	Calc	Other	Pr3mID	Clean Up	

$$\begin{aligned} & \begin{bmatrix} 3 & 2 \\ 2 & -5 \end{bmatrix} \rightarrow ma & \begin{bmatrix} 3 & 2 \\ 2 & -5 \end{bmatrix} \\ \hline & [3,2;2,-5] \rightarrow ma \\ \hline \text{MAIN} & \text{RAD AUTO} & \text{FUNC} & 1/30 \end{aligned}$$

F1▼	F2▼	F3▼	F4▼	F5	F6▼	
Tools	Algebra	Calc	Other	Pr3mID	Clean Up	

$$\begin{aligned} & \begin{bmatrix} 3 & 2 \\ 2 & -5 \end{bmatrix} \rightarrow ma & \begin{bmatrix} 3 & 2 \\ 2 & -5 \end{bmatrix} \\ \hline & [3,2;2,-5] \rightarrow ma \\ \hline \text{MAIN} & \text{RAD AUTO} & \text{FUNC} & 1/30 \end{aligned}$$

Then the `simult` command can be used on the variables or on the matrices themselves to solve the system. Or the two coefficient matrices (either the variables that represent them or the matrices themselves) can be combined using the `augment` command, and then the `rref` command (row-reduced echelon form) can be used to solve the system.

F1▼	F2▼	F3▼	F4▼	F5	F6▼	
Tools	Algebra	Calc	Other	Pr3mID	Clean Up	

$$\begin{aligned} & [13] \rightarrow mc & [13] \\ & \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow mb & \begin{bmatrix} x \\ y \end{bmatrix} \\ & \text{simult}(ma, mc) & \begin{bmatrix} 81/19 \\ -17/19 \end{bmatrix} \\ \hline & [3,2][2,-5], [[11][13]] \\ \hline \text{MAIN} & \text{RAD AUTO} & \text{FUNC} & 4/30 \end{aligned}$$

F1▼	F2▼	F3▼	F4▼	F5	F6▼	
Tools	Algebra	Calc	Other	Pr3mID	Clean Up	

$$\begin{aligned} & \text{simult}(ma, mc) & -17/19 \\ & \text{simult}\left(\begin{bmatrix} 3 & 2 \\ 2 & -5 \end{bmatrix}, \begin{bmatrix} 11 \\ 13 \end{bmatrix}\right) & \begin{bmatrix} 81/19 \\ -17/19 \end{bmatrix} \\ \hline & [3,2][2,-5], [[11][13]] \\ \hline \text{MAIN} & \text{RAD AUTO} & \text{FUNC} & 5/30 \end{aligned}$$

F1▼	F2▼	F3▼	F4▼	F5	F6▼	
Tools	Algebra	Calc	Other	Pr3mID	Clean Up	

$$\begin{aligned} & \begin{bmatrix} 81/19 \\ -17/19 \end{bmatrix} \\ & \text{augment}(ma, mc) & \begin{bmatrix} 3 & 2 & 11 \\ 2 & -5 & 13 \end{bmatrix} \\ \hline & \text{augment}(ma, mc) \\ \hline \text{MAIN} & \text{RAD AUTO} & \text{FUNC} & 6/30 \end{aligned}$$

F1▼	F2▼	F3▼	F4▼	F5	F6▼	
Tools	Algebra	Calc	Other	Pr3mID	Clean Up	

$$\begin{aligned} & \begin{bmatrix} 2 & -5 & 13 \end{bmatrix} \\ & \text{rref}\left(\begin{bmatrix} 3 & 2 & 11 \\ 2 & -5 & 13 \end{bmatrix}\right) & \begin{bmatrix} 1 & 0 & 81/19 \\ 0 & 1 & -17/19 \end{bmatrix} \\ \hline & \text{rref}([3,2,11][2,-5,13]) \\ \hline \text{MAIN} & \text{RAD AUTO} & \text{FUNC} & 7/30 \end{aligned}$$

Students can also solve a system of equations “by hand” using the row operation commands on the TI-89. For example, starting with the augmented matrix $\begin{bmatrix} 3 & 2 & 11 \\ 2 & -5 & 13 \end{bmatrix}$, the `mRow` command can be used to multiply the first and second rows by 2 and -3, respectively.

```

┌───┬───┬───┬───┬───┬───┬───┐
│ F1  │ F2  │ F3  │ F4  │ F5  │ F6  │   │
│ Tools│Algebra│Calc│Other│Pr3mID│Clean Up│   │
├───┴───┴───┴───┴───┴───┴───┘
                                [0 1 -17/19]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ 2,  │ [3 2 11] │ [2 -5 13] │ , 1 │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
                                [6 4 22]
                                [2 -5 13]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ 2,  │ [[3,2,11][2,-5,13]],1) │   │   │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
MAIN          RAD AUTO          FUNC          8/30

```

```

┌───┬───┬───┬───┬───┬───┬───┐
│ F1  │ F2  │ F3  │ F4  │ F5  │ F6  │   │
│ Tools│Algebra│Calc│Other│Pr3mID│Clean Up│   │
├───┴───┴───┴───┴───┴───┴───┘
                                [2 -5 13]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ -3, │ [6 4 22] │ [2 -5 13] │ , 2 │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
                                [6 4 22]
                                [-6 15 -39]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ -3, │ [[6,4,22][2,-5,13]],2) │   │   │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
MAIN          RAD AUTO          FUNC          9/30

```

Then the ROWAdd command can be used to obtain a zero in the first column of the second row, followed by the mRow command again to obtain a one in the second column of the second row.

```

┌───┬───┬───┬───┬───┬───┬───┐
│ F1  │ F2  │ F3  │ F4  │ F5  │ F6  │   │
│ Tools│Algebra│Calc│Other│Pr3mID│Clean Up│   │
├───┴───┴───┴───┴───┴───┴───┘
                                [-6 15 -39]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ rowAdd │ [6 4 22] │ [-6 15 -39] │ , 1, 2 │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
                                [6 4 22]
                                [0 19 -17]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ ... │ [[6,4,22][-6,15,-39]],1,2) │   │   │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
MAIN          RAD AUTO          FUNC          10/30

```

```

┌───┬───┬───┬───┬───┬───┬───┐
│ F1  │ F2  │ F3  │ F4  │ F5  │ F6  │   │
│ Tools│Algebra│Calc│Other│Pr3mID│Clean Up│   │
├───┴───┴───┴───┴───┴───┴───┘
                                [0 19 -17]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ mRow │ 1/19, [6 4 22] │ [0 19 -17] │ , 2 │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
                                [6 4 22]
                                [0 1 -17/19]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ ... │ [[6,4,22][0,19,-17]],2) │   │   │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
MAIN          RAD AUTO          FUNC          11/30

```

The mRowAdd command can now be used to obtain a zero in the second column of the first row. Finally, the mRow command can be used one more time to obtain a one in the first column of the first row, and thus obtain a solution to the system of equations.

```

┌───┬───┬───┬───┬───┬───┬───┐
│ F1  │ F2  │ F3  │ F4  │ F5  │ F6  │   │
│ Tools│Algebra│Calc│Other│Pr3mID│Clean Up│   │
├───┴───┴───┴───┴───┴───┴───┘
                                [0 1 -17/19]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ mRowAdd │ -4, [6 4 22] │ [0 1 -17/19] │ , 2, 1 │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
                                [6 0 486/19]
                                [0 1 -17/19]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ ... │ [[4,22][0,1,-17/19]],2,1) │   │   │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
MAIN          RAD AUTO          FUNC          12/30

```

```

┌───┬───┬───┬───┬───┬───┬───┐
│ F1  │ F2  │ F3  │ F4  │ F5  │ F6  │   │
│ Tools│Algebra│Calc│Other│Pr3mID│Clean Up│   │
├───┴───┴───┴───┴───┴───┴───┘
                                [0 1 -17/19]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ mRow │ 1/6, [6 0 486/19] │ [0 1 -17/19] │ , 1 │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
                                [1 0 81/19]
                                [0 1 -17/19]
┌───┬───┬───┬───┬───┬───┬───┐
│ ... │ ... │ ... │ ... │ ... │ ... │ ... │
│ ... │ [[486/19][0,1,-17/19]],1) │   │   │   │   │   │
├───┴───┴───┴───┴───┴───┴───┘
MAIN          RAD AUTO          FUNC          13/30

```

Try a few system of equations problems.

- Solve this system of equations in two (or more) different ways:
 $x + y + z = -1$ $2x + 4y + z = 1$ $3x - y - z = -15$

- Solve the following system of equations:

$$a_{11}x + a_{12}y = b_1 \qquad a_{21}x + a_{22}y = b_2$$

(Does this look familiar? Can it be done with a general system for 3 equations and 3 unknowns?)

3. Determine an equation for a polynomial function that passes through the following points:
 (1, -5), (-2, 1), (3, 31), (-4, -25)

4. Solve the following system of equations:
$$\begin{cases} \frac{1}{y} + \frac{3}{z} = 4 \\ \frac{3}{x} + \frac{1}{z} = 5 \\ \frac{1}{x} + \frac{3}{y} = 9 \end{cases}$$
 (Try it directly to see what happens.)

Activity: Finding Inverse Functions

The `SOLVE` command can also be used to determine the inverse of a function. After defining a function $f(x)$ using the `STO` \triangleright or `Define` commands, the equation $x = f(y)$ can then be solved for y to determine the inverse function, $y = f^{-1}(x)$.

```

F1+  F2+  F3+  F4+  F5  F6+
Tools Algebra Calc Other Pr3mID Clean Up

■ 2·x - 1
  4 - 3·x → f(x) Done
■ solve(x = f(y), y)
  y = 4·x + 1
  3·x + 2
solve(x=f(y), y)
MAIN RAD AUTO FUNC 2/30
  
```

```

F1+  F2+  F3+  F4+  F5  F6+
Tools Algebra Calc Other Pr3mID Clean Up

■ x = f(y) x = -(2·y - 1)
  3·y - 4
x=f(y)
MAIN RAD AUTO FUNC 1/30
  
```

The alternative method is to solve the equation $x = f(y)$ for y in a manner similar to that used to solve the linear equation.

```

F1+  F2+  F3+  F4+  F5  F6+
Tools Algebra Calc Other Pr3mID Clean Up

■ (x = -(2·y - 1)
  3·y - 4) · (3·y - 4)
  x · (3·y - 4) = -(2·y - 1)
■ expand(x · (3·y - 4) = -(2·y - 1)
  3·x·y - 4·x = 1 - 2·y
expand(ans(1))
MAIN RAD AUTO FUNC 3/30
  
```

```

F1+  F2+  F3+  F4+  F5  F6+
Tools Algebra Calc Other Pr3mID Clean Up

■ expand(x · (3·y - 4) = -(2·y - 1)
  3·x·y - 4·x = 1 - 2·y
■ (3·x·y - 4·x = 1 - 2·y) + 2·y
  3·x·y + 2·y = 4·x + 1
■ factor(3·x·y + 2·y = 4·x + 1)
  (3·x + 2) · y = 4·x + 1
ans(1)/(3x+2)
MAIN RAD AUTO FUNC 5/30
  
```

Experiment with other functions to see how this technique works and what pitfalls might be encountered when using it to determine inverse functions. Such work should help give students a deeper understanding of inverse functions and help them avoid getting bogged down in algebraic manipulations that detract from the learning that occurs as students' work to find inverses.

```

F1+  F2+  F3+  F4+  F5  F6+
Tools Algebra Calc Other Pr3mID Clean Up

■ factor(3·x·y + 2·y = 4·x + 1)
  (3·x + 2) · y = 4·x + 1
■ (3·x + 2) · y = 4·x + 1
  3·x + 2
  y = 4·x + 1
  3·x + 2
ans(1)/(3x+2)
MAIN RAD AUTO FUNC 6/30
  
```


Activity: Connecting Algebra to Graphs and Modeling

Consider the following problem situation:

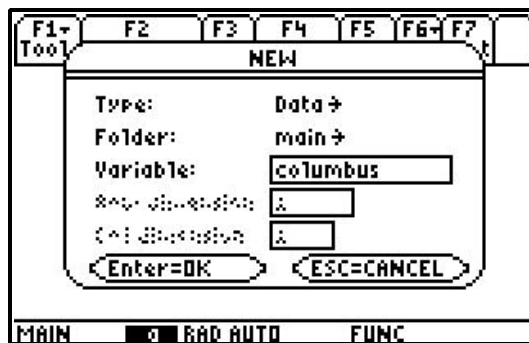
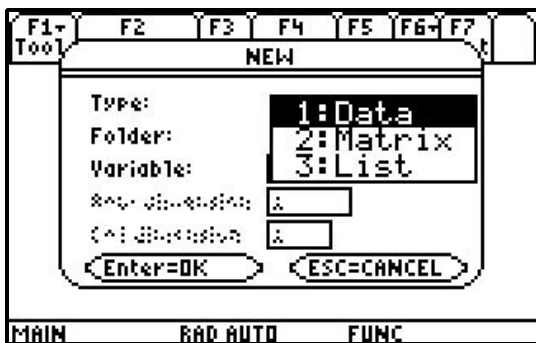
An airline transports about 1700 people per week between Columbus and Los Angeles. A round-trip ticket for this flight costs \$362. The company is doing market research to determine whether it should increase the fare on this flight. Research indicates that for each \$5 increase in price, 19 fewer passengers will take the flight. What ticket price should the airline charge?

In solving this problem, the biggest hurdle for most students is coming up with an equation to model the problem situation. Even the students who do come up with the equation often do not know exactly what the equation represents or why the equation models the problem situation. The statistical and graphical capabilities of the TI-89 can be used to solve the problem and to derive an equation that models the problem situation. The symbolic algebra capabilities of the TI-89 then provides students with a means for discovering how to derive an equation that represents this and similar problem situations *without* using data or graph.

To begin, we need to create a TI-89 Data variable. To do this, press the APPS key and select 6:Data/Matrix Editor from this menu. A submenu will appear. Choose 3:New.



A dialog box will appear. You want to choose a Data variable. Give your Data variable a name (like columbus), then press ENTER twice to accept your selections.



Note that on the TI-89, you have three options in this editor. You have a List variable, which is simply a single list, a Data variable, which is a set of any number of lists all under one variable name, and a Matrix variable, which is a matrix you enter using this editor rather than doing so from the HOME screen (more on this later). A Data variable consists of columns headed C1, C2, C3, and so on. You can adjust the size of the columns through the Format Menu which can be accessed by press F1 followed by 8:Format or by pressing ♦ F (the “|” key).

The figures at the right shows a TI-89, created for this problem. Column c1 (Incrs) contains different values for the number of \$5 price increases the airline could impose of the ticket price. Columns c2 (Peopl) and c3 (Tpric) are the number of people who will still take this flight and the ticket price corresponding to the number of \$5 price increases in column c1. Finally, column c4 (Trev) is the revenue generated by ticket sales corresponding to the number of \$5 price increases in column c1.

F1- Tools	F2 Plot Setup	F3 Cell	F4 Header	F5 Calc	F6 Util	F7 Stat
DATA	incrs	numbr	tpric	trev		
	c1	c2	c3	c4		
1	0	1700	362	6154...		
2	1	1681	367	6169...		
3	2	1662	372	6182...		
4	3	1643	377	6194...		

c1=

MAIN RAD AUTO FUNC

The data here was generated by having students calculate the number of people who would fly, along with the corresponding ticket price and revenue, for specific numbers of \$5 price increases.

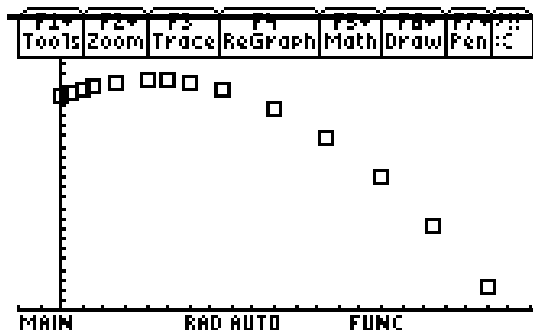
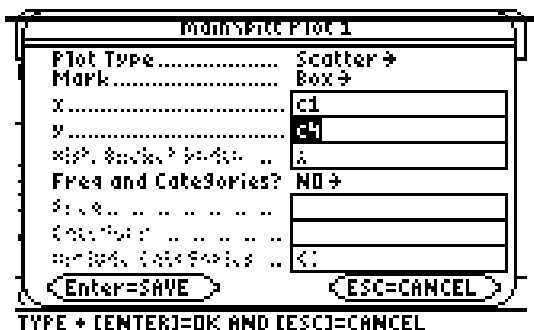
During this process, students may recognize the patterns in the data being input and suggest that entries in each column could be “globally” defined (like a spreadsheet). For example, column c4 can be defined as the product of columns c2 and c3, as shown in the figure at the right. What students do not usually recognize, at least initially, is that these “global definitions” form the basis for an equation that represents the problem situation.

F1- Tools	F2 Plot Setup	F3 Cell	F4 Header	F5 Calc	F6 Util	F7 Stat
DATA	incrs	numbr	tpric	trev		
	c1	c2	c3	c4		
1	0	1700	362	6154...		
2	1	1681	367	6169...		
3	2	1662	372	6182...		
4	3	1643	377	6194...		

c4=c2*c3

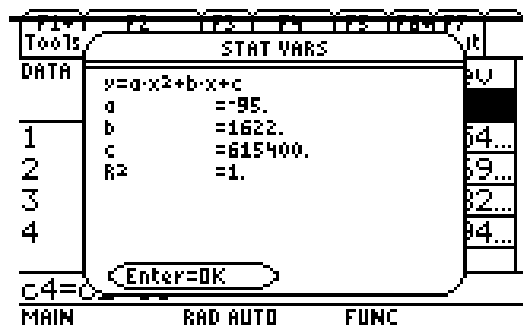
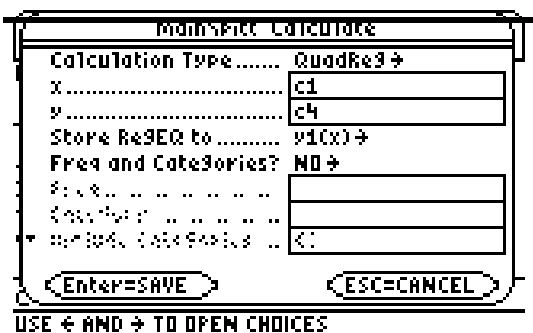
MAIN RAD AUTO FUNC

Once the data has been entered, a scatter plot of columns c1 versus c4 can be generated. To access the statistical plot menu, press F2 (Plot Setup), then F1 (Define). Select Scatter for the Plot Type and Box for Mark (or change it to a different mark by scrolling to the Mark line, pressing the “right arrow” key to access the submenu that shows the different marks you can use, and then selecting a new type of mark). Then designate column c1 for the x-values and column c4 for the y-values.



To draw the scatter plot, press \blacklozenge F1 to access the Y = Editor, turn off any extra functions or plots that are on, and select 9:ZoomData from the F2:Zoom menu. To adjust viewing window for the scatter plot, press \blacklozenge F2 (Window), change the window settings to your liking, then press \blacklozenge F3 (Graph).

The graph suggests that the relationship between the number of \$5 price increases and the ticket sales revenue is quadratic. Performing a quadratic regression on the data can test this conjecture. To do this, again press the APPS key then select 6:Data/Matrix Editor followed by 1:Current to return to the cricket data.. To calculate a quadratic regression, select the F5:Calc menu, set Calculation Type to QuadReg, designate column C1 for the x-values and column C4 for the y-values, and finally set Store RegEq to y1(x) (scroll down to this line, press the “right arrow” key and select y1(x) or some other variables in the Y= Editor if you wish). Now press ENTER to save your selections. The STAT VARS dialog box will appear, showing the results of your regression. At the same time, this regression equation has been entered in the Y= Editor under y1(x) (or whatever variable you chose). The quadratic regression on columns C1 versus C4 produces the equation $y = -95x^2 + 1622x + 615400$.



Based on this equation, it appears that the airline could generate maximal ticket sales by increasing the ticket price to: (a) about \$404.68 (allowing for any price change based on information provided) or (b) \$407 (allowing for only an integral number of \$5 price increases).

By factoring $-95x^2 + 1622x + 615400$ (using the TI-89 if desired), students can see that the equation for this problem situation can be written as:

$$y = -(5x + 362)(19x - 1700) \quad \text{or} \quad y = (5x + 362)(1700 - 19x),$$

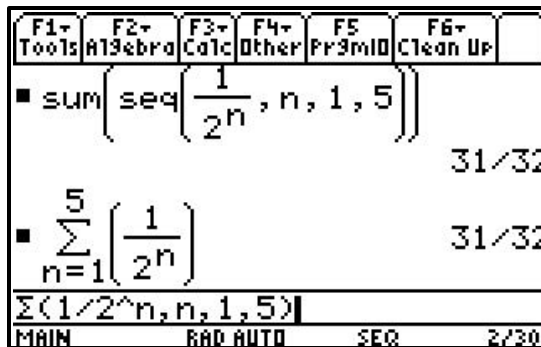
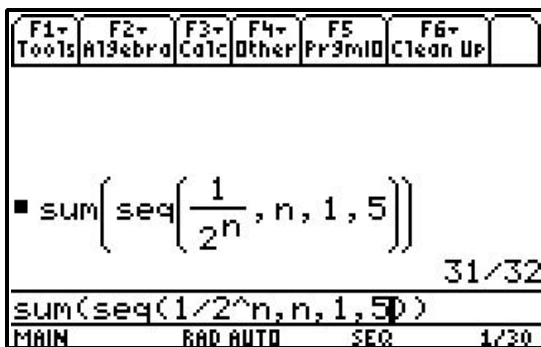
which is exactly the equation we would have expected them to derive (or have derived for them). The difference here is that the equation should be more meaningful to them since it was derived from actual data and in the context of solving the problem.

Another Algebra-Graph Connection Problem

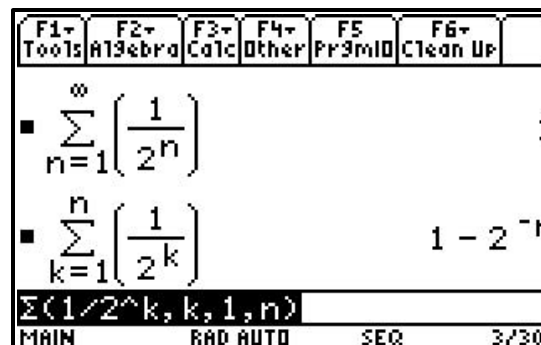
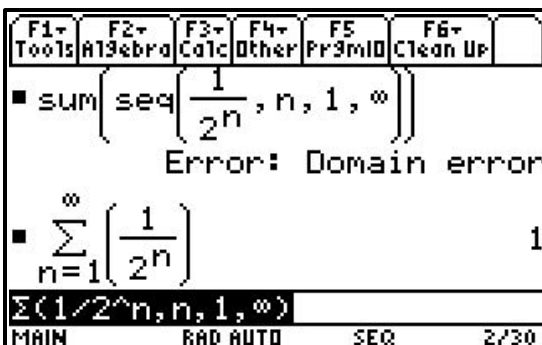
Graph the function $f(x) = x^2 + bx + 2$ for different values of b . For each, estimate the coordinates of the minima of the graph. Use these points (and any other minima you may have estimated) to find a function $f^*(x)$ such that if (x, y) is a minimum point of the graph of f for some value of b , then (x, y) is also a point on the graph of f^* . Use analytically methods to justify your choice of $f^*(x)$.

Activity: Summations

On calculators like the TI-83, you could do summations on a finite number of terms using the sum and seq(commands (both of which can be found quickly on the TI-89 under CATALOG). On the TI-89, there is a special Σ (sum command, which can be found under the F3 (Calculus) menu or under CATALOG, that also handles such calculations.



What distinguishes the Σ (sum) command from the sum and seq(commands is that it can handle infinite sums and finite sums with a variable number of terms.



Try a few of the summations shown below and then discuss the implications of hand-held technology that can perform these types of calculations (such as how such capabilities might be used to help students better understand certain types of mathematical induction proofs!).

$$\sum_{k=1}^n k \quad \sum_{k=1}^n k^3 \quad \sum_{k=1}^n k(k+1) \quad \sum_{k=1}^n a + (k-1)d \quad \sum_{k=1}^{\infty} ar^{k-1}$$

$$\sum_{k=1}^n ar^{k-1} \quad \sum_{k=0}^{\infty} \frac{1}{k^2} \quad \sum_{k=0}^{\infty} \frac{1}{k!} \quad \sum_{k=1}^{\infty} \frac{1}{k(k+1)} \quad \sum_{k=1}^{\infty} \frac{1}{k(k+2)}$$

Activity: Using Patterns to Develop Formulas:

Many different “rules” for exponents, logarithms, rationalizing denominators, and factoring can be explored and developed by having students look for patterns when the TI-89 does the simplification. For example, use the TI-89 to find values for $\ln(2) + \ln(3)$, $\ln(5) + \ln(7)$, $\ln(6) + \ln(x)$, and the like. Based on these results, what conjectures might be made about the value of $\ln(x) + \ln(y)$ where $x > 0$ and $y > 0$ (followed by a discussion of how the conjecture might be proved).

Many formulas for derivatives and antiderivatives can be developed through pattern recognition. For example, use the TI-89 to find the derivative of $y = x + \sin x$; of $y = x^2 + \sin x$; and of $y = x^3 + \sin x$. Based on your observations, predict the derivative of $y = x^4 + \sin x$; of $y = f(x) + g(x)$. (You can do the same with subtraction, multiplication, division and composition of functions!). Try this exploration for other favorite formulas to see which might work and which might not work so well.

Activity: Parametrics and Trigonometry:

According to MLB records, during a 1919 exhibition game between the Boston Red Sox and New York Giants, Babe Ruth hit the longest homerun in major league history. It has been conjectured that the ball hit off Babe's bat at a trajectory of 45° and an initial speed of approximately 137 feet/sec.

The ball's path can be represented parametrically as follows:

$$x(t) = (s \cos A) t \quad y(t) = -16t^2 + (s \sin A) t \quad (s = \text{initial speed } A \text{ and } A = \text{initial trajectory})$$

- Plot the path of the baseball.
- Determine how far the ball was from home plate when it hit the ground.
- Determine the greatest height achieved by the ball.
- The center field fence at the old Polo Grounds was 460 feet from home plate and was approximately 15 feet tall. Would the ball have cleared this fence?
- Determine the steepest and shallowest trajectory that the ball could have taken off the bat and still cleared the Polo Ground's center field fence.
- Determine the slowest initial speed the ball could have had and still cleared the Polo Ground's center field fence.

Activity: Developing the power rule from graphs of $y = x^n$:

Graph $y = x^n$ for specific values of n . For each graph, use the derivative command, 6:Derivatives under the F5:Math menu in the Graph editor to find the value of the derivative of y at a specific point on the graph. Press \blacklozenge , to download the value of the derivative into the data variable `sysData` in the Data/Matrix Editor. Now access `sysData` in the Data/Matrix Editor and the column next to the value of the derivative, enter `XC`, thus entering the value of the x -coordinate of the specific point at which you found the derivative. Do this for a number of points on the graph of y and then use the regression features of the TI-89 to determine the equation of y' . Do enough of these and students will "discover" the power rule for derivatives (i.e., if $y = x^n$ where n is a positive integer, then $y' = x^{n-1}$)

Activity: Miscellaneous Calculus Fun

Prove or Disprove: If a cubic polynomial has three real roots, then the tangent line to the graph of this polynomial at a point whose x -coordinate is the average of two of the roots will intersect the graph at the third root. [See if it works for a specific case such as $f(x) = (x - 2)(x + 3)(x - 4)$.]

Prove or Disprove: If P is any point on the graph of $f(x) = x^3$ such that the tangent to the graph of $f(x)$ at P intersects the graph at another point Q , then slope of the graph of $f(x)$ at Q is four times the slope of the graph at P . [Again, see if it works for a specific case such as P being the point $(2,8)$.]

Draw a tangent line to any cubic polynomial at (almost) any point (say $x = a$). Determine the second point (say $x = b$) where this tangent intersects the cubic. Determine the area of the region bounded by the tangent and the cubic. Next, draw the tangent line to the cubic at $x = b$ and determine second point (say $x = c$) where this tangent intersects the cubic. Determine the area of the region bounded by this second tangent and the cubic. Determine the ratio of the larger area to the smaller area? Prove this result.

Find the volume of the largest right circular cone inscribed in a sphere of radius r .

The AP Examinations

The Calculus AB and BC Examinations seek to assess how well a student has mastered the concepts and techniques of the subject matter of the corresponding courses. Each examination consists of two sections, as described below.

Section I: a multiple-choice section testing proficiency in a wide variety of topics

Section II: a free-response section requiring the student to demonstrate the ability to solve problems involving a more extended chain of reasoning

The examination time for the AP Calculus Examinations is 3 hours and 15 minutes. The multiple-choice section of each examination consists of 45 questions in 105 minutes. Part A of the multiple-choice section (28 questions in 55 minutes) does not allow the use of a calculator. Part B of the multiple-choice section (17 questions in 50 minutes) contains some questions for which a graphing calculator is required.

Beginning in May 2000, the free-response section of each examination will have two parts: one part requiring graphing calculators and a second part not allowing graphing calculators. This change in format is an effort to respond to heightened concerns with equity as more students may use graphing calculators with computer algebra system (CAS) features. The AP examinations are designed to accurately assess student mastery of both the concepts and techniques of calculus. The two-part format for the free-response section provides greater flexibility in the types of questions that can be asked while ensuring fairness to all students taking the exam, regardless of the graphing calculator used. A list of approved graphing calculators is available at College Board Online <www.collegeboard.org/ap>.

The free-response section of each examination consists of 6 questions in 90 minutes. Part A of the free-response section (3 questions in 45 minutes) will contain some questions or parts of questions for which a graphing calculator is required. Part B of the free-response section (3 questions in 45 minutes) will not allow the use of a calculator. During the second timed portion of the free-response section (Part B), students will be permitted to continue work on questions in Part A, but they will not be permitted to use a calculator during this time. In determining the grade for each examination, the scores for Section I and Section II are given equal weight. Since the examinations are designed for full coverage of the subject matter, it is not expected that all students will be able to answer all the questions.

Graphing Calculator Capabilities for the Exams

Examination questions are designed based on the assumption that all students have access to four basic calculator capabilities used extensively in calculus. Beginning in May 2000, a graphing calculator appropriate for use on the examinations is expected to have the built-in capability to:

- 1) plot the graph of a function within an arbitrary viewing window,
- 2) find the zeros of functions (solve equations numerically),
- 3) numerically calculate the derivative of a function, and
- 4) numerically calculate the value of a definite integral.

One or more of these capabilities should provide the sufficient computational tools for successful development of a solution to any question that requires the use of a calculator. Questions have been designed so as to not favor students who use graphing calculators with more extensive built-in features.

Technology Restrictions on the Exams

Nongraphing scientific calculators, computers, devices with a QWERTY keyboard, and electronic writing pads are not permitted for use on the AP Calculus Examinations. Student should be thoroughly familiar with the operation of the calculator he or she plans to use on the examination. Calculators may not be shared, and communication between calculators is prohibited during the examination. Students may bring to the examination one or two (but no more than two) graphing calculators from the approved list. Calculator memories will not be cleared. Students are allowed to bring to the examination calculators containing whatever programs they want. Students must not use calculator memories to take test materials out of the room. Students should be warned that their scores will be invalidated if they attempt to remove test materials from the room by any method.

Showing Work on Free-Response Sections of the Exams

Students are expected to show all their work. According to the general instructions for Section II of the 2000 Exam,

[Students] will be graded on the correctness and completeness of [their] methods as well as the accuracy of [their] answers. Correct answers without supporting work may not receive credit.

For results obtained using one of the four required calculator capabilities, students are required to write the setup (e.g., the equation being solved, or the derivative or definite integral being evaluated) that leads to the solution, along with the result produced by the calculator. For example, if the student is asked to find the area of a region, the student is expected to show a definite integral (i.e., the setup) and the answer. The student need not compute the antiderivative; the calculator may be used to calculate the value of the definite integral without further explanation. For solutions obtained using a calculator capability other than one of the four required ones, students **must** also show the mathematical steps that lead to the answer. A calculator result is not sufficient. For example, if the student is asked to find a relative minimum value of a function, the student is expected to use calculus and show the mathematical steps that lead to the answer. It is not sufficient to graph the function or use a built-in minimum finder.

When a student is asked to justify an answer, the justification must include mathematical reasons, not merely calculator results. Functions, graphs, tables, or other objects that are used in a justification should be clearly identified.

Activity: AP Problem with a Bit of Everything

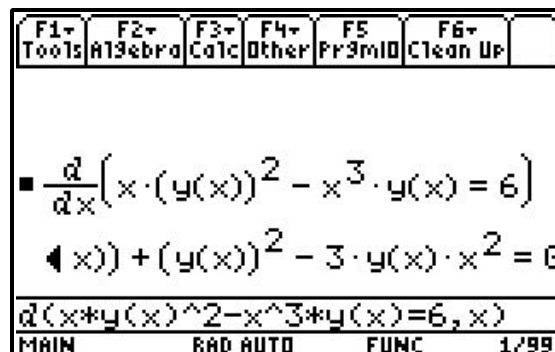
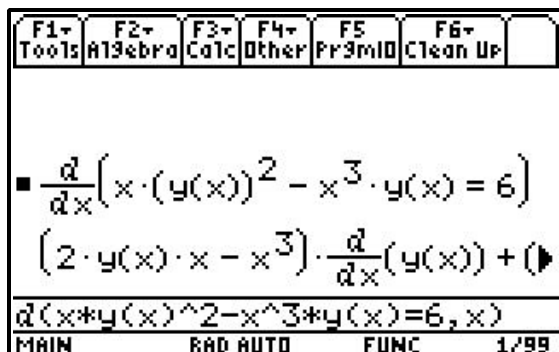
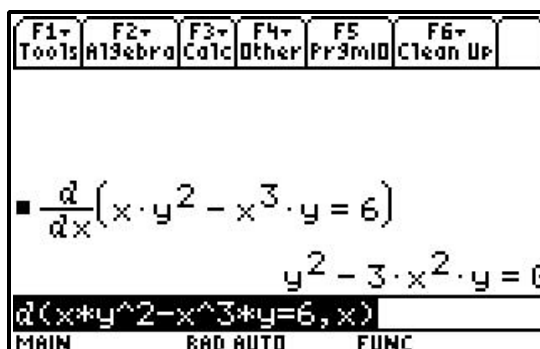
Problem #5 in the 2000 AB and BC Free Response Sections is as follows.

Consider the curve given by $xy^2 - x^3y = 6$.

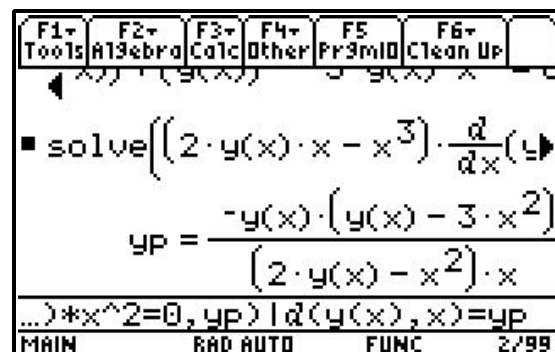
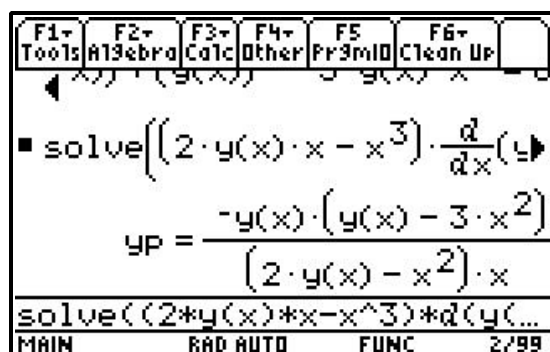
- Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.
- Find all points on the curve whose the x-coordinate is 1, and write an equation for the tangent line at each of these points.
- Find the x-coordinate of each point on the curve where the tangent line is vertical.

Part a presents difficulties for the TI-89's `d` (differentiate) command if you attempt to use it to find $\frac{dy}{dx}$ implicitly. The difficulty here is that the

TI-89 does not recognize that y is a function of x . This difficulty can be avoided by using the variable $y(x)$, which the TI-89 does recognize as a function of x , instead of y .



The TI-89 cannot solve the latter equation for $\frac{d}{dy}(y(x))$ since it does not recognize this as a variable. To handle this difficulty, set $\frac{d}{dy}(y(x))$ equal to some other variable that the TI-89 will recognize (say y_p) and solve the equation for this new variable.



That's almost what they asked for, but not quite. However, using the `expand` command doesn't help, unless that is followed by the `comDenom` command and then all is well.

F1+	F2+	F3+	F4+	F5	F6+
Tools	Algebra	Calc	Other	Pr3mid	Clean Up
$\text{expand}\left(y_p = \frac{-y(x) \cdot (y(x) - 3)}{(2 \cdot y(x) - x^2)}\right)$ $y_p = \frac{5 \cdot x^3}{4 \cdot (2 \cdot y(x) - x^2)} - \frac{y(x)}{2 \cdot x}$					
$\dots 3 \cdot x^2 / ((2 \cdot y(x) - x^2) \cdot x)$					
MAIN		RAD AUTO		FUNC 3/99	

F1+	F2+	F3+	F4+	F5	F6+
Tools	Algebra	Calc	Other	Pr3mid	Clean Up
$\text{comDenom}\left(y_p = \frac{5 \cdot x^3}{4 \cdot (2 \cdot y(x) - x^2)}\right)$ $y_p = \frac{3 \cdot y(x) \cdot x^2 - (y(x))^2}{2 \cdot y(x) \cdot x - x^3}$					
$\dots -x^2) - y(x) / (2 \cdot x) + 5 \cdot x / 4)$					
MAIN		RAD AUTO		FUNC 4/99	

For part b, the SOLVE(command quickly finds all the points whose x-coordinate is 1. In part c, this command can also be used to determine for what values of x and y the tangent line is vertical (assuming the student knows this can occur when the denominator of $\frac{dy}{dx}$ is equal to zero).

F1+	F2+	F3+	F4+	F5	F6+
Tools	Algebra	Calc	Other	Pr3mid	Clean Up
$y_p = \frac{3 \cdot y(x) \cdot x^2 - (y(x))^2}{2 \cdot y(x) \cdot x - x^3}$					
$\text{solve}(x \cdot y^2 - x^3 \cdot y = 6, y) \rightarrow$ $y = 3 \text{ or } y = -2$					
$\dots \text{olve}(x \cdot y^2 - x^3 \cdot y = 6, y) x = 1$					
MAIN		RAD AUTO		FUNC 5/99	

F1+	F2+	F3+	F4+	F5	F6+
Tools	Algebra	Calc	Other	Pr3mid	Clean Up
$\text{solve}(x \cdot y^2 - x^3 \cdot y = 6, y) \rightarrow$ $y = 3 \text{ or } y = -2$					
$\text{solve}(2 \cdot x \cdot y - x^3 = 0, y)$ $y = \frac{x^2}{2} \text{ or } x = 0$					
$\text{solve}(2 \cdot x \cdot y - x^3 = 0, y)$					
MAIN		RAD AUTO		FUNC 6/99	

For the solution $y = \frac{x^2}{2}$, substituting for y in the original equation and then solving for x should produce the desired answers. For the solution $x = 0$, hopefully students recognize the need to substitute $x = 0$ back into the original equation to verify that this solution is valid. Doing so shows such is not the case.

F1+	F2+	F3+	F4+	F5	F6+
Tools	Algebra	Calc	Other	Pr3mid	Clean Up
$x \cdot y^2 - x^3 \cdot y = 6 x = 0$ false					
$\text{solve}(x \cdot y^2 - x^3 \cdot y = 6, x) \rightarrow$ $x = -3 \frac{1}{5}, 2 \frac{3}{5}$					
$\dots x \cdot y^2 - x^3 \cdot y = 6, x) y = x^2 / 2$					
MAIN		RAD AUTO		FUNC 8/99	

F1+	F2+	F3+	F4+	F5	F6+
Tools	Algebra	Calc	Other	Pr3mid	Clean Up
$\text{solve}(2 \cdot x \cdot y - x^3 = 0, y)$ $y = \frac{x^2}{2} \text{ or } x = 0$					
$x \cdot y^2 - x^3 \cdot y = 6 x = 0$ false					
$x \cdot y^2 - x^3 \cdot y = 6 x = 0$					
MAIN		RAD AUTO		FUNC 7/99	

So what is the point of all this work with the TI-89 considering that many of the uses made of the TI-89 in this activity are “illegal” on the AP Exam? The point is that it allows students to focus on coming up with the correct set-up, equations, and justifications without having to worry about doing the symbol manipulation correctly. The necessary symbol manipulation skills can be handled separately. It also points out that use of the TI-89 to do some of the symbol manipulation is more difficult or time-consuming than doing it by-hand.

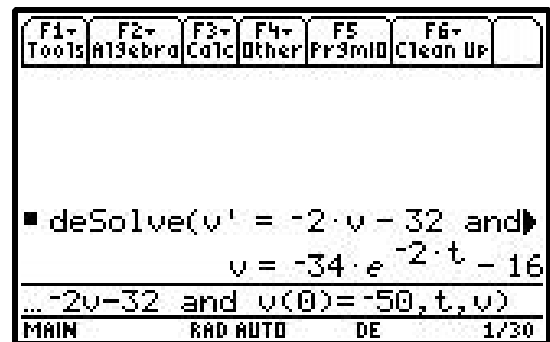
Activity: A Couple More AP Problem of Interest

Below are the sequence of steps one can perform on the TI-89 to “solve” parts of two more problems on recent AP Exams.

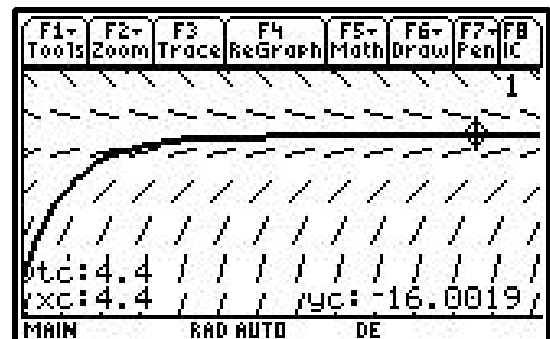
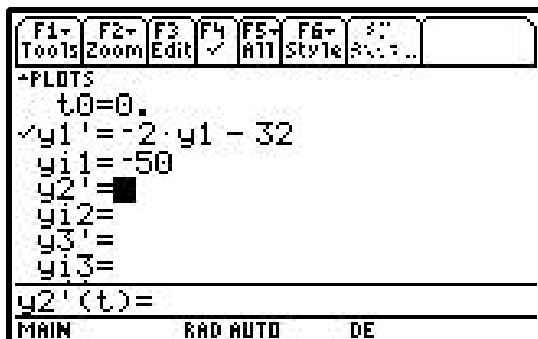
1997 BC Exam #6

Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t = 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$ with initial condition $v(0) = -50$.

- Use separation of variables to find an expression for v in terms of t , where t is measured in seconds.
- Terminal velocity is defined as $\lim_{t \rightarrow \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second



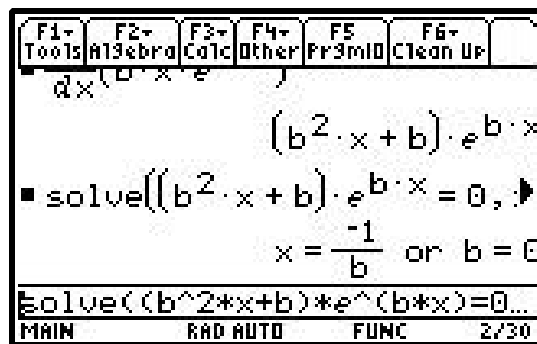
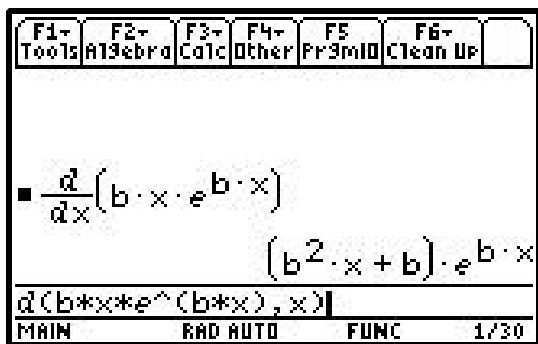
Use $\text{deSolve}()$ command (rather than separation of variables) to find the equation for $v(t)$.



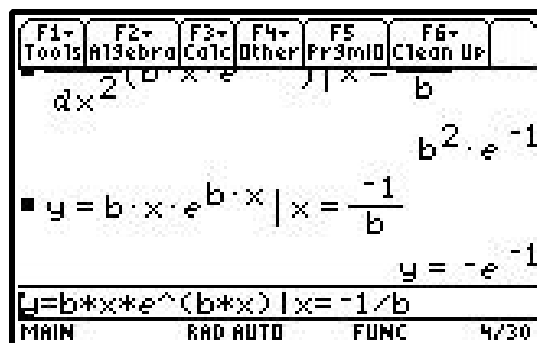
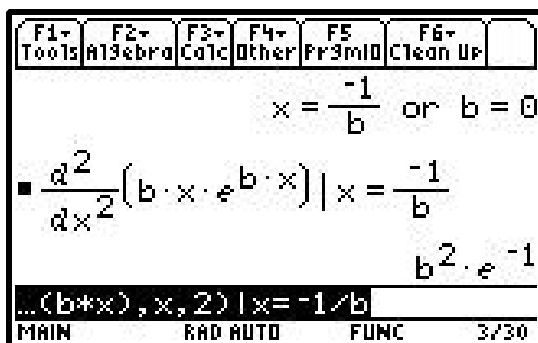
Graph the differential equation (in DIFF EQUATIONS mode) to estimate the terminal velocity.

1998 BC Exam #2d

Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b .



Determine where the derivative of $y = bxe^{bx}$ is equal to 0.



Show that y'' is always positive and that the value of y where y' is 0 is always the same.

This activity sheet is designed to find a general rule for differentiating the composition of two or more functions. As a review, let's consider some of the rules for derivatives we have seen so far that involve variable expressions other than just x . Fill in each of the following (assume k is a constant).

$$\begin{array}{ll}
 y = (x + k)^n \Rightarrow y' = \underline{\hspace{2cm}} & y = (kx)^n \Rightarrow y' = \underline{\hspace{2cm}} \\
 y = e^{x+k} \Rightarrow y' = \underline{\hspace{2cm}} & y = e^{kx} \Rightarrow y' = \underline{\hspace{2cm}} \\
 y = \ln(x + k) \Rightarrow y' = \underline{\hspace{2cm}} & y = \ln(kx) \Rightarrow y' = \underline{\hspace{2cm}} \\
 y = \sin(x + k) \Rightarrow y' = \underline{\hspace{2cm}} & y = \sin(kx) \Rightarrow y' = \underline{\hspace{2cm}}
 \end{array}$$

Now let's step things up a bit. Fill in each blank with what you believe will be the derivative of the given function. Then in the space provided, write in the derivative provided by the TI-89/92 (hopefully they match).

	Your derivative	TI-89/92 derivative
(1.1) $y = (3x + 4)^{10}$	$y' = \underline{\hspace{2cm}}$	$y' = \underline{\hspace{2cm}}$
(1.2) $y = \frac{2}{(5x - 7)^4}$	$y' = \underline{\hspace{2cm}}$	$y' = \underline{\hspace{2cm}}$
(1.3) $y = (x^2 - 7x + 1)^5$	$y' = \underline{\hspace{2cm}}$	$y' = \underline{\hspace{2cm}}$
(1.4) $y = \sqrt{x^3 + 1}$	$y' = \underline{\hspace{2cm}}$	$y' = \underline{\hspace{2cm}}$
(2.1) $y = 2^{(3x+7)}$	$y' = \underline{\hspace{2cm}}$	$y' = \underline{\hspace{2cm}}$
(2.2) $y = e^{(4x^3+2x)}$	$y' = \underline{\hspace{2cm}}$	$y' = \underline{\hspace{2cm}}$
(2.3) $y = \ln(7 - 5x)$	$y' = \underline{\hspace{2cm}}$	$y' = \underline{\hspace{2cm}}$
(2.4) $y = \ln\left(\sqrt[4]{x^3}\right)$	$y' = \underline{\hspace{2cm}}$	$y' = \underline{\hspace{2cm}}$

	Your derivative	TI-89/92 derivative
(3.1) $y = \cos(11x + 5)$	$y' =$ _____	$y' =$ _____
(3.2) $y = \sec(9 - 3x)$	$y' =$ _____	$y' =$ _____
(3.3) $y = \sin\left(\frac{-2}{x^3}\right)$	$y' =$ _____	$y' =$ _____
(3.4) $y = \tan(x^3 + 4x + 1)$	$y' =$ _____	$y' =$ _____
(4.1) $y = (\ln(x))^5$	$y' =$ _____	$y' =$ _____
(4.2) $y = \cos^7(x)$	$y' =$ _____	$y' =$ _____
(4.3) $y = \sqrt[3]{\tan(x)}$	$y' =$ _____	$y' =$ _____
(4.4) $y = \frac{5}{\ln^3(x)}$	$y' =$ _____	$y' =$ _____
(5.1) $y = e^{\tan x}$	$y' =$ _____	$y' =$ _____
(5.2) $y = \ln(\sec x)$	$y' =$ _____	$y' =$ _____
(5.3) $y = \cos(e^x)$	$y' =$ _____	$y' =$ _____
(5.4) $y = 5 \sin^3(2x)$	$y' =$ _____	$y' =$ _____

Given what you have seen so far, write a description of how you could take the derivative of the composite of two functions. Then complete the statement at the bottom of the page.

Chain Rule: If $k(x) = f(g(x))$, then $k'(x) =$