Monopoly - Part 1

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Answers

7 8 9 **10** **11** 12

TI-Nspire™ | Activity | Student | 90 min

Introduction

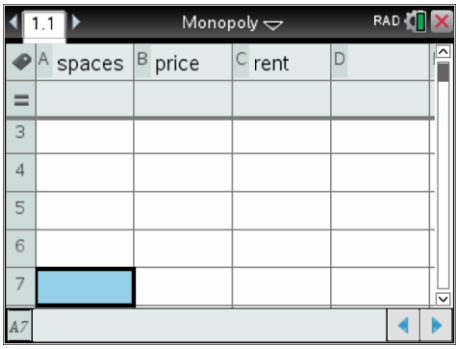
Monopoly™ is one of the most popular board games in the world. While the word monopoly means 'exclusive possession or control', a quick internet search of the word most frequently references the game than the definition. When playing the game, most people have their favourite property set to collect; without doubt, the most popular property set is the blue pair adjacent to Go. At $350 and $400 they are the most expensive on the board, but do these properties represent value for money? If you're going to be a property tycoon, then it is worth doing your homework first. This investigation looks at different ways to explore which properties on the Monopoly board represent the best value for money.

Equipment

- Monopoly Board
- TI-Nspire Calculator

Instructions

Start a new TI-Nspire document and insert a spreadsheet.

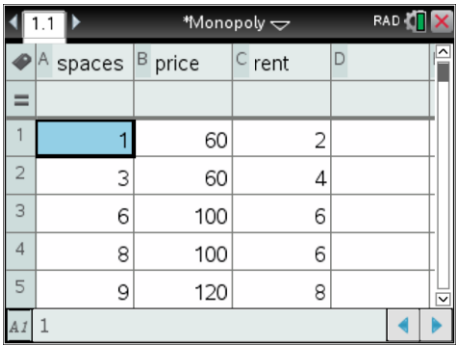


Create the following list names:

- Spaces
- Price
- Rent

Save the file as: "Monopoly"

In this investigation ONLY property prices will be recorded, utilities and railroads will not be included. The "Go" square represents 0, so the first property appears on the first space, the second property on the third and so on.



Record the location (spaces), price and rent for all the available properties on the board. The first side of the board has been completed in the spreadsheet shown opposite.

Author: P. Fox

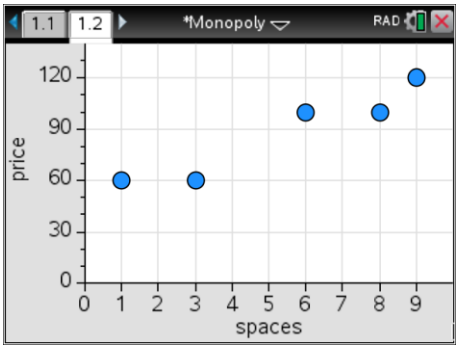
Question: 1.

Describe the general trend for property prices and rental return as a player progresses around the board.

The further a player moves around the board the higher the property price.

Insert a Data & Statistics application and produce a scatter plot with ***spaces*** on the independent (horizontal) axis and ***price*** on the dependent (vertical) axis.

Data for the first side of the board is shown opposite, the axis have been adjusted using the Window Zoom option so as to include the origin.

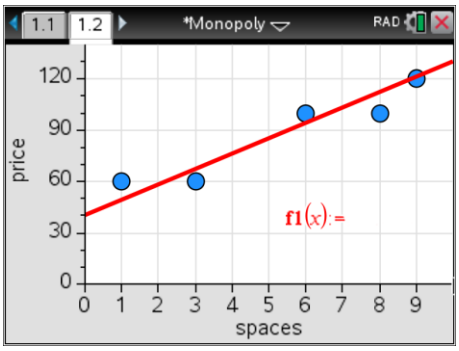


By default the graph zooms in on the data being displayed. For this data, zero will not be displayed on the *price* or *space* axes. Use the Window - Zoom option to adjust x-min and y-min to include zero. Including the zero on both axes helps identify the relative location of the data points and therefore understand the relationship.

Imagine a straight line passing as close as possible to all the data points. (It is not possible for a single straight line to pass through all the points).

Determine, by approximation only, the equation to a line that would best represent or fit the data. This equation can be changed manually by experimentation.

A line representing the first couple of data points has been shown opposite; the equation itself has been hidden.



Question: 2.

Explain how you determined your original equation, the subsequent adjustments you made and record your final equation.

(ie: What estimates were made to help determine the gradient and y intercept of the original equation?)

Answers will vary: Typical strategies include using the price of Mayfair (or its equivalent) as $400 being 40 spaces from Go and subsequently estimating the gradient as 10. This approach assumes prices are directly proportional to spaces. The data clearly shows that Go is not equivalent to a property value of $0.00, so the final gradient estimates should be less than 10.

- Using the first and last property values with their respective locations produces a gradient of approximately 8.7
- Averaging the first property set price and location and applying a similar strategy to the last property set produces a gradient of approximately 8.5

Author: P. Fox

- Both of the above strategies assume that Mayfair and Parklane form a reasonable fit, applying the above strategies to other property groups produces a gradient much closer to a value of 8.
- Using the gradient as a value between 8 and 10 means the y – intercept should fall within the range: $30 to $52. These values can be achieved by using reasonable gradient values (as described above) and extrapolating back to 'Go' (y – intercept) from combinations of the first three spaces on the board.
  A final equation of best fit, estimated by students should appear close to: $y = 8.5x + 43$

By default the plotted function is recorded in f1(x). This function can be called upon in any application within this problem. For example in the calculator page type: f1(11) to estimate the value of the first property on the second side of the board.

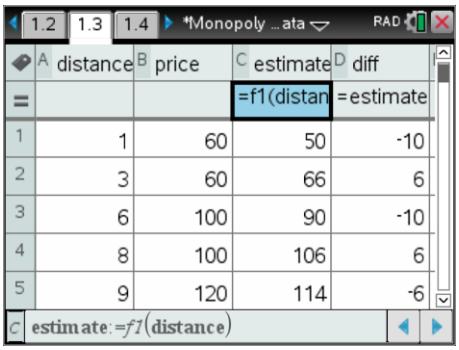
Question: 3.

Reproduce the table below for **all sides** of the Monopoly board. Record the actual and estimated property values (estimated from your rule) in the table. [Difference = Estimate - Actual]

Spaces	1	3	6	8	9	11	13	14
Actual	60	60	100	100	110	140	140	160
Estimate								
Difference								

Answers will vary depending on the student's equation. Students should consider using technology, as shown opposite to determine the result, this will also make Question 4 easier!

The example shown uses: $f_1(x) = 8x + 42$

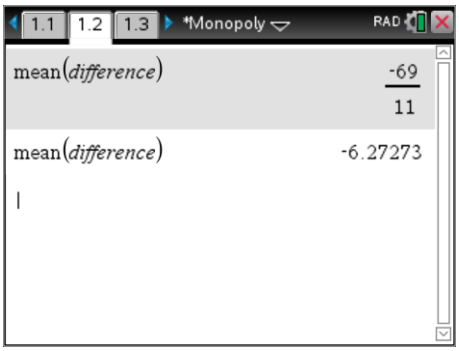


Question: 4.

Calculate and record the 'mean' difference. Explain why this calculation is not a true representation of the accuracy of your line.

Answers will vary depending on the student's equation. Students can use the calculator application to determine the mean (average) difference.

A quick look through the difference list shows that almost all differences are 6 or higher. The negative values reduce the mean and in the case shown opposite make the difference negative.



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Question: 5.

Square all the differences, compute the *mean* of the squared values and then *square-root* the result.

(i) Record this new computed value.

Answers will vary, the value can be computed using: mean(difference^2) from the calculator application when the differences have been computed in a list and labelled accordingly. Typically this value will be of the order 150+, after square-rooting the result, approximately: 12.4

(ii) Explain the effect that 'squaring' has on the differences.

Squaring removes the negatives, but the process also makes the numbers much larger. Visually the result would be to produce a 'square' where the side length is measured from the actual point to the line… 'a residual square'.

(iii) Explain why the 'square-root' step is included.

The square-root is designed to 'undo' the squaring operation.

Question: 6.

Return to the Statistics application. From the **Analyse** menu, select **Residuals** followed by **Show Residual Plot**. Move the cursor over the data points on this plot. What does this plot represent?

Click on a point in the residual plot to see the corresponding point in the scatter plot.

The residual plot shows how far the actual data point is compared with the value determined by the defined rule.

Question: 7.

From the **Analyse** menu select **Regression** followed by **Linear Regression.** Explain what this line does and compare it with your line including specific reference to the 'residuals'.

The regression line is a statistical measure of the 'line of best fit' by minimising the residuals. (Minimises the sum of the squared residuals).

Question: 8.

Does the relationship between the property price and distance from 'Go' provide any information about which property represents the best value?

No. The purpose of this question is to highlight that just because a correlation exists, doesn't give it meaning. In contrast, actual property prices decrease as you move further away from the CBD (Central Business District). An investigation into this is provided in the "House Prices" activity.

Author: P. Fox

Question: 9.

Investigate the relationship between property price and rental return.

(i) Determine an equation of best fit for the data and explain the practical meaning of the equation by use of examples.

Answers will vary. The regression equation is: $y = 0.12x - 6.3$

The 0.12 (gradient) and -6.3 (y intercept) mean that the rental return is approximately 12% of the property value less approximately $6. For example the rent on Trafalgar Square ($240) should be approximately $22. The actual rent is $20.

(ii) If a property lies below the line of best fit, what does it suggest about either its price or rental return.

If the property lies below the line the rental return is below the estimated value, so either the price is too high or the return is too low.

(iii) As properties occur in groups, which property group represents the best value for money? Justify your answer.

This problem can be explored in a number of ways. One way is to add all the property prices within the group and then compare this amount with the rental return (doubled when all properties are owned) for each property within the group. This approach simulates a player's financial commitment to purchase the property group followed by an opponent landing on a specific property within the group. This is a very practical approach but also provides a level of bias toward the first and last property groups as there are only two properties required to make a set. This approach yields 7.5 (Mayfair) and 10.7 (Park Lane). These values interpret as: "If you bought both of these properties an opponent would need to land on Mayfair 7.5 times to repay the total investment or 10.7 times for Park Lane." These returns are better than any other property group. The next best property on the board is "Whitechapel Road" with a yield value of 15, however "Old Kent" which is in the same property group has the worst yield on the board at 30. This outcome suggests that an 'averaging' needs to be considered.

So, an alternative way of looking at the problem is to determine the average property price for the group and the average rental return for the group. The pattern in these results is very clear, the further you move away from go the better the rental return with Park Lane and Mayfair topping the list once again. The average property price is $375 and the average rent (using double when both properties are owned) is $85. The ratio in this case is 4.4, however the practical meaning of this value is more difficult to interpret. "An opponent would need to land on both properties 4.4 times to repay the total investment." This scheme makes the first duo (Old Kent and Whitechapel) the slowest to repay the original investment!

Both of the above methods conclude that the final property group (Mayfair and Park Lane) are the best property group in which to invest, however it does considerably shift the importance of the other property groups.

Author: P. Fox

(iv) If the railroad sites were changed to 'properties' determine the following:

a. Individual property price for each railroad, based on its location.
Based on the location, the equation for property price as a function of the distance from Go (spaces) should be used. Prices will vary depending on whether students have used a linear regression or their own equation.
Example: $y = 8x + 42$

Kings Cross: (5 spaces) = $82.00

Marylebone: (15 spaces) = $182.00

Fenchurch: (25 spaces) = $282.00

Liverpool: (35 spaces) = $382.00

b. Rental return for each railroad, based on its price.

Based on the property price … (prices will vary depending on the equation used.)
Example: $y = 0.12x - 6.3$

Kings Cross: ($82.00) = $3.54

Marylebone: ($182.00) = $15.54

Fenchurch: ($282.00) = $27.54

Liverpool: ($382.00) = $39.54

Notes to Teachers

There are numerous other investigations worth exploring. Consider for example exploring capital improved values, where houses and hotels are placed on a property group, is there an ideal number of houses for any given property group that maximises the return on dollars spent?

From a probability perspective, are there any squares on the board that have a greater likelihood of being landed upon? "Advance token to nearest railroad" and "Advance token to Pall Mall" appear on Chance and Community Chest cards. Does this impact on the value of the property in regards to rental return? What about "Go to Jail"? Even though this doesn't return any rent; Bow, Marlborough and Vine streets are 6, 8 and 9 spaces away respectively and these numbers collectively have more than a 33% chance of appearing when the two dice are rolled!

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