

## Radian Measure — An Analytic Approach

by – Paul W. Gosse

### Activity overview

*This activity describes the ratio between distance traveled along the unit circle and its radius, and the ratio between distance traveled along a circle of variable radius and its radius. Students will develop the relationship 1 radian is approximately 57.3 degrees, and should be able to generalize that 2 pi radians is 360 degrees. What sets this activity apart is it utilizes the arcLen( command, found in the Calculations-Calculus menu as Arc Length, which analytically measures the distance along a curve in the process of completing the activity. Using arcLen(, students can generate the pi-circumference relationship using the tools of calculus. Finally, creating a scatterplot of radian measure versus degrees from an interactive,*

*analytic construction, allows students to legitimately develop  $1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ$  from real-time*

*measurements with a scatterplot and linear regression. Since the actual use of the integral which calculates arc length is 'hidden' in this activity (as an integral at least), this activity is suitable for students encountering radian measure for the first time.*

### Concepts

*Equation of circle center origin radius r, radian measure, pi, arc length.*

### Teacher preparation

*Students should understand the functional form for the equation of a semi-circle with center the origin and radius r. The command arcLen( is used (it is embedded in a spreadsheet and lightly explained in the activity) however no calculus is needed to interact with this activity. A TI-Nspire CAS system, though, is. Note: the arcLen( command does not easily lend itself to working with lists, multiple inputs of any kind, or data captures. Hence, the formulae, seen in the spreadsheet were inserted in c1 and d1 and filled downward to c10 and d10. Should you desire further data captures, simply fill the formula down further.*

### Classroom management tips

*This activity independently develops the relationship between radian and degree measure as an investigative and data-gathering exercise. It would be useful for students to work in pairs in order to exchange insights as they work through the activity.*

### TI-Nspire Applications

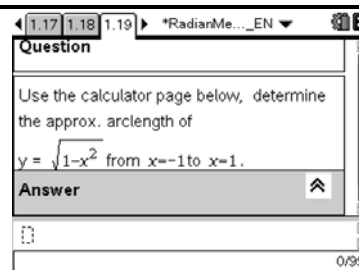
*Notes (including Q&A), G&G, L&S, D&S, Calculator.*

### Step-by-step directions

*The activity is self-explanatory and is an exploration.*

*Note: Arc length is a calculus topic involving an integral. In this activity, the presence of the integral is invisible and students can focus on arcLen( strictly as a function which returns the distance along a curve from one x value to another.*

Page 1.19 is particularly relevant as students determine that the arc length of  $y = \sqrt{1-x^2}$  from  $-1$  to  $1$  is  $\pi$ . In other words, the ratio of the distance along the unit semi-circle from  $-1$  to  $1$ , to its diameter (the distance from  $-1$  to  $1$ ) is  $\frac{\pi}{2}$ . Hence, the ratio of the distance along the whole unit circle to its diameter is  $\pi$ .



## Assessment and evaluation

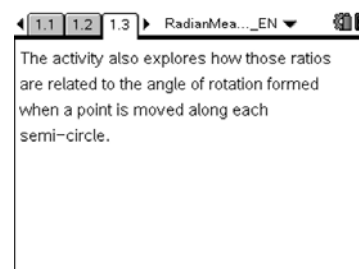
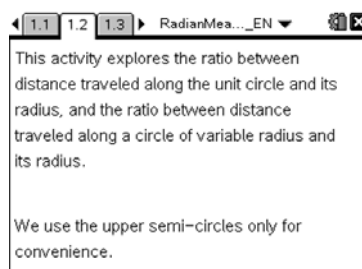
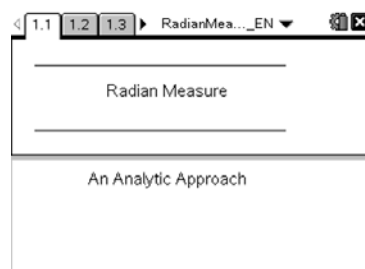
- This activity is exploratory in nature. Therefore, assessment could consist of a journal activity inquiring about what was learned about radian measure versus degrees. Students could be asked to write the approximate and exact radian, or degree equivalent, of a given angle measure.
- Answers: [1.13]  $r_1$  and  $r_2$  are the same. It appears as if, for the same angle of rotation, that the ratio of arc length along the circumference of a circle to the radius is constant; [1.14] the relationship appears linear with slope 57.3 and y-intercept 0; [1.18] It represents half the circumference of a circle radius 1; [1.19] The result should be about 3.14159; [1.20]  $\pi$ ; [1.21]  $\pi$  is the name, and Greek letter, assigned to the ratio of circumference around any circle and its diameter. On a circle radius 1, the semi-circumference is  $\pi$  and the full circumference is  $2\pi$ .

## Activity extensions

- Arc length is calculated using integral calculus. As an extension, students could explore how this integral works.
- Students who have explored the limit of  $\sin(x)$  over  $(x)$  as  $x$  approaches 0 could explore whether the ratio of the arc lengths of  $y=\sin(x)$  and  $y=(x)$  between  $x = 0$  and  $x = a$ , as  $a$  approaches 0, approach the same limit as the ratio of the functions.

## Student TI-Nspire Document

*RadianMeasure\_AnalyticApproach\_EN.tns*



1.2 1.3 1.4 RadianMea...\_EN

The arc length command,  $\text{arclen}()$ , determines the distance a point actually travels when moved along a function from  $x=a$  to  $x=b$ .

It is available using the **Calculus** button on a calculator page. The actual syntax is:

$\text{arclen}(\text{function rule or name, variable, start value, stop value})$ .

1.3 1.4 1.5 RadianMea...\_EN

Although it seems as if  $\text{arclen}()$  would apply only to a circle, try it using  $y=x$  from  $x=0$  to  $x=1$  by running the command below (press Enter). Then try  $y=2x$  on the same domain. You should be able to understand each result.

$\text{arclen}(x,x,0,1)$

0/99

1.4 1.5 1.6 RadianMea...\_EN

The two semi-circles used in this activity have been defined using the commands below.

Define  $f1(x)=\sqrt{1-x^2}$  Done

Define  $f2(x)=\sqrt{\text{rad}^2-x^2}$  Done

2/99

1.5 1.6 1.7 RadianMea...\_EN

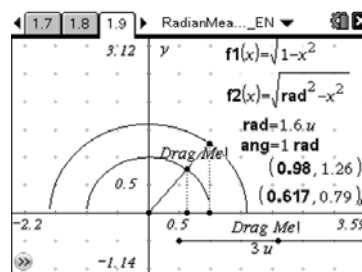
Visit page 1.9 (noting the points below). Then go to page 1.8.

- A slider is set up on the bottom right to vary **rad** (the variable radius of the second circle) from 0 to 3 units. It is initially set at about 2 units.
- The coordinates of a point on each semi-circle are shown as well as the actual radius and angle of rotation.

1.6 1.7 1.8 RadianMea...\_EN

Drag the indicated point on the unit circle and note the updated values on the screen. Vary **rad** using the lower slider and drag the indicated point again noting the updated values on screen.

When done, slide **rad** back to about 2 units and the angle to around  $20^\circ$  and go to page 1.10.



1.8 1.9 1.10 RadianMea...\_EN

Page 1.12 is a spreadsheet set up to manually capture the x-coord. of the rotating points as **x1** & **x2**, and the angle of rotation **a**, for different locations of the drag point.

The spreadsheet will also calculate the ratio (**r1** & **r2**) of arc length to radius for each semi-circle using the  $\text{arcLen}()$  command.

1.9 1.10 1.11 RadianMea...\_EN

Being careful not to move too near the 'edges' of each semi-circle so values 'disappear', drag the indicated point pressing **ctrl+**. (Ctrl and period at the same time) to manually capture data.

Do this 10 times in total which will fill the spreadsheet with 10 captured and calculated values.

1.10 1.11 1.12 RadianMea...\_EN

|   | A x1           | B x2           | C r1 | D r2 |
|---|----------------|----------------|------|------|
| 1 | =capture('x1') | =capture('x2') |      |      |
| 2 |                |                |      |      |
| 3 |                |                |      |      |
| 4 |                |                |      |      |
| 5 |                |                |      |      |

A1

1.11 1.12 1.13 RadianMea...\_EN

**Question**

What do you notice about the values for **r1** & **r2**?

What conclusion can you draw?

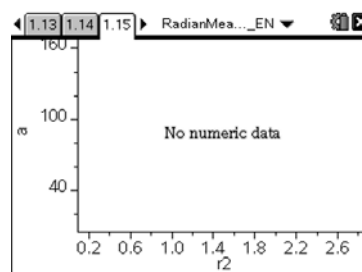
**Answer**

1.12 1.13 1.14 RadianMea...\_EN

**Question**

The next page shows a scatterplot of angle of rotation, **a**, against **r2** (which we now know is the same as **r1** for all values of **a**). Use **ZoomData** if necessary. What do you notice?

**Answer**



1.14 1.15 1.16 RadianMea...\_EN

Using the previous page, run an appropriate regression analysis.

How is the angle of rotation related to the arclength to radius ratio for any semi-circle?

1.15 1.16 1.17 RadianMea...\_EN

Extension

Discovering  $\pi$

1.16 1.17 1.18 RadianMea...\_EN

**Question**

What does the arclength of  $y = \sqrt{1-x^2}$  from  $x=-1$  to  $x=1$  represent?

**Answer**

1.17 1.18 1.19 \*RadianMe...\_EN

**Question**

Use the calculator page below, determine the approx. arclength of  $y = \sqrt{1-x^2}$  from  $x=-1$  to  $x=1$ .

**Answer**

0/99

1.18 1.19 1.20 \*RadianMe...\_EN

**Question**

The approximate value 3.14159 is an approximation of what famous mathematical constant?

**Answer**

1.19 1.20 1.21 \*RadianMe...\_EN

**Question**

How is your answer to the previous question related to the formula  $C=\pi d$ ?

**Answer**