TI-*nspire*™

Grade level: 9-12
Subject: mathematics
Time required: 45 minutes

Radian Measure — An Analytic Approach

by – Paul W. Gosse

Activity overview

This activity describes the ratio between distance traveled along the unit circle and its radius, and the ratio between distance traveled along a circle of variable radius and its radius. Students will develop the relationship 1 radian is approximately 57.3 degrees, and should be able to generalize that 2 pi radians is 360 degrees. What sets this activity apart is it utilizes the arcLen(command, found in the Calculations-Calculus menu as Arc Length, which analytically measures the distance along a curve in the process of completing the activity. Using arcLen(, students can generate the pi-circumference relationship using the tools of calculus. Finally, creating a scatterplot of radian measure versus degrees from an interactive, analytic construction, allows students to legitimately develop 1 rad = $\left(\frac{180}{\pi}\right)^{\circ}$ *from real-time measurements with a scatterplot and linear regression. Since the actual use of the integral which calculates arc length is 'hidden' in this activity (as an integral at least), this activity is suitable for students encountering radian measure for the first time.*

Concepts

Equation of circle center origin radius r, radian measure, pi, arc length.

Teacher preparation

Students should understand the functional form for the equation of a semi-circle with center the origin and radius r. The command arcLen(is used (it is embedded in a spreadsheet and lightly explained in the activity) however no calculus is needed to interact with this activity. A TI-Nspire CAS system, though, is. Note: the arcLen(command does not easily lend itself to working with lists, multiple inputs of any kind, or data captures. Hence, the formulae, seen in the spreadsheet were inserted in c1 and d1 and filled downward to c10 and d10. Should you desire further data captures, simply fill the formula down further.

Classroom management tips

This activity independently develops the relationship between radian and degree measure as an investigative and data-gathering exercise. It would be useful for students to work in pairs in order to exchange insights as they work through the activity.

TI-Nspire Applications

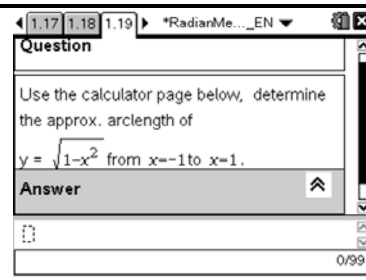
Notes (including Q&A), G&G, L&S, D&S, Calculator.

Step-by-step directions

The activity is self-explanatory and is an exploration.

Note: Arc length is a calculus topic involving an integral. In this activity, the presence of the integral is invisible and students can focus on arcLen(strictly as a function which returns the distance along a curve from one x *value to another.*

Page 1.19 is particularly relevant as students determine that the arc length of $y=\sqrt{1-x^2}$ *from* -1 *to* 1 *is* π*. In other words, the ratio of the distance along the unit semi-circle from* -1 *to* 1*, to its diameter (the distance from* -1 *to* 1*)* *is* $\frac{\pi}{2}$*. Hence, the ratio of the distance along the whole unit circle to its diameter is* π*.*



Assessment and evaluation

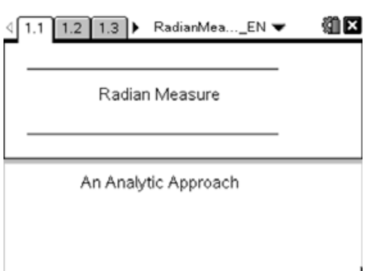
- *This activity is exploratory in nature. Therefore, assessment could consist of a journal activity inquiring about what was learned about radian measure versus degrees. Students could be asked to write the approximate and exact radian, or degree equivalent, of a given angle measure.*
- *Answers: [1.13] r1 and r2 are the same. It appears as if, for the same angle of rotation, that the ratio of arc length along the circumference of a circle to the radius is constant; [1.14] the relationship appears linear with slope 57.3 and y-intercept 0; [1.18] It represents half the circumference of a circle radius 1; [1.19] The result should be about 3.14159; [1.20] pi; [1.21] Pi is the name, and Greek letter, assigned to the ratio of circumference around any circle and its diameter. On a circle radius 1, the semi-circumference is pi and the full circumference is 2pi.*

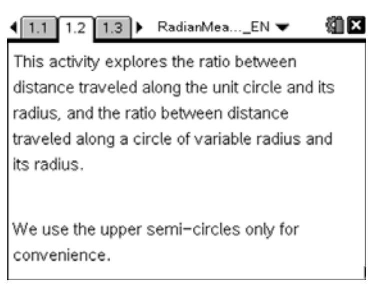
Activity extensions

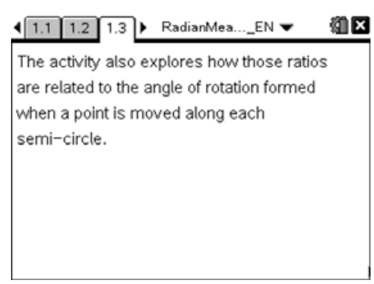
- *Arc length is calculated using integral calculus. As an extension, students could explore how this integral works.*
- *Students who have explored the limit of sin(x) over (x) as x approaches 0 could explore whether the ratio of the arc lengths of y=sin(x) and y=(x) between x = 0 and x = a, as a approaches 0, approach the same limit as the ratio of the functions.*

Student TI-Nspire Document

RadianMeasure_AnAnalyticApproach_EN.tns

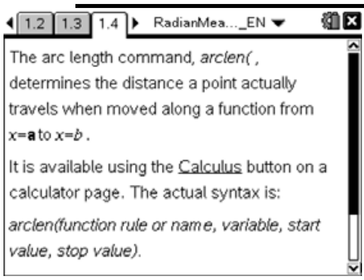


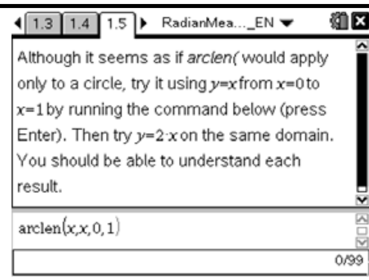


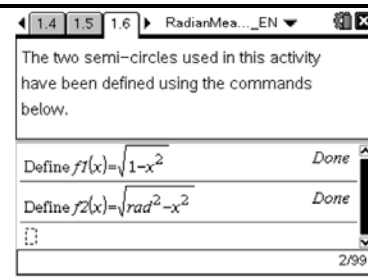


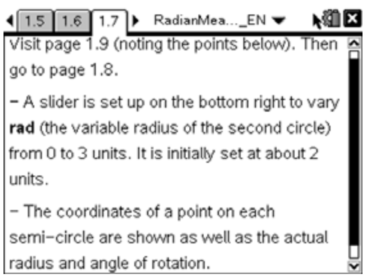
by: Paul W. Gosse
Grade level: secondary
Subject: mathematics
Time required: 45 to 90 minutes

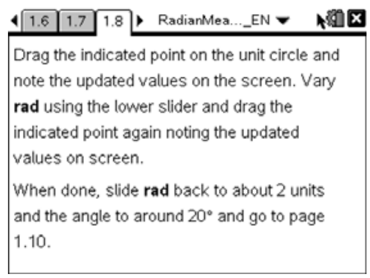
Materials: TI-Nspire CAS

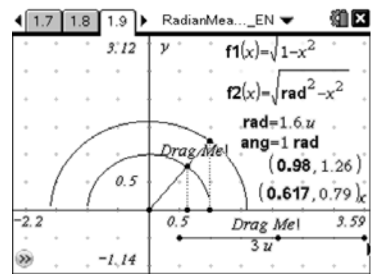


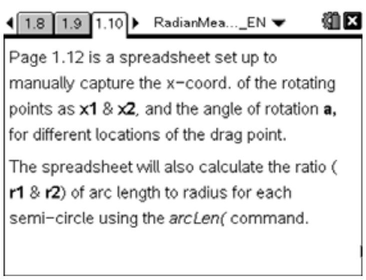


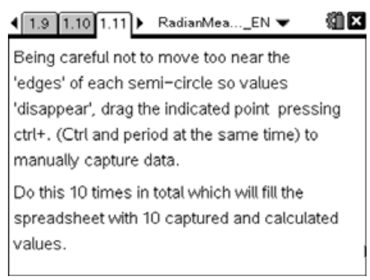


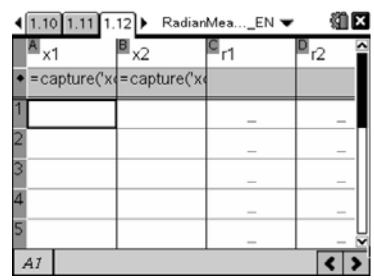


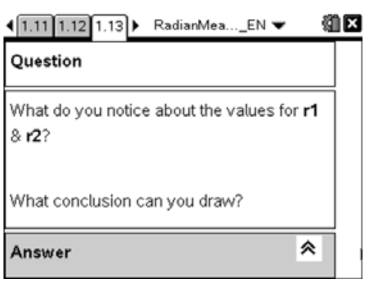


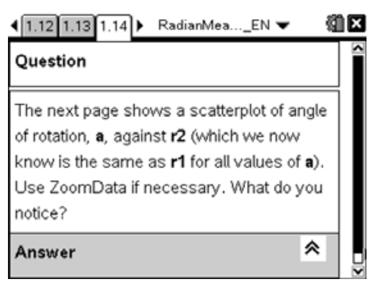


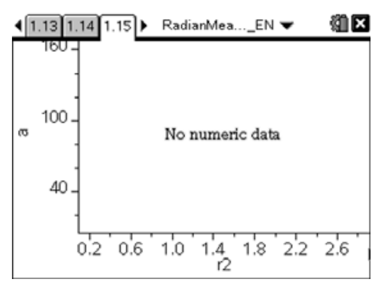


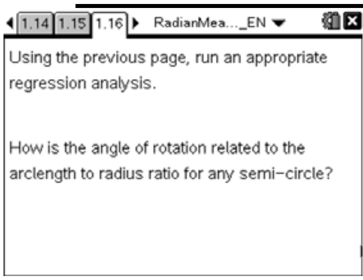


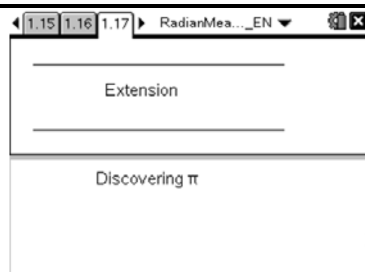


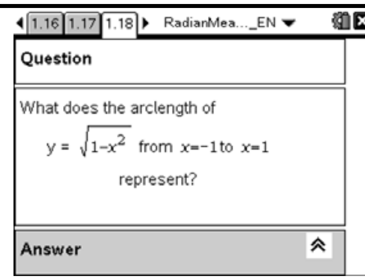


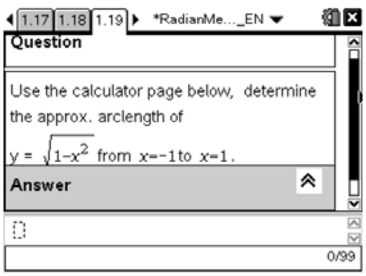


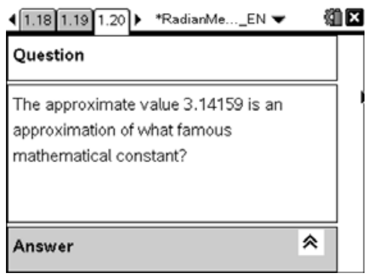


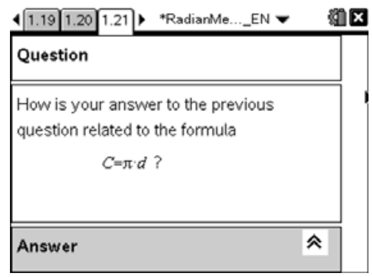

1.14 1.15 1.16 RadianMea..._EN
Using the previous page, run an appropriate regression analysis.
How is the angle of rotation related to the arclength to radius ratio for any semi−circle?


1.15 1.16 1.17 RadianMea..._EN
Extension
Discovering π


1.16 1.17 1.18 RadianMea..._EN
Question
What does the arclength of
y = $\sqrt{1-x^2}$ from x=−1 to x=1
represent?
Answer


1.17 1.18 1.19 *RadianMe..._EN
Question
Use the calculator page below, determine the approx. arclength of
y = $\sqrt{1-x^2}$ from x=−1 to x=1.
Answer
0/99


1.18 1.19 1.20 *RadianMe..._EN
Question
The approximate value 3.14159 is an approximation of what famous mathematical constant?
Answer


1.19 1.20 1.21 *RadianMe..._EN
Question
How is your answer to the previous question related to the formula
C=π·d ?
Answer