E X P L O R A T I O N S ™

Activity 1

Objectives

- Use the Probability Simulation Application (App) to roll a fair die to explore the Law of Large Numbers informally

- Explore features of the Probability Simulation App

Law of Large Numbers:
Equal Opportunities

Introduction

How many rolls of a fair, six-sided die do you think it would take so that each face turns up the same number of times? Do you think if you rolled the die six times that each face would come up exactly once? If you rolled the die twelve times, would each face come up twice? Or do you think you would need to roll the die many more times than that, say 36, 360, or 3,600 times?

In this activity, you conduct probability experiments that involve tossing a fair die. By graphing the experimental results obtained each time, you can compare the experimental probability of each outcome to its theoretical probability and see how the Law of Large Numbers affects the results.
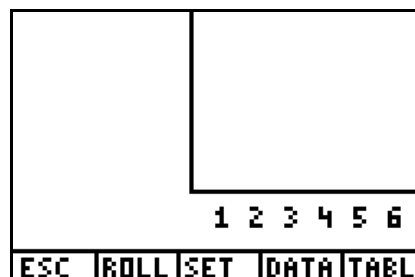
Problem

How many times can you roll a fair, six-sided die so that the frequencies of the six outcomes are equal (or nearly equal)? In other words, how large is "large" in the Law of Large Numbers?

Exploration

Getting Started

1. Press APPS, select **Prob Sim**, and then press any key to display the **Simulation** menu.

```
Simulation
1.Toss Coins
2.Roll Dice
3.Pick Marbles
4.Spin Spinner
5.Draw Cards
6.Random Numbers
 OK      OPTN ABOUT QUIT
```

2. Select **Roll Dice**.

3. Select **ROLL**.

The animation shows the rolling of a die and the result of the roll. A bar appears above the number rolled on the corresponding bar graph. The height of the bar indicates its frequency, which in this case is equal to 1.
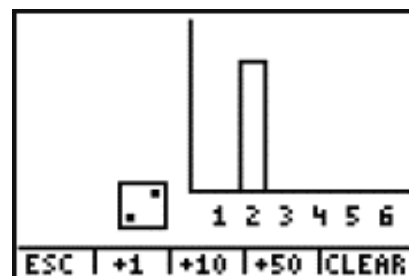
Notice the options that appear at the bottom of the screen. You can select **+1**, **+10**, or **+50** to continue to roll one, 10, or 50 rolls at a time.

The experimental results appear on the bar graph. When you roll multiples of 10 and 50, a counter appears briefly to show the total number of rolls represented by the bars on the graph.

4. Select **CLEAR** to clear the first roll.

5. Select **YES** to confirm that you want to clear the first roll, and then select **+10** to roll the die ten times.
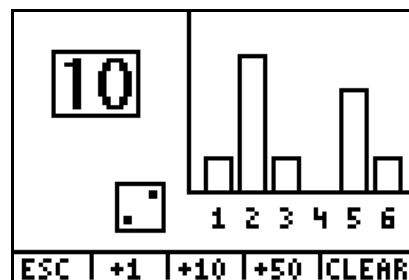
6. Press ▶ to highlight each bar to see the frequency of each outcome.

7. After you have practiced rolling the die and viewing the results, clear the screen to continue.

8. Select **ESC** two times, and then select **YES** to return to the **Simulation** menu.

First Rolls of the Die

1. Select **Roll Dice** to begin a new group of rolls.

2. Select **ROLL** and then select **+1** to roll the die by ones until you have rolled it six times.

Record your results in # 1 on the Student Worksheet.

3. Select **+10** to roll the die 10 times more until you have rolled the die a total of 36 times.

Respond to the remaining questions in # 1 on the Student Worksheet.

One way to compare the frequencies of each outcome in an experiment is to create a Qualitative Scale of Evenness (QSE). To create a QSE, you rate the bar graphs using a scale that ranges from 1 to 5, with 1 indicating that the heights of the bars are not very simmilar, and 5 indicating that the heights of the bars are exactly the same. It is important that everyone in the class agrees on the QSE to be used because everyone will use it to rate other graphs as you work through this activity.

Respond to # 2 on the Student Worksheet.

After you and your classmates have agreed on a QSE, apply the QSE to the bar graphs that are created when you conduct experiments using the Probability Simulation App.

More Rolls of the Die

Your graphing calculator should still display the bar graph showing the results of rolling a fair die 36 times. In this new investigation, you set the number of trials from memory.

1. Select **ESC** twice, and then select **YES** to return to the **Simulation** menu.

2. Select **Roll Dice**.

3. Select **SET** and change the settings as shown. (Press ⊡ to change the **Update** setting.)

4. Select **OK**.

5. Select **ROLL**. The die is rolled 204 times, and the corresponding bar graph is created. Notice that 204 is a multiple of 6, which allows the graph to have even bars.

Hint: Press ▶ to highlight each bar of the graph. The total frequency for that number is displayed at the top of the highlighted bar. Select **TABL** *to see the results of every roll in a table. Select* **GRPH** *to return to the graph view.*

Record your results in # 3 on the Student Worksheet.

6. Select **ROLL** again to collect data for an additional 204 rolls. The graph now displays the cumulative results of 408 rolls.

> Continue to select **ROLL** until you have rolled the die 1,020 times. Record your results in # 3 on the Student Worksheet for each roll.

> Respond to # 4 on the Student Worksheet.

Displaying the Results

While you rolled the die 1,020 times, all of your classmates also rolled a die 1,020 times. Combining the results from all students' experiments gives a much larger number of trials (for example, 25 × 1,020 trials). Along with your classmates, enter your data for your 1,020 rolls into a single graphing calculator that your teacher supplies.

1. Quit the Probability Simulation App if it is still running. To quit the App, select **ESC** two times, select **YES** to clear all data from lists, select **QUIT**, and then select **YES**.

2. Press STAT, select **1:Edit,** and then enter your frequencies for the outcomes from 1 to 6 in **L1** through **L6**, respectively.

   *Note: Before entering any data, make sure the lists are clear of any previous data. To clear data, highlight **L1** and then press CLEAR ENTER. Repeat these steps for each list that needs to be cleared.*

3. Press 2nd [QUIT] to go to the Home screen.

4. Press 2nd [LIST], and then press ▶ twice to display the **MATH** menu.

5. Select **5:sum**.

6. Press 2nd [L1]) ENTER. The total for List 1 is displayed.

7. Repeat this procedure for each list by pressing [2nd] [ENTRY] [◄] [◄] [2nd] [list number] [ENTER].

```
                              3892
sum(L4)
                              4075
sum(L5)
                              4068
sum(L6)
                              3928
```

Record the results in # 5 on the Student Worksheet.

Graphing the Results

1. Return to the Stat List Editor and clear the lists as described in step 2 above.

2. Enter the numbers 1, 2, 3, 4, 5, 6 into L1 and the corresponding frequency totals into L2.

```
L1      L2      L3      2
1       4016    ------
2       4053
3       3892
4       4075
5       4068
6       3928
------
L2(7) =
```

3. Press [2nd] [STAT PLOT], select **4: PlotsOff**, and then press [ENTER] to turn all of the plots off.

```
STAT PLOTS
2↑Plot2…Off
   ∠ L1    L2    ▫
3:Plot3…Off
   ∠ L1    L2    ▫
4▪PlotsOff
5:PlotsOn
```
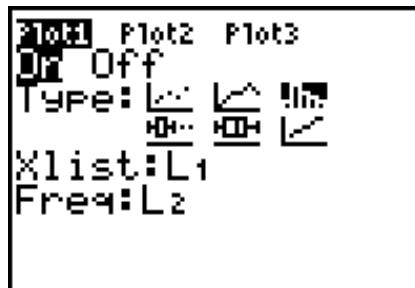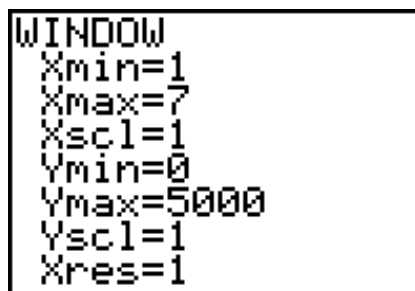
```
                              179
sum(L5)
                              158
sum(L6)
                              175
PlotsOff
                             Done
```
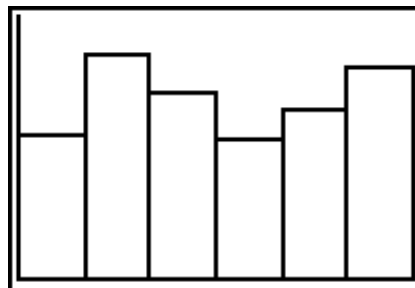
4. Press Y=. Highlight the equation and then press CLEAR to erase equations from the Y= editor.

5. Press 2nd [STAT PLOT], select **1: Plot1**, and then set up Plot1 as shown.

6. Press WINDOW and enter the values as shown. Set Ymax to the greatest sum (from number 5 on the Student Worksheet) rounded up to the next 1,000.

7. Press GRAPH. A bar graph of the data is displayed.

8. Press TRACE and then press ◄ or ► to display the frequency of each outcome.

Respond to # 6 – 10 on the Student Worksheet.

Extensions

There are other choices in the Probability Simulation App in addition to Roll Dice to generate random numbers. Use one of the other simulations—Toss Coins, Pick Marbles, or Spin Spinner—to demonstrate the Law of Large Numbers. Share your findings with the class.

Respond to Extension 1 on the Student Worksheet.

The Law of Large Numbers that you investigated in this Exploration is referred to in the activities that follow. If you need more experience to feel confident with the application, explore each simulation: Toss Coins, Roll Dice, Pick Marbles, or Spin Spinner. After you run each simulation, you see a bar graph that you can evaluate using the QSE that the class agreed upon. As you become more familiar with these models, your prediction of the probability of any outcome will improve.

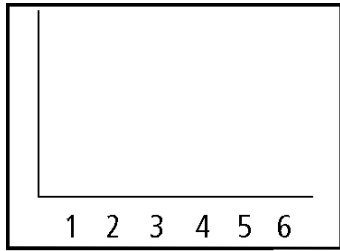Respond to Extensions 2 and 3 on the Student Worksheet.

Student Worksheet

Name _____

Date _____

Law of Large Numbers: Equal Opportunities

1. Are the bars on the bar graph the same height after 6 rolls?_____ After 36 rolls? _____ Sketch your graph after 36 rolls.

Compare your graph to those of your classmates. How do the graphs compare?

2. Create a QSE that you can use to judge the results of each probability experiment. In each of the blank graphs below, sketch an example of a bar graph that represents a rating from 1 through 5. For a rating of 1, the heights of the bars should all be different; for a rating of 5, the heights of the bars should be equal.
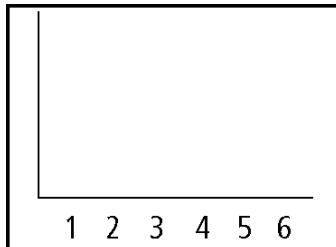

Rating of 1

Rating of 2

Rating of 3

Rating of 4

Rating of 5

3. In the table below, record your data from the rolls of the die. Be sure to record the QSE rating of the resulting bar graph before checking and recording the frequency of each outcome.

		Frequency of Outcomes on a 6-Sided Fair Die					
Number of Rolls	QSE Rating	1	2	3	4	5	6
204							
408							
612							
816							
1,020							

4. How did the QSE ratings change as the number of rolls increased?

Have these five experiments provided sufficient data to allow you to predict the theoretical probability of each outcome? Would a greater number of rolls make your prediction more reliable? How closely do the experimental (empirical) and theoretical probabilities agree in this model?

5. Record the cumulative totals for all students using the frequency data for the 1,020 rolls.

List 1 total   _____

List 2 total   _____

List 3 total   _____

List 4 total   _____

List 5 total   _____

List 6 total   _____

6. Sketch the graph of the class results below.


```
1  2  3  4  5  6
```

7. Record the QSE rating of the bar graph that you created (in step 7 of the *Graphing the Results* section) with the frequency totals gathered from the class . Give your reasoning for giving the graph that rating score. Do you think a greater number of rolls would produce a different graph? Why or why not?

8. Do you think the class has enough data to come to a valid conclusion about predicting the theoretical probability of any one outcome when rolling a fair die?

9. Use the cumulative class data to calculate the experimental (empirical) probability for each of the six possible outcomes on the die. How do these probabilities compare with the theoretical probabilities?

10. Summarize the results of the class experiment in terms of the Law of Large Numbers. The following terms may be helpful in your explanation: theoretical probability, experimental probability, frequency, outcome, trial.

Extensions

1. Predict the probability of the outcomes in each of the other models. Do you think that the Law of Large Numbers applies to these probability simulations as well?

2. Considering the steps and explorations in this activity, what is your response to the question, "How many times must an experiment involving a fair model be repeated before you can reliably determine the probability of any outcome?" That is, how large is "large" in the Law of Large Numbers?

3. Imagine that you rolled the die as many times as the cumulative number of rolls done by the class. (For example, if all 25 students roll a die 1,020 times, then the total number of rolls is 25 × 1,020 = 25,500 times.) Do you think the bar graph of the data would look like the one that represented the class result? Go ahead and roll the die the same number of times as your class to verify your answer.

    Note: *Set your settings trial to 999—that is as high as it can be set—and repeatedly select ROLL. You may wish to do this while working on another assignment because it will take some time for the application to complete the experiment.*

Teacher Notes

A c t i v i t y   1

Law of Large Numbers: Equal Opportunities

Preparation

Each student enters his or her data for the 1,020 rolls in the same graphing calculator (preferably one that can be used with a ViewScreen™ panel or TI-Presenter™ video adapter).

This activity addresses the Data Analysis and Probability Standard of the NCTM Principles and Standards for School Mathematics. This activity is appropriate for students in Algebra 1 and Algebra 2.

A probability is a ratio that compares the number of favorable outcomes to the total number of possible outcomes:

$$P(\text{outcome}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

Theoretical probability is determined by logical reasoning which might use a formula or a representation such as an area model, a tree diagram, or an organized list. Empirical probability is determined by conducting an experiment or simulation to collect data. It is often referred to as experimental probability.

Using the Probability Simulation App, students roll a fair die. As the number of rolls increases, the experimental (empirical) probability of an outcome gets closer to its theoretical probability. These experiments help students understand the distinction between theoretical probability and experimental probability. The experiment also highlights the role that large numbers play in bridging the two sets of values.

The Law of Large Numbers asserts that the greater the number of trials in a probability experiment, the closer the results are to the theoretical probability of any outcome. In this activity, therefore, students are asked to imagine just how large the number of trials has to be to draw a valid conclusion about the probability of an outcome.

The Exploration and classroom discussions of this activity are important to completing the rest of the activities in this book. They not only lay the foundation for concepts explored in later activities, but also acquaint students with the App's features.

To help your students better understand the distinction between theoretical and experimental probabilities, have students compare the frequencies of the outcomes after each trial using a visual scale called a Qualitative Scale of Evenness (QSE). It is important to note that students are working with qualitative scales, not quantitative scales; they are not calculating experimental probabilities until the end of the activity. Using the QSE that they create, students rate the bar graph distribution created in the Probability Simulation App using a scale from 1 to 5, where 1 indicates that none of the bars are close to the same height and 5 indicates that all of the bars have exactly the same height (a highly unlikely, but perfect distribution).

It is important that the entire class determines and agrees upon the ratings to be used in their QSE. After the students have reached a consensus, have the rating scale available for all to reference, either as a hand-out or on a large visual display. Remind students that they all are to use this rating system when evaluating their graphs.
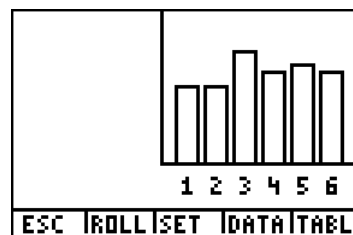
Below is an example of a possible QSE with a description and visual for each rating.
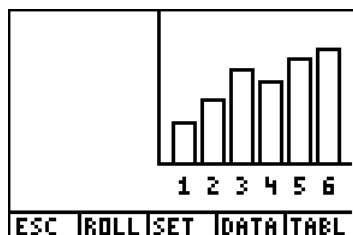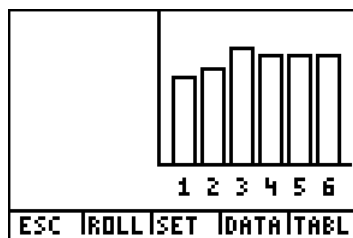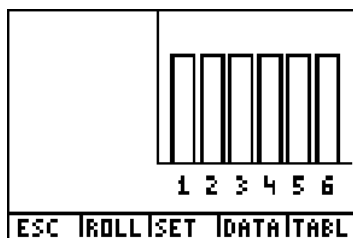
Rating of 1 – No bars have the same height
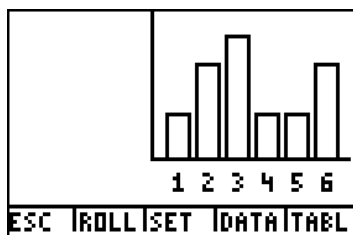
Rating of 2 – A few bars have the same height

Rating of 3 – Most bars close to the same height

Rating of 4 – Bars very similar in height

Rating of 5 – All bars have exactly the same height

Management

Have a graphing calculator available for your students to enter data for their 1,020 rolls. This makes it easier to consolidate the class data to make the bar graph for the cumulative totals. If the graphing calculator can be connected to a ViewScreen™ panel or TI-Presenter™ video adapter, the entire class can view the results at the same time.

Helpful Hints

Following are some graphing calculator hints for working with the Probability Simulation App.

1. When students press APPS, the **Applications** menu is displayed. Depending on the handheld that students are using, they may or may not see **Prob Sim** immediately; they may need to scroll down to locate it. Once students are familiar with their graphing calculator, they will be able to access the application efficiently. If your graphing calculator has a long list of Apps, press ALPHA [P] to jump to the Apps beginning with the letter P.

2. To select one of the options displayed across the bottom of the screen, press the corresponding graphing key below the option. For example, to select **ROLL**, press WINDOW.

3. The graph that is displayed on the graphing calculator automatically defaults to show the frequency of each outcome. If you want the graph to show the experimental probability of each outcome, you must change the settings. Select **SET**, then select **Prob** for the Graph setting.

4. You can pause the simulation at any time by pressing ALPHA. To resume the simulation, select the option corresponding number of rolls selected (for **+10**, press ZOOM; for **+50**; press TRACE. Select **ROLL** (press WINDOW) if the number of rolls has been set in the **Settings Menu**).

5. The heights of the bars on the graph indicate the frequency of each outcome. The tallest bar represents the most frequently occurring outcome. Remind students that the heights of the bars are related to one another and are based on the total number of rolls in the experiment. That is, the sum of the frequencies of the six outcomes must equal the total number of rolls. To display the frequency of each outcome, students can highlight each bar by pressing ◀ or ▶.

6. With a large number of rolls of the die, the difference among the heights of the bars may appear very small. This is due to the small screen size, screen resolution, and the interval scale used. To adjust the interval scale, students can access the viewing window and can change Ymin and Ymax. Set the Ymin to be 200 less than the lowest frequency and set the Ymax to be 200 more than the greatest frequency. Your students can still move the cursor from bar to bar by pressing ◀ or ▶ to see the actual frequency of an outcome.

Answers to Student Worksheet

1. Answers will vary. There may be a wide variety of graphs. A few graphs may be similar.

2. See examples in *Teacher Notes* section.

3. Answers will vary.

4. Answers will vary. Sample answer: Yes, the QSE rating increases. No, the number of rolls is not large enough. I predict that more rolls will produce a graph where the heights of the bars are closer to being equal.

5. Answers will vary; however, all responses within a class should be the same.

6. Sketches will vary; however, all sketches within a class should be the same.

7. Answers will vary.

8. Answers will vary. Students should understand that the more trials run (the larger the number), the closer the experimental probability approximates the theoretical probability.

9. Answers will vary; however, all responses within a class should be the same. Each experimental probability should approximate a theoretical probability of 0.1667.

10. Answers will vary.

Answers to Extensions

1. Answers will vary. Students should understand that the more trials run (the larger the number), the closer the experimental probability comes to theoretical probability.

2. Answers will vary. Ask students to explain their thinking.

3. Answers will vary.