NUMB3RS Activity: Sudoku Puzzles
Episode: "All's Fair"

Topic: Sudoku Puzzles **Grade Level:** 8 - 10

Objective: Explore sudoku puzzles and Latin squares

Time: 60 - 90 minutes

Introduction

In "All's Fair," Alan, Larry, and Charlie play and discuss a sudoku puzzle. Sudoku puzzles were designed by Howard Grans and first published in 1979. Popular in Japan since the 1980's, the puzzle has gained attention in the United States and Europe in recent years and many versions are now published in newspapers and on the Internet.

Sudoku, which means "single number," is a logic puzzle typically played on a 9 × 9 grid. With several values given, the goal is to fill in the grid with the numbers 1 through 9 so that every number occurs exactly once in each row, once in each column, and once in each 3 × 3 box outlined in bold. An example of a sudoku puzzle is shown below.

3					8	2	4	7
6	9			3				
	4						3	
2			9	8				
	3	8				1	7	
				7	3			5
	5						9	
				6			5	1
7	2	9	3					4

[Generated by **http://www.opensky.ca/~jdhildeb/software/sudokugen**]

One possible strategy used to solve the puzzles is to begin by looking at the first three columns of numbers, and work through the possibilities in some type of order. For example, there is a nine in column 3 and column 2; there must be a 9 in column 1. The nine must be located in the second 3 × 3 box. Therefore, it can go in one of two places. You can write a small 9 in both of those boxes to show that it is a possibility and move on. The strategy continues by looking at the columns 4–6, and then 7–9. This process repeats for the rows and for the boxes.

In this *NUMB3RS* episode, Charlie notes that there are over 6.7 sextillion (6.67×10^{21}) different completed sudoku 9 × 9 grids. This activity asks students to analyze smaller grids that are only 4 × 4, although there is a 9 × 9 puzzle to try. The analysis to determine how many puzzles there are, involves a look at a related topic, Latin squares. A Latin square is an $n \times n$ grid filled with the numbers 1 through n, such that every number occurs exactly once in each row and in each column. An example of a 5 × 5 Latin square is shown at the right.

1	2	3	4	5
3	4	5	1	2
5	1	2	3	4
2	3	4	5	1
4	5	1	2	3

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After determining the number of 4 × 4 grids possible, students explore large numbers, trying to get a handle on the magnitude of 6.7 sextillion. Extensions involve magic squares and the brute force method of proof, including reference to the four-color theorem.

Discuss with Students

1. Complete the 9 × 9 sudoku puzzle on the previous page.

2. With no restrictions on where numbers 1 through 4 are placed or how often they are used in the grid, how many different 4 × 4 grids could be created using only those digits?

Discuss with Students Answers:

1.

3	1	5	6	9	8	2	4	7
6	9	2	7	3	4	5	1	8
8	4	7	5	1	2	9	3	6
2	7	1	9	8	5	4	6	3
5	3	8	4	2	6	1	7	9
9	6	4	1	7	3	8	2	5
1	5	6	8	4	7	3	9	2
4	8	3	2	6	9	7	5	1
7	2	9	3	5	1	6	8	4

2. 4^{16}

Student Page Answers:

1. *Answers will vary*

2a. *There is already a 1 in the rightmost column.* **2b.** *The 1 cannot go in the rightmost column. See the completed puzzle below.*

1	2	4	3
3	4	2	1
2	1	3	4
4	3	1	2

3.

1	2	3	4
2	4	1	3
3	1	4	2
4	3	2	1

4a. *There is only one, shown below.*

1	2
2	1

There is only one reduced 2 × 2 Latin square because 1 must go in the upper left corner and 2 must complete the first row and column. There is no choice for the number in the bottom right cell, so there is exactly one 2 × 2 Latin square.

4b. *There is only one, shown below.*

1	2	3
2	3	1
3	1	2

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4c. *There are four, shown below.*

1	2	3	4
2	4	1	3
3	1	4	2
4	3	2	1

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

1	2	3	4
2	1	4	3
3	4	2	1
4	3	1	2

1	2	3	4
2	3	4	1
3	4	1	2
4	1	2	3

4d. *There are two 2s in the upper left box. The 12 possible reduced 4 × 4 grids are shown below*

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

1	2	3	4
3	4	1	2
2	3	4	1
4	1	2	3

1	2	3	4
3	4	1	2
4	1	2	3
2	3	4	1

1	2	3	4
3	4	1	2
4	3	2	1
2	1	4	3

1	2	3	4
3	4	2	1
2	1	4	3
4	3	1	2

1	2	3	4
3	4	2	1
4	3	1	2
2	1	4	3

1	2	4	3
3	4	2	1
2	1	3	4
4	3	1	2

1	2	4	3
3	4	2	1
2	3	1	4
4	1	3	2

1	2	4	3
3	4	2	1
4	1	3	2
2	3	1	4

1	2	4	3
3	4	2	1
4	3	1	2
2	1	3	4

1	2	4	3
3	4	1	2
2	1	3	4
4	3	2	1

1	2	4	3
3	4	1	2
4	3	2	1
2	1	3	4

5. *Earth weighs 6 sextillion metric tons according to* ***http://www.ecology.com/earth-at-a-glance/earth-at-a-glance-feature/*** *; there are only 31,536,000 (3.15×10^7) seconds in a year and 3.15×10^9 seconds in 100 years; there are 10 times as many stars as completed sudoku grids, 70 sextillion stars, according to* ***http://www.cnn.com/2003/TECH/space/07/22/stars.survey/***

6.

3	6	2	8	1	7	5	9	4
1	9	8	4	6	5	7	2	3
4	7	5	9	3	2	8	1	6
2	8	4	1	5	9	3	6	7
7	5	6	2	8	3	1	4	9
9	3	1	7	4	6	2	5	8
6	4	7	3	2	1	9	8	5
8	1	3	5	9	4	6	7	2
5	2	9	6	7	8	4	3	1

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Name ____ Date ______________

NUMB3RS Activity: Sukoku Puzzles

In "All's Fair," Alan, Larry, and Charlie solve and discuss a sudoku puzzle. A 4 × 4 sudoku grid is filled with numbers so that every row, every column, and each of the four highlighted boxes contain the numbers 1 through 4. Below is an example of a completed 4 × 4 sudoku puzzle.

1	2	4	3
3	4	1	2
4	3	2	1
2	1	3	4

1. Fill in your own 4 x 4 puzzle on the blank grid below.

2. In a sudoku puzzle, some of the numbers are given and the goal is to fill in the blank cells. Consider the puzzle below.

1		4	
			1
2		3	
	3		

a. There must be a 1 somewhere in the third row, and the only empty cells in the row are in the 2nd and 4th columns. Why can't the 1 go in the 4th column? Fill in the entire third row.

b. Next consider the lower right-hand box, which now needs a 1 and a 2. There is only one choice for the 1 (why?). Fill in the remainder of the grid.

A sudoku puzzle is a variation of a puzzle called a Latin square. A Latin square is an $n \times n$ grid filled with numbers so that the numbers 1 through n occur exactly once in each row and in each column. Below is an example of a 5 × 5 Latin square.

1	2	3	4	5
3	4	5	1	2
5	1	2	3	4
2	3	4	5	1
4	5	1	2	3

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3. Complete the following 4 × 4 Latin square.

	2		4
2			3
	1	4	
			1

4. The 4 × 4 Latin square in question 3 is called "reduced," because the numbers are in increasing order across the top row and down the leftmost column.

a. Show that there is only one reduced 2 × 2 Latin square.

b. How many reduced 3 × 3 Latin squares are there?

c. How many reduced 4 × 4 Latin squares are there? Start by considering the circled cell, in the square below, which must have a 1, 3, or 4. Try each possibility on a different chart below and continue to fill in each.

1	2	3	4
2	◯		
3			
4			

1	2	3	4
2			
3			
4			

1	2	3	4
2			
3			
4			

1	2	3	4
2			
3			
4			

d. Challenge We cannot use the same definition of "reduced" for a sudoku puzzle as we did for a Latin square. (What is wrong with the sudoku grid on the left below?) Instead, a "reduced" sudoku will look like the grid on the right – the upper left square must contain the numbers in the order shown. Use a similar process to that used in question 4c to list the 12 reduced 4 × 4 sudoku grids.

1	2	3	4
2			
3			
4			

1	2		
3	4		

Sudoku puzzles are usually 9 × 9 grids. Counting the number of 9 × 9 grids involves similar methods used above, but the number of possibilities is substantially higher. In the *NUMB3RS* episode, Charlie remarks that there are 6.67 sextillion, or 6.67×10^{21}, solutions to the 9 × 9 sudoku puzzle.

5. Which of the following values do you think is closest to 6.67 sextillion?

a. the number of seconds in a year
b. the number of seconds in a 100 years
c. the earth's mass in metric tons
d. the number of stars in the known universe

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6. Below is a 9 × 9 sudoku puzzle. How can the cells be filled?

3				1	7	5	9	
	9		4		5	7		
4					2			6
						3		7
	5			8			4	
9		1						
6			3					5
		3	5		4		7	
	2	9	6	7				1

[Generated by **http://www.opensky.ca/~jdhildeb/software/sudokugen/**]

One strategy to try is to identify an empty cell and determine which numbers could fill it. For example, the cell in the sixth column of the bottom row is empty. The row contains the numbers 1, 2, 6, 7 and 9; the column contains 2, 4, 5 and 7; and the box contains 3, 4, 5, 6 and 7. The only number not in any of the three lists is 8, so that number must go into that cell.

Another strategy is to choose a number missing from a row, column, or box and try to place it correctly. For example, the first column needs a 7 and has blank cells in the 2nd, 4th, 5th, 8th, and 9th rows. But there is already a 7 in rows 2, 4, 8 and 9. So the 5th cell in the first column must be a 7. Continue with these and other strategies you find to fill in the remainder of the puzzle.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Extension 1
A magic square is a Latin square whose row, column, and diagonal sums are equal. Can the boxes of a 9 × 9 sudoku grid all be magic squares?

Extension 2
The Latin squares counted in this activity were in reduced form. Develop a formula that relates the total number of Latin squares to the reduced number of Latin squares. Use the Web site below for more information.
http://en.wikipedia.org/wiki/Latin_square

Extension 3
The method used to count the number of sudoku grids is brute force. Brute force is a relatively recent method of mathematical proof, and is met with skepticism by some mathematicians. Brute force was most famously used to prove the four-color theorem in graph theory. To learn more about the four-color theorem, see the Web site below.
http://en.wikipedia.org/wiki/Four_color_theorem

Other Resources

- This Web site gives more information about sudoku puzzles.
  http://en.wikipedia.org/wiki/Sudoku

- This article by Bertram Felgenhauer and Frazer Jarvis discusses the number of possible sudoku grids.
  http://www.afjarvis.staff.shef.ac.uk/sudoku/

- This Web site is one of many Web sites where you can generate and solve sudoku puzzles.
  http://www.opensky.ca/~jdhildeb/software/sudokugen/

- Go to the Web site below and search for "sudoku." You will find several programs that generate sudoku puzzles on a TI-83/84 Plus calculator.
  http://www.ticalc.org/archives/files/recent.html

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