NUMB3RS Activity: Exploring Parabolas
Episode: "Structural Corruption"

Topic: Parabolas   **Grade Level:** 9 - 10
Objective: Students will investigate parabolas of the form $y = a(x - b)^2 + c$ and state the
    effect each coefficient has on the parabola's graph.
Time: 20 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator, Derive™ 6 for the Extensions

Introduction
When a Cal Sci student apparently commits suicide by jumping off of a bridge, Charlie
investigates and suspects foul play. He believes the parabolic path followed by the
student terminates farther from the bridge than it should for a jumper. While Charlie was
ultimately wrong in his assumption of foul play, he was correct that the body would follow
a parabolic path in its descent.

This activity is intended as a means for students to discover the effects the parabola's
coefficients have on its graph. This activity makes extensive use of the Transformation
Graphing App to explore properties of parabolas. This App can be downloaded for free
from **http://education.ti.com/educationportal/sites/US/productDetail/
us_transg_83_84.html**.

The Extensions section of this activity uses Derive 6. To download the picture used for the
background, go to **http://education.ti.com/exchange** and search for "7721." For more
information about Derive 6, including information on how to purchase the program, go to
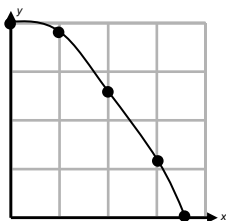**http://education.ti.com/derive**.

Discuss with Students
This activity focuses on a family of graphs called parabolas. Parabolas are a subset of
the conic sections (the others being ellipse, hyperbola, and circle). While parabolas have
many properties, the focus of this activity will be to observe the changes that graphs of
the form $y = a(x - b)^2 + c$ undergo when the values of the coefficients *a, b*, and *c* are
changed.

The extensions activity assumes that the scale used is the same as that of the
coordinate system superimposed over the picture.

Student Page Answers:

1.

x	y
0	200
1	184
2	136
3	56
4	–56

2. *(0,200)* **3.** *±3.5* **4.** *When B is negative, the graph moves
B units to the left. When B is positive, the graph moves
B units to the right.* **5.** *When C is negative, the graph
moves C units down. When C is positive, the graph moves
C units up.* **6.** *When A is negative, the parabola is concave
down. When A is positive, the graph is concave up. When
−1 < A < 1, the graph gets wider, otherwise it gets skinner.*
7. *(2, 5)* **8.** *Positive values of A make the graph open
upwards, and negative values of A make the graph open
downwards. As the absolute value of A increases, the graph
becomes narrower.*

9. *The graph becomes wider as the values for A approach zero i.e. 0.5, –0.01, etc.* **10.** *When
A = 0, the equation $y = A(X - B)^2 + C$ reduces to $y = C$. The graph of any constant, C, is always
a horizontal line.* **11.** $y = -\dfrac{1}{2}(X - 1)^2 + 8$

Name: _____   Date: _____

NUMB3RS Activity: Exploring Parabolas

When a Cal Sci student apparently commits suicide by jumping off of a bridge, Charlie investigates and suspects foul play. He believes the parabolic path followed by the student terminates farther from the bridge than it should for a jumper. While Charlie was ultimately wrong in his assumption of foul play, he was correct that the body would follow a parabolic path in its descent.

What exactly is a parabolic path? A parabolic path follows a certain trajectory that can be described by an equation of the form $y = ax^2 + bx + c$. The paths made by a baseball in a game of catch, a cannon fired at a target, or even an object thrown off a bridge are parabolas.

1. Suppose the person jumping off the building followed the equation $y = -16x^2 + 200$ where *x* is time (seconds) and *y* is distance above the ground (feet). Complete the table below and graph the parabolic path.

| *x*	*y*
0	____
1	____
2	____
3	____
4	____
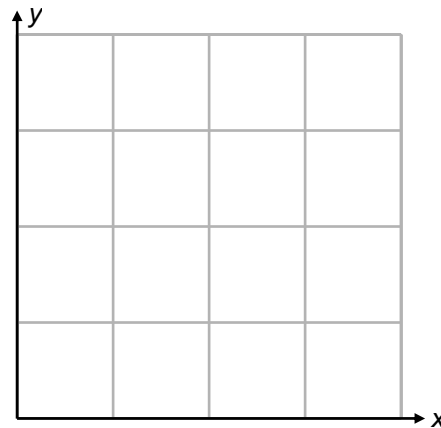
This equation can also be graphed using your graphing calculator. Press `Y=` and enter $y = -16x^2 + 200$ in equation Y_1. Now press `WINDOW` and use the following settings: `Xmin = 0`, `Xmax = 4`, `Ymin = 0`, and `Ymax = 200`. Now press `GRAPH`.

2. The highest (or lowest) point a parabola reaches is called its vertex. What is the vertex for the parabola above? _____

3. The *x*-coordinates of the points where the parabola crosses the *x*-axis are called the *x*-intercepts. What are the x-intercepts of the equation above? _____

While any equation can be graphed by plotting points, it would be much easier if the graph could be determined directly from the equation. Use the Transformation Graphing App to determine if the vertex and shape of the parabola can be obtained directly from the equation.

Press the $\boxed{\text{APPS}}$ key and select **Transfrm** (press any key to start the App). One this is done, press $\boxed{\text{Y=}}$. Notice that to the left of each equation, there is a new symbol. This shows that the Transformation Graphing App is running. (It also means that you can only graph one equation at a time while the App is running). This App allows the changing of the coefficients in an equation to explore how each value affects the graph.

Example: Enter $A(X-B)^2+C$ into Y_1. (Remember to use the $\boxed{\text{ALPHA}}$ key to type letters.) Press $\boxed{\text{ZOOM}}$ and select **6:ZStandard** to display the following:
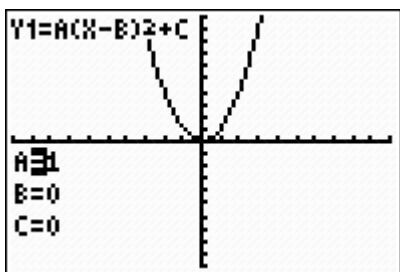
The equation is shown at the top left of the window, and the values of A, B, and C are shown below. Use the up and down arrow keys to select different values for A, B, and C. Use the right and left arrow keys to change the values by an increment. Observe how the graph changes. When finished, choose values to match the screen to the left.

4. What happens to the graph when the B coefficient is changed?
    _____

5. What happens to the graph when the C term is changed? _____

6. What happens to the graph when the A coefficient is changed?
    _____

7. Using the Transformation Graphing App, find the vertex of the parabola when B = 2 and C = 5. (Use any nonzero integer for A.) _____

8. Using integers, describe what effect the value of A has on the graph.
    _____

9. Experiment with values of A and decide how to make the graph wider
    _____

10. Explain why a horizontal line is formed when A = 0. _____
    _____

11. Use the Transformation Graphing App to find an equation whose graph has zeros at –3 and 5, has a vertex at (1, 8), and opens downwards.
   _____

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research

Extensions

Activity: Using Derive™ 6 to Approximate Parabolas

Introduction

This extension will focus on using the Derive program to approximate points along a picture of a water fountain following a parabolic path. These points will then be entered into your graphing calculator to obtain the parabolic equation.


[Source: **http://www.xeix.org/IMG/jpg/parabola.jpg**]
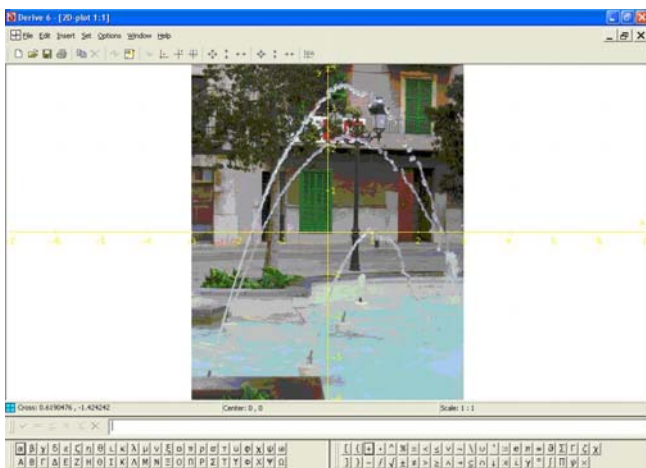
After starting Derive, program, choose **Insert** and select **2D-Plot Object...**.

Right-click on the displayed grid and choose **Display Options...**. Under each tab, set all of the colors to yellow. Under the **Grids** tab, set the **Horizontal Interval** to 14. Under the **Color** tab, set the **Background Display** to **Center**. Click the ... box next to **Picture** and insert the picture of the fountain (your teacher will provide this image).
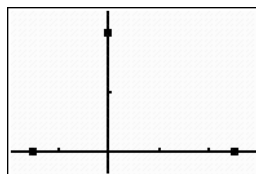
Using the displayed graph, approximate three points on one of the parabolic streams Then, follow the steps below on your calculator.
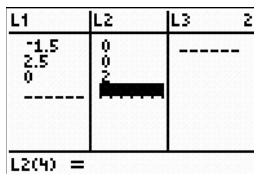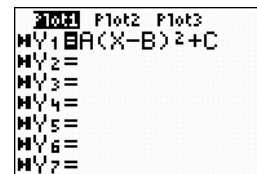

Press STAT and select **1:Edit...**. Insert your three points into lists L_1 and L_2.

Press 2nd [STAT PLOT], choose **Plot 1**, and use the settings shown above.

Press ZOOM and select **9:ZoomStat**. Notice the points are now displayed.

Press Y= and enter the general equation of a parabola.

Now use what you learned in the activity to find the equation of a parabola that fits the scatterplot. (Hint: for a better fitting curve, consider experimenting with non-integer values of A, B, C.)

Additional Resources

- Explore other conic sections on your calculator. Download the Conic Graphing App from the Web site below.
  http://education.ti.com/educationportal/sites/US/productDetail/ us_conic_83_84.html

For the Student

- Other objects called catenaries or "hanging chains," closely resemble parabolas, but have a slightly different shape and a different type of equation. You can learn more about catenaries at **http://mathworld.wolfram.com/Catenary.html**.

- Another way to determine the equation for the parabola given three points is using the Quadratic regression capabilities of the calculator. To learn more, consult your calculator's guidebook or go to **http://education.ti.com**.

Related Topic

- For another approach to looking at parabolas in parametric form, see the *NUMB3RS* activity "Where is the Bullet." This activity can be downloaded for free by going to **http://education.ti.com/exchange** and searching for "6181."

- To learn how to determine the equation of a parabola given three points, see the *NUMB3RS* activity "Parabolic Food Fight." This activity can be downloaded for free by going to **http://education.ti.com/exchange** and searching for "6969."