Uniform Circular Motion – ID: 9746

By Irina Lyublinskaya

Time required
45 minutes

Activity Overview

In this activity, students explore uniform circular motion and solve problems that deal with kinematics and dynamics of this motion. They explore the relationship between the velocity of an object moving along a circular path, the radius of the path, and the centripetal acceleration the object experiences.

Concepts

- *Uniform circular motion*
- *Centripetal acceleration*

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Notes, Calculator, Lists & Spreadsheet

Teacher Preparation

Before carrying out this activity, you should review with students the concepts of velocity, speed, and acceleration. Students should also be familiar with Newton's laws of motion.

- *The screenshots on pages 2–7 demonstrate expected student results. Refer to the screenshots on pages 8 and 9 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9746" in the search box.***

Classroom Management

- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration during this activity:

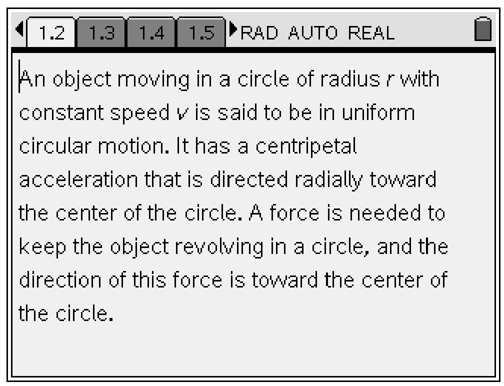
- What is the relationship between the magnitude and direction of velocity and acceleration in uniform circular motion?
- How can we use the formula $a = \frac{v^2}{r}$ to solve problems dealing with kinematics and dynamics of uniform circular motion?

The purpose of this activity is to provide students with an opportunity to explore uniform circular motion and use the formula for centripetal acceleration, $a = \frac{v^2}{r}$, to solve kinematics and dynamics problems. TI-Nspire technology provides students with a dynamic environment for explorations and tools for graphical and numerical analysis for a variety of situations.

This activity consists of three problems. In the first problem, students explore the relationship between velocity, the radius of a circle, and centripetal acceleration. In the second and third problems, students use TI-Nspire technology to solve kinematics and dynamics problems, respectively.

Problem 1 – Exploring the relationship between velocity, radius, and centripetal acceleration

Step 1: Students should open the file **PhyAct28_CircularMotion_EN.tns,** read the first two pages, and then answer questions 1 and 2.



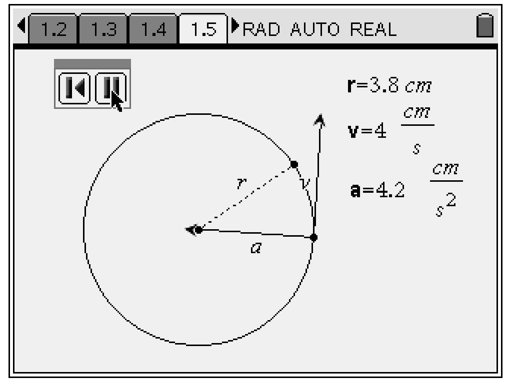
Q1. How can an object undergoing uniform circular motion be accelerating if its speed is constant?

A. *Acceleration is defined as rate of change of velocity. Even though the magnitude of velocity (speed) stays the same in uniform circular motion, the direction changes. Therefore, the object is accelerating. Alternatively, it is necessary to apply an unbalanced force to an object to keep it moving in a circle. According to Newton's first law, unbalanced forces cause changes in motion, or acceleration.*

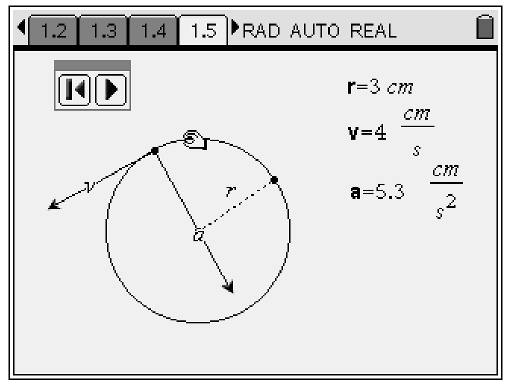
Q2. What is the relationship between the direction of velocity and the direction of acceleration in uniform circular motion? Explain your answer.

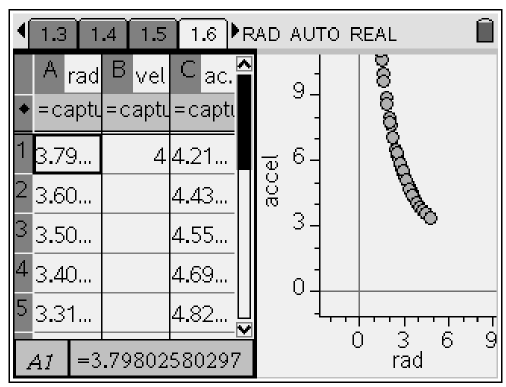
A. *The velocity and acceleration vectors are perpendicular to each other at all times. Because the acceleration involves a change in direction but not in magnitude, there is no component of acceleration in the direction of velocity.*

Step 2: Next, students should read the text on page 1.4 before moving to page 1.5, which shows an animation of uniform circular motion. Students should use the animation control buttons to observe the directions and magnitudes of the velocity and acceleration vectors as the point moves along the circle.



Step 3: Next, students should capture values of radius and acceleration. The *Lists & Spreadsheet* application on page 1.6 is already set up to collect the data. On page 1.5, students should move the cursor over the circle, press ctrl, and then drag the circle with NavPad arrows to vary the radius of the circle. Students then should move to page 1.6 to observe the data on the graph. They can press ctrl tab to move between the spreadsheet and the graph. The graph should be set up to show **accel** vs. **rad**. Students should examine the graph of acceleration vs. radius. Then, they should answer questions 3 and 4.





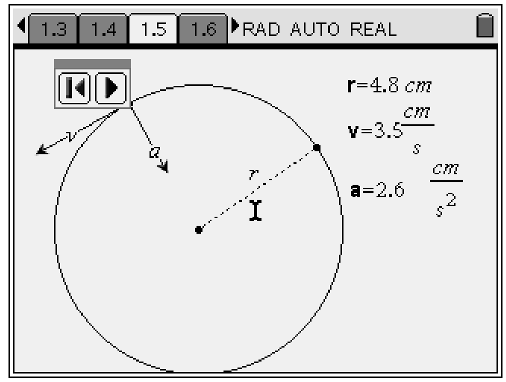
Q3. How does the magnitude of acceleration depend on the radius of the circle?

A. *As radius increases, the magnitude of the acceleration decreases.*

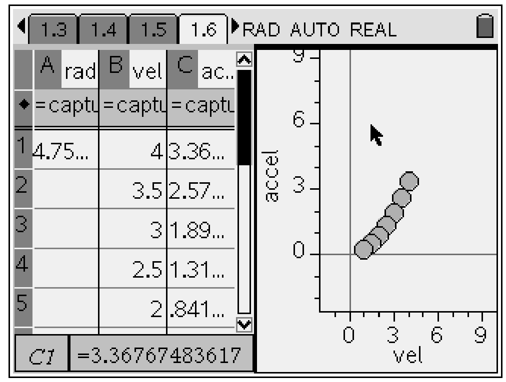
Q4. Do these data agree with the equation for centripetal acceleration, $a = \frac{v^2}{r}$? Explain your answer.

A. *Speed is constant, so the equation relating acceleration and radius is an inverse relationship (that is, acceleration is equal to the inverse of radius multiplied by a constant). The graph of* ***accel*** *vs.* ***rad*** *has the form* $y = \frac{k}{x}$, *which is an inverse relationship. Therefore, the data agree with the centripetal acceleration equation.*

Step 4: Next, students will graph **accel** vs. **vel.** First, they should erase the acceleration and radius data on page 1.6 by moving the cursor to the diamond line of each column and pressing enter twice. In order to analyze acceleration vs. velocity, students should move to the *Graphs & Geometry* application, click on the independent variable, and choose **vel** from the variables menu.



Step 5: Next, students should move back to page 1.5 and move the cursor over the value of velocity. Students should double-click on the velocity and then type different values for **v**. They should enter several different values (at least six) and observe the effects on the circle. After data are collected, students should move back to page 1.6 and observe the graph of **accel** vs. **vel**. Then, they should answer questions 5–7.



Q5. How does the magnitude of acceleration depend on the magnitude of velocity (speed)?

A. *The magnitude of acceleration is directly proportional to the square of speed.*

Q6. Do these data agree with the equation for centripetal acceleration, $a = \frac{v^2}{r}$? Explain your answer.

A. *Radius is constant, so the equation relating acceleration and speed is a direct square (that is, acceleration is equal to a constant multiplied by the square of the speed). The graph of* ***accel*** *vs.* ***vel*** *has the form* $y = kx^2$*, which is a direct-square relationship. Therefore, the data agree with the centripetal acceleration equation.*

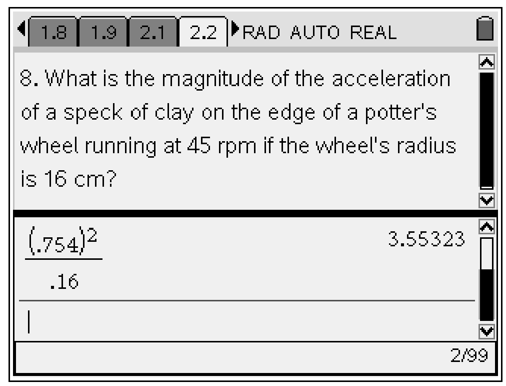
Q7. A motorcycle travels around a curve at a constant speed of 50 mph. Will the acceleration of the motorcycle be the same when it travels around a sharp curve as when it travels around a gentle curve at the same speed?

A. *The sharper turn means a smaller radius of curvature. If radius gets smaller, the acceleration increases.*

Problem 2 – Solving kinematics problems involving uniform circular motion

Step 1: Next, students should move to page 2.1 and read the text there. They should then solve the kinematics problems given on pages 2.2 and 2.3. Each problem page has a *Calculator* application for students' use. To move between the problem and the *Calculator* application, students should press ctrl tab.

Q8. What is the magnitude of the acceleration of a speck of clay on the edge of a potter's wheel running at 45 rpm if the wheel's radius is 16 cm?



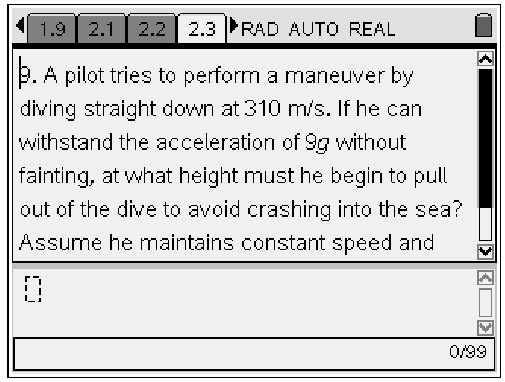
A. *To solve this problem, first find the speed, as shown below:*

$$v = \frac{45\ \cancel{rev}}{\cancel{min}} \bullet \frac{2\pi\ rad}{\cancel{rev}} \bullet \frac{1\ \cancel{min}}{60\ sec} \bullet 0.16\ m = 0.754\ m/s$$

(Students may need to be reminded that radians are dimensionless and therefore can be left out of the units of velocity.) Then, use the centripetal acceleration equation to calculate acceleration, as shown below:

$$a = \frac{v^2}{r} = \frac{(0.754\ m/s)^2}{0.16\ m} = 3.6\ m/s^2$$

Q9. A pilot tries to perform a maneuver by diving straight down at 310 m/s. If he can withstand the acceleration of 9*g* without fainting, at what height must he begin to pull out of the dive to avoid crashing into the sea? Assume he maintains constant speed and circular motion during the recovery.

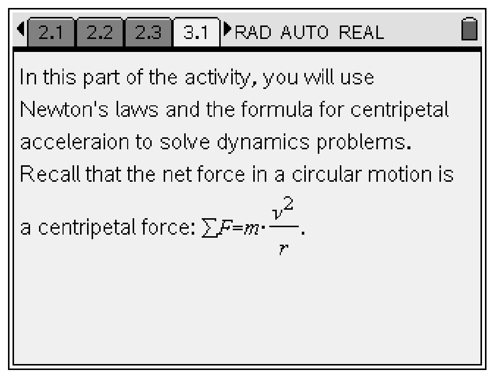


A. *The maximum acceleration of the plane is* $a = 9 \cdot g = 9 \cdot 9.8\ m/s^2 = 88.2\ m/s^2$ *. The path of the plane as it pulls out of the dive can be thought of as a circle with radius* r, *where* r *is the minimum height above sea level at which the plane must pull up. Therefore, to solve this problem, calculate the height above the sea, and rearrange the centripetal acceleration equation to solve for* r, *as shown below:*

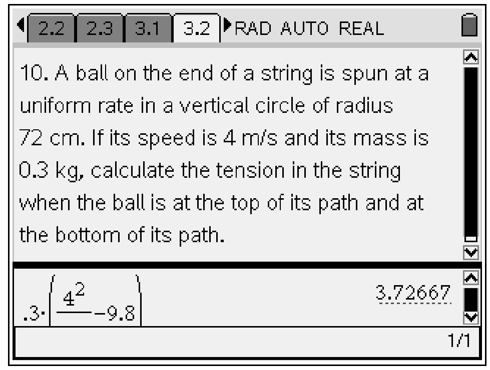
$$r = \frac{v^2}{a} = \frac{(310\ m/s)^2}{88.2\ m/s^2} = 1.1 \times 10^3\ m$$

Problem 3 – Solving dynamics problems

Step 1: Next, students should move to page 3.1 and read the text there. They should then solve the dynamics problems given on pages 3.2 and 3.3. Each problem page has a *Calculator* application for students' use. To move between the problem and the *Calculator* application, students should press ctrl tab. Note: The dynamics problems on pages 3.2 and 3.3 require students to be able to connect several different physics concepts to the concepts they have learned in this activity. Encourage students to discuss the problems and work together to figure out how to solve them. Alternatively, work through the questions as a group, guiding the students stepwise through the solutions. If you wish, you may assign this problem as an extension for more advanced students.



Q10. A ball on the end of a string is spun at a uniform rate in a vertical circle of radius 72 cm. If its speed is 4 m/s and its mass is 0.3 kg, calculate the tension in the string when the ball is at the top of its path and at the bottom of its path.



A. *According to Newton's second law, at the top of the ball's path, the total force acting on the ball is the sum of its weight (mg) and the tension in the string (T), as shown here:*

$\sum F = mg + T = m\frac{v^2}{r}$. *Therefore,*

$T = m\left(\frac{v^2}{r} - g\right)$. *Substituting the given values yields the following:*

$$T = (0.3\ kg)\left(\frac{(4\ m/s)^2}{0.72\ m} - 9.8\ m/s^2\right) = 3.73\ N.$$

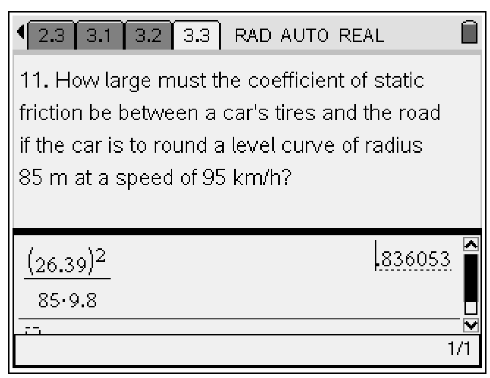
Conversely, at the bottom of the ball's path, the total force on the ball is the difference between the tension and the weight:

$\sum F = T - mg = m\frac{v^2}{r}$, *so* $T = m\left(\frac{v^2}{r} + g\right)$.

Substituting the given values yields the following:

$$T = (0.3\ kg)\left(\frac{(4\ m/s)^2}{0.72\ m} + 9.8\ m/s^2\right) = 9.61\ N.$$

Q11. How large must the coefficient of static friction be between a car's tires and the road if the car is to round a level curve of radius 85 m at a speed of 95 km/h?



A. *First, convert speed into m/s:*

$$95\frac{\cancel{km}}{\cancel{hr}} \bullet \frac{1000\ m}{\cancel{km}} \bullet \frac{\cancel{hr}}{3600\ sec} = 26.39\ m/s.$$

According to Newton's second law,

$\sum F = F_{friction} = m\frac{v^2}{r} = \mu N = \mu mg$, *where μ is the coefficient of friction. Therefore,*

$$\mu = \frac{v^2}{rg} = \frac{(26.39\ m/s)^2}{(85\ m)(9.8\ m/s^2)} = 0.84.$$

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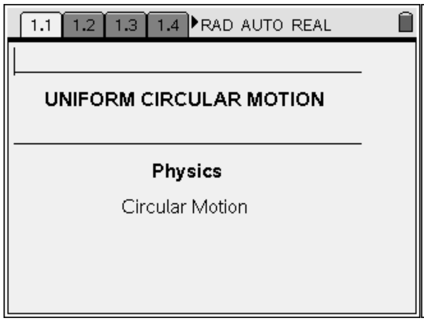
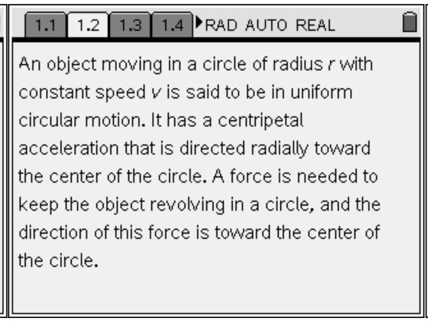
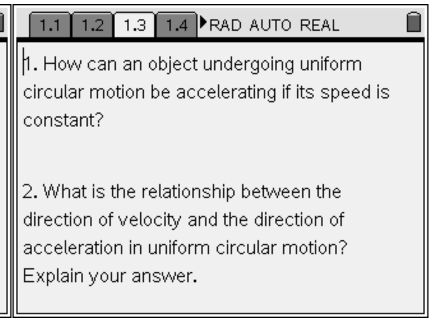
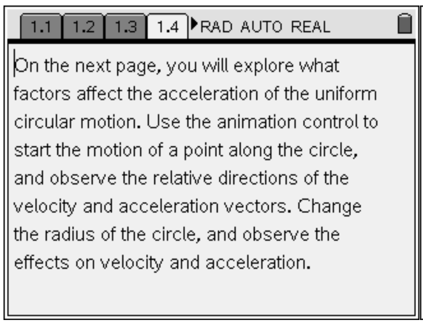
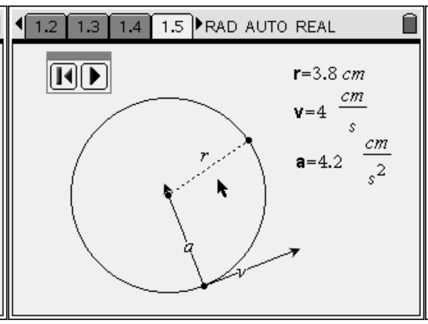
(Student)TI-Nspire File: *PhyAct28_CircularMotion_EN.tns*

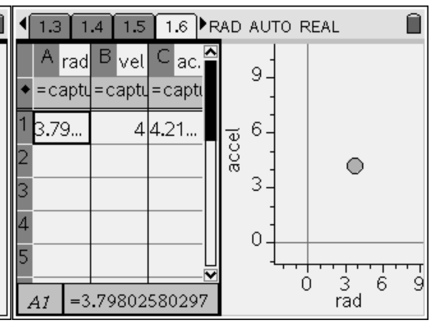


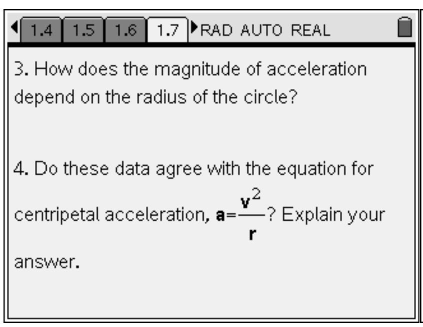


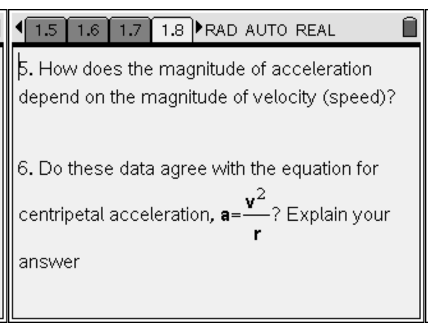


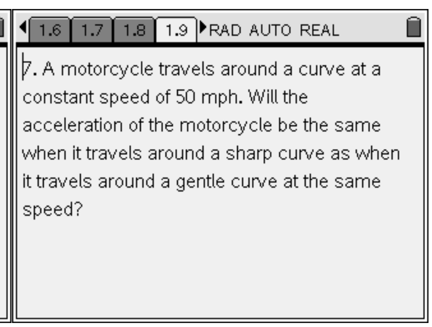


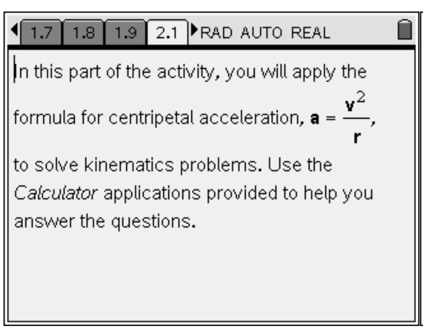


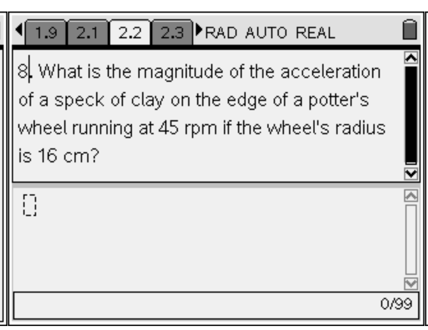


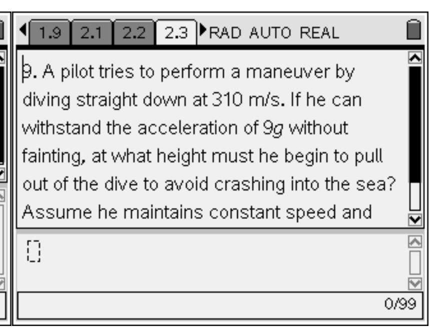


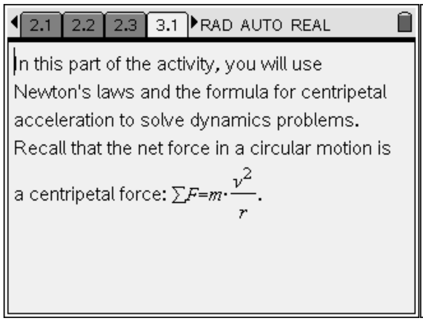


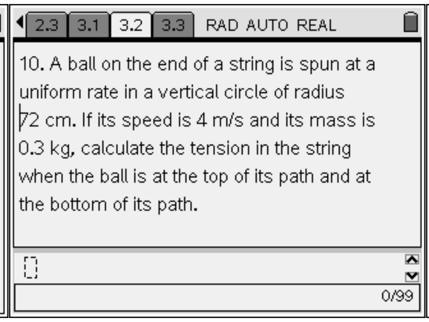









2.1 2.2 2.3 3.1 RAD AUTO REAL
In this part of the activity, you will use Newton's laws and the formula for centripetal acceleration to solve dynamics problems. Recall that the net force in a circular motion is a centripetal force: $\sum F=m\cdot\frac{v^2}{r}$.


2.3 3.1 3.2 3.3 RAD AUTO REAL
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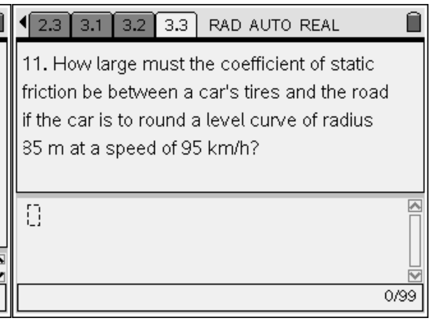
2.3 3.1 3.2 3.3 RAD AUTO REAL
11. How large must the coefficient of static friction be between a car's tires and the road if the car is to round a level curve of radius 85 m at a speed of 95 km/h?
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