Laws of Sines and Cosines

ID: 9849

Time required
45 minutes

Activity Overview

Through out history, mathematicians from Euclid to al-Kashi to Viète have derived various formulas to calculate the sides and angles of non-right (oblique) triangles. al-Kashi used these methods to find the angles between the stars back in the 15th century. Both the famous Laws of Sines and Cosines are used extensively in surveying, navigation, and other situations that require triangulation of non-right triangles. In this activity, students will explore the proofs of the Laws, investigate various cases where they are utilized, and apply them to solve problems.

Topic: Trigonometry

- *Proofs of the Laws of Sines and Cosines*
- *Deriving algebraic solutions*
- *Applying the Laws of Sines and Cosine*
- *Right triangle trigonometry*

Teacher Preparation and Notes

- *This activity is designed for use in a precalculus classroom. Students should already be familiar with algebraic symbol manipulation, right triangle trigonometry, and properties of congruent triangles from Geometry.*
- ***To download the student and solution TI-Nspire documents (.tns files) and the student worksheet, go to education.ti.com/exchange and enter "9849" in the quick search box.***

Associated Materials

- *LawSineCosine_Student.doc*
- *LawSineCosine.tns*
- *LawSineCosine_Soln.tns*

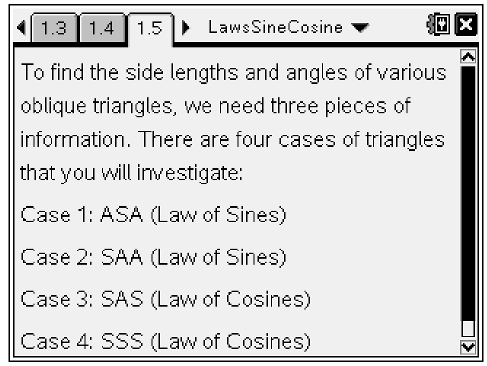
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Sine. It's the Law. (TI-Nspire technology) — 11852*
- *Ain't No River Wide Enough (TI-Nspire technology) — 9886*
- *From 0 to 180: Rethinking the Cosine Law with Data (TI-Nspire technology) — 9833*

Problem 1 – Review of Geometry

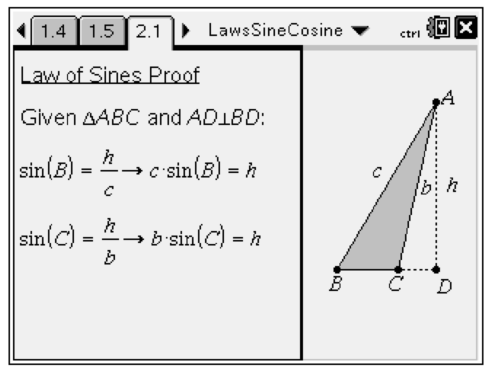
The first problem is an introduction to the activity. On pages 1.2–1.3, students are provided with historical information about the Laws of Sines and Cosines. They are also asked to recall from Geometry what SAS, ASA, SAA, SAS, SSS, and SSA mean and which one does not always work. This is an opportunity for a review of triangle congruency and class discussion. Page 1.5 outlines the specific cases of when the Laws of Sines and Cosines are applied.



Problem 2 – Proof of the Law of Sines

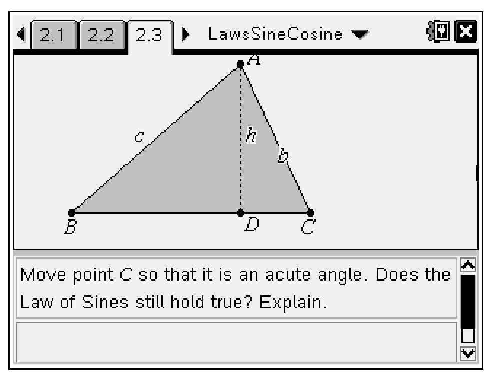
Pages 2.1–2.3 cover the proof of the Law of Sines. The proof involves using right triangle trigonometry.

The angle C refers to the angle ACD.



On page 2.3, students are instructed to grab and drag point C such that the obtuse triangle becomes a right triangle and then an acute triangle.

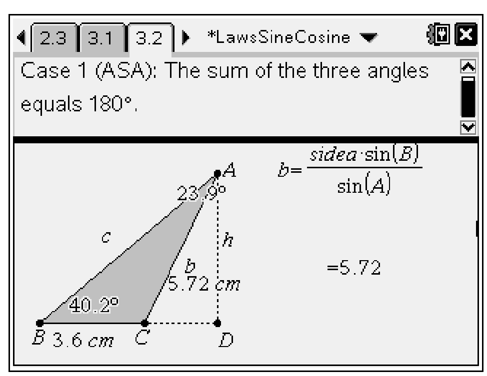
Students are asked if the Law of Sines still holds true. (yes because h is always the height of the triangle no matter what its shape)



Problem 3 – ASA and SAA cases

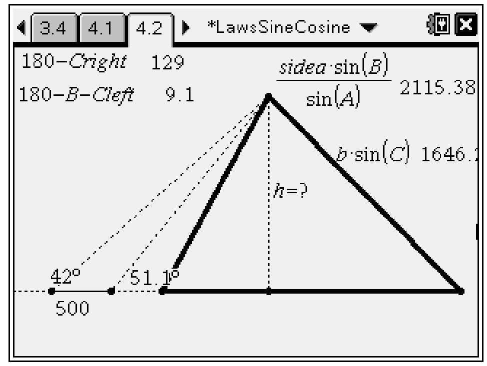
On page 3.2 and 3.3, students will use the Law of Sines to find the length of b. They will need to use the **Calculate** tool from the Actions menu and then select the formula that is displayed on the screen.

Students can use the **Length** tool from the Measurement menu to verify their calculation of side b. They should move point C to see if the measurement and calculation stays the same or changes.



Problem 4 – Law of Sines Problem

On page 4.1, students are given the following problem: *A surveyor took two angle measurements to the peak of the mountain 500m apart. What is the height of the mountain?*



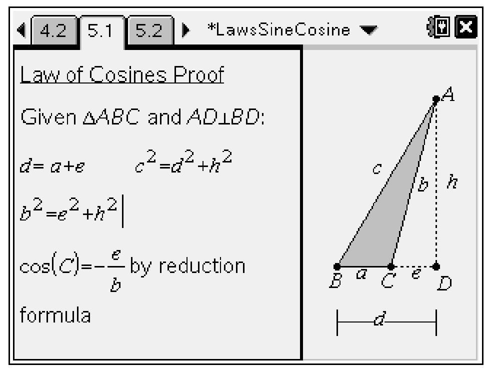
Students can see a picture (not to scale) of the problem on the next page. To solve the problem, they can either use the **Calculate** tool or insert a *Calculator* page.

Cright refers to the angle 51.1° and *Cleft* is the supplementary angle of *Cright*.

Solution: 1646.28 m

Problem 5 – Proof of the Law of Cosines

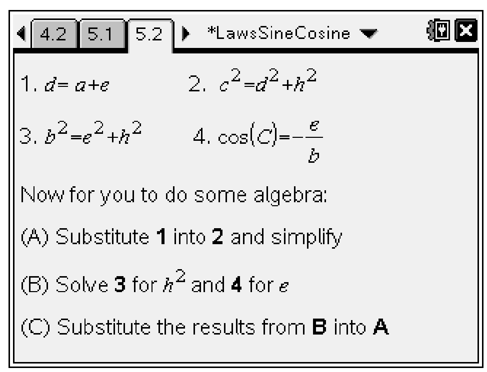
Using the same triangle as the Law of Sines, students explore the proof of the Law of Cosines on pages 5.1–5.3.



Again, the angle C refers to the angle ACD.

Note: The reduction formula $\cos(\theta) = -\cos(\pi - \theta)$ is used to obtain $\cos(C) = -\dfrac{e}{b}$.

Students are asked to use algebra to complete the proof:



(A) $c^2 = (a+e)^2 + h^2$

$c^2 = a^2 + 2ae + e^2 + h^2$

(B) $h^2 = b^2 - e^2$

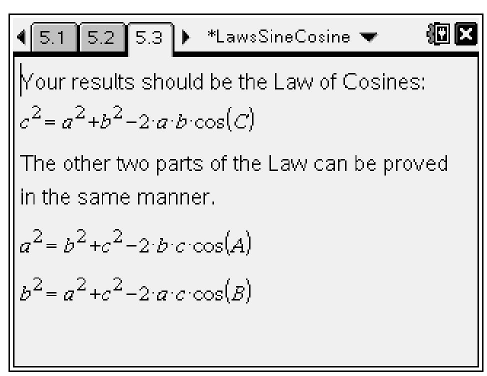
$e = -b \cdot \cos(C)$

(C) $c^2 = a^2 + 2ae + e^2 + b^2 - e^2$

$c^2 = a^2 + 2ae + b^2$

$c^2 = a^2 + 2a(-b \cdot \cos(C)) + b^2$

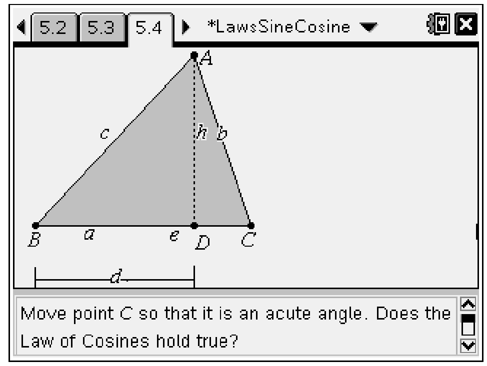
$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$



Students are told that they can prove the other two parts of the Law of Cosines is a similar manner. You may want to discuss with them how to do so.

Students are instructed to grab point *C* again and move the triangle to a right triangle and then an acute triangle.

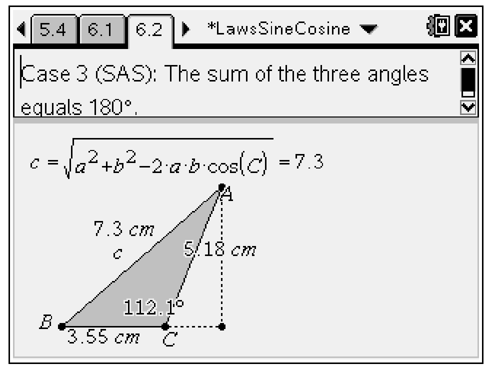
They are asked: *Does the Law of Cosines hold true for all oblique triangles?* (yes)



Problem 6 – SAS and SSS Cases

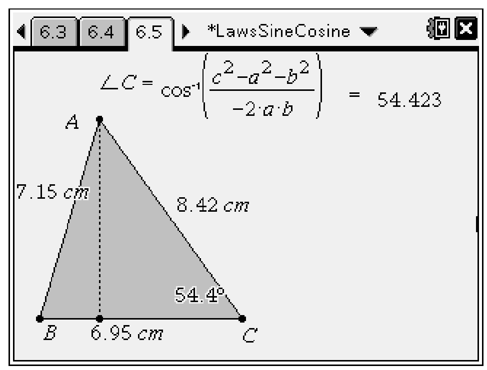
On page 6.2, students will use the Law of Cosines to find the length of side *c*. They will need to use the **Calculate** tool from the Actions menu and then select the formula already displayed on the screen.

Students can use the **Length** tool to verify their calculation of side *c*. They should then move point *C* to see if the measurement and calculation stays the same or changes.



On page 6.5, students will find the measure of angle *C*. They will need to use the inverse cosine function to calculate the angle.

To verify the angle, students should use the **Angle** tool from the Measurement menu.

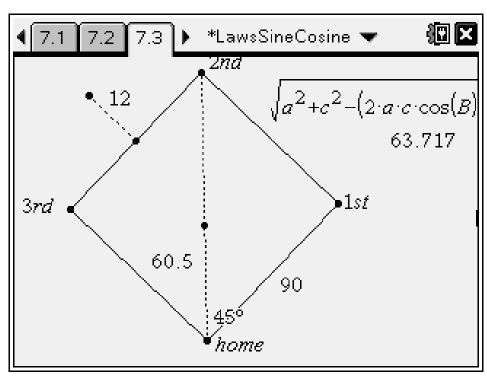


Problem 7 – Law of Cosines Problem

On page 7.1, students are given the following problem:

A Major League baseball diamond is a square with each side measuring 90 feet. The pitching mound is located 60.5 feet from home plate on a line joining home plate and second base.

The solution for the first part of *a* is shown in the screenshot at the right.



a) 63.7 ft; 66.8 ft

b) 92.8°

c) 111.48 ft