Improving the mathematics skills of our citizenry has been a major concern for educators, policy makers, and the general public since long before Sputnik ushered in “new math.” With the most recent decade of education reform and the advent of “new-new math,” advances in mathematics research and education have led to both fruitful exchanges of ideas and challenging debates. Never before has it been so clear that mathematical literacy is vital for our nation’s economic growth, security, and civic progress. And never has the call to bring all children to high levels of mathematical literacy been sounded so forcefully. Yet, though our culture, our country, and our schools by and large expect all adults to be able to read, we do not expect all adults to be proficient in mathematics. (How often does someone utter, “I was never good at math,” only to be met with nods of understanding and compassion?) By and large, Americans accept the kinds of results that come from the widespread belief that not all children can learn mathematics beyond “arithmetic.”

Believing that all children can learn mathematics, and, indeed, that they must, the Council of Chief State School Officers and Texas Instruments Incorporated, have joined together in a partnership to respond to the clarity of purpose and urgency of mission felt in the states today around mathematics education. This partnership will investigate the influences on mathematics education and develop recommendations for effective state actions to lead to improved student performance in mathematics. This paper is the introduction to a series of papers designed to analyze the imperatives and opportunities in several critical areas of mathematics education. The papers will explore the depth and type of mathematical knowledge that students will need to be successful in today’s society; how that depth and type of mathematical knowledge is best taught and what this implies for schools and classrooms; and the conditions that need to be established to create this kind of teaching and learning in every classroom. Specific topics that will be addressed by this series include

- The Imperative of Mathematical Literacy
- Standards, Curriculum, Instruction, and Assessment
- Teacher Preparation and Professional Development
- Teacher Recruitment, Assignment, and Retention
- Opportunities for Support and Partnerships

In the first paper of this series, we made the case for why all students need to be literate in mathematics. High quality standards, curriculum, instruction, and assessment—the focus of this paper—is one set of tools necessary to improving mathematics achievement.

These briefing papers are developed specifically to be disseminated and used by those working to improve mathematics education. Permission is granted to reproduce and to quote items from the papers, as long as references to the authors and sponsoring organizations are provided. For this edition, the recommended citation would be: Stumbo, Circe, and Susan Follett Lusi, (2005), Standards-Based Foundations for Mathematics Education: Standards, Curriculum, Instruction, and Assessment in Mathematics, (Washington, DC: Council of Chief State School Officers and Texas Instruments).

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The mathematics classes that students need in order to acquire the ambitious knowledge and skills required in today’s world are significantly different from the mathematics instruction many of us remember and the classes many of our children experience. In order to assure all students succeed as workers, democratic citizens, and lifelong learners, mathematics education should teach children how to solve both routine and non-routine problems. A routine problem requires a student to use at least one of the four basic arithmetic operations (or ratio) to solve a problem that is practical in nature. A non-routine problem is concerned with developing a student’s mathematical reasoning power and emphasizes heuristics (strategies) rather than practical applications. Different strategies are effective for solving routine as opposed to non-routine problems. Recognizing or creating strategies requires number sense—that is, “having an intuitive feel for number size and combinations, and the ability to work flexibly with numbers in problem situations in order to make sound decisions and reasonable judgments”—and proficiency in computing, geometry, algebra, data analysis, probability, and statistics.

In what ways do we teach students to ensure they acquire this knowledge and set of skills? This question has long been the subject of debate in the United States. Forty years ago, the launching of Sputnik ushered in “new math” as a response to the advances in science and technology enjoyed by the Soviet Union, a nationwide fear that we were falling behind the Soviets in mathematics achievement, and a fear of the repercussions of inferior mathematical literacy. “New math” switched the emphasis on mathematics instruction from teachers “telling” students about mathematics and having students “recite” what they learned back to the teacher to student “inquiry” and “discovery.” New math emphasized abstraction and set theory as an approach to teaching basic mathematics. These changes were difficult for teachers and parents alike, as they struggled with new ways to teach mathematics and felt hamstrung in their own ability to help students with their work. Where it was implemented, new math endured for only about a decade. Though the focus on new math spurred public sentiment around the importance of mathematics education, in the end, new math neither revolutionized mathematics instruction nor helped to close achievement gaps between groups of students.

Whether or not “new math” revolutionized mathematics education, student achievement in mathematics has been rising steadily in the past thirty years, largely due to increasing
expectations as a result of the Sputnik scare. The challenge we face is that the rise in achievement has not been as great as international, economic, and social conditions demand. In the past decade, the collision of a number of impulses—lackluster improvements in student performance, the National Council of Teachers of Mathematics’ (NCTM) development of a set of standards for what students should know and be able to do in mathematics, technological advances, and research on how students learn—has inspired another round of innovations in mathematics instruction, often called “new-new” math. These innovations have spurred yet another, often contentious debate. While there are many different aspects to these “math wars,” the fundamental differences of opinion reside in the relative balance promoted between the teaching of computational skills and the development of problem solving skills. Proponents of focusing teaching on computational skills stress the importance of students knowing how to do mathematical operations and having a strong command of number sense and operations. These reformers fear, for example, that the widespread use of tools such as calculators might allow students to get by without developing strong number sense. Meanwhile, proponents of a focus on problem solving skills stress that not only do students arrive at answers by vastly different means, their beliefs about what numbers represent vary widely. Focusing teaching on identifying those beliefs and strategies, then, had been the domain of the latter set of education reformers.

Today, most mathematics educators agree that a balanced approach to teaching mathematics that honors the need for students to attain proficiency in performing both simple and complex computational skills, as well as knowing how and when to apply those skills when presented with a problem, is called for. As James Milgram, a prominent mathematician concerned with the quality of mathematics education, puts it, “Some schools in our country teach nothing but arithmetic, some nothing but something they call problem solving and mathematical reasoning. Both call what they teach mathematics. Both are wrong.”

Like in reading, mathematics educators do not believe students separate computation from problem solving. Students cannot do their job well if they do not understand the stepping stones of basic computation, yet computation is best learned, most meaningful and motivational, and most likely to produce results when learned in the presence of real-life problems in mathematics. While the relative balance in priority between the two camps remains fodder for the “math wars,” balanced approaches are beginning to prevail nationwide. The emerging consensus around mathematics education has been described by the North Central Regional Educational Laboratory:

- All children should have equal access to appropriately challenging materials.
- The mathematics process includes problem solving, reasoning and proof, communications, and the ability to understand how mathematical elements are represented in different ways.
- Mathematical problems may be solved in multiple ways.
- Technology does not replace the teacher or basic skills. Rather, it is an essential teaching tool; it influences the mathematics that is taught and enhances student learning.

The coming together of divergent paths has been a critical step forward in the improvement of mathematics education. Upon reaching agreement on the need for a balance among curricular and pedagogical approaches, the heavy lifting has just begun, however.
Education leaders throughout the United States are still searching for the most effective levers to ensure each child has access to high quality opportunities to learn mathematics. A recent article in Education Week describes findings from an analysis of results from the 2003 Trends in International Mathematics and Science Study (TIMSS), which suggests at least three areas where the United States lags behind Singapore, the most competitive nation in terms of student performance in mathematics. “Singapore’s domination over the United States in students’ math performance stems from the Southeast Asian country’s uniform expectations for student learning, its use of textbooks rich with problem-solving exercises, and a commitment to producing well-trained teachers….”

Tackling issues in the United States surrounding standards, curriculum, instruction, and assessment should provide the basis of a statewide action agenda for improving mathematical literacy among all children.

What Should All Students Know and Be Able to Do?

Standards, Curriculum, and Instruction

A high quality mathematics curriculum needs to be designed around ambitious standards that reflect the current and future needs of society and include knowledge of content and processes, as well as expectations for each grade. The curricula that flow from these standards needs to be rigorous for all students and it needs to be balanced between teaching computation skills and problem solving. The rigor of the curriculum needs to be thought about in terms of the depth and coherence with which content is taught, rather than the number of discrete topics covered. Assessments of student performance on the standards and curriculum must be timely, informative, valid, and reliable.

Revising Standards in Michigan

In 1998, with the help of Achieve, Inc., a non-profit, non-partisan group created by the nation’s governors to raise academic standards, the Michigan Department of Education took a close look at its content standards and related curricula. Achieve’s report gave Michigan two important findings:

1. The Michigan academic standards are too broad and general, and crucial content found in the benchmark documents is missing in the Michigan standards.
2. Michigan’s assessments embody a more comprehensive and demanding set of expectations for Michigan’s students than might be assumed from reading the state standards alone.

The state of Michigan immediately began reforming their mathematics and English language arts curricula. After a rocky start, Michigan’s government, school, and business leaders came together and convened a group of experts, led by Dr. Bill Schmidt from Michigan State University to revise standards. The resulting mathematics standards were reviewed by Achieve and approved by the Michigan State Board of Education in November 2003. After the board’s approval, Michigan held workshops across the state to inform educators and to hear their feedback on the new standards.

The new mathematics standards are organized into five areas: numbers and operations; algebra; measurement; geometry; and data and probability. As part of its compliance with The No Child Left Behind Act, Michigan also changed from defining its standards in clusters of grades (K–2, 3–5, and 6–8) to defining distinct standards for each grade level. The new standards determine content on which students will be assessed, which in turn influences the instruction students receive in the classroom.

<table>
<thead>
<tr>
<th>STANDARDS—Broad Content Learning Statements</th>
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<tr>
<td>BENCHMARKS—More Discrete Grade Span Learning Expectations</td>
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<td>GLCE—Grade Level Content Performance Expectations</td>
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<td>PACING GUIDE—Time Sequenced Performance Expectations</td>
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<td>INSTRUCTIONAL DESIGN</td>
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Locally Based Assessments

Source for figure: http://www.michigan.gov/mde/0,1607,7-140-47065--,00.html

K–8 Mathematics and English Language Arts Grade Level Content Expectations (GLCE)
These components of standards-based reform have been in development for over the past decade in the United States. Yet, while national and international studies tell us that student performance in mathematics is improving in the United States, we have a long way to go.

In terms of standards, research shows that state or district content standards are driving factors behind teachers’ curricular choices. Seven out of 10 lessons nationally are reported by teachers as being influenced by the standards. However, the quality of state mathematics standards, which are present in 49 states, is variable at best. The American Diploma Project has demonstrated that state content standards in mathematics (as well as English) are not sufficiently aligned with the expectations of the workplace and of postsecondary institutions. The Fordham Foundation’s recent review of state mathematics standards finds them to be inadequate to meet 21st century needs, with only three states being identified as worthy of emulation. As state leaders strive to improve student performance system wide, refinements in state standards will be necessary, yet few states have tackled the task with aplomb.

In terms of curriculum, it has also been shown that the majority of school systems do not have the alignment, coherence, or depth to their mathematics curricula needed to prepare students for today’s economy and civic responsibility. Analysis of the 1999 TIMSS data (student performance data gathered through the Third International Mathematics and Science Study-Repeat) shows that we try to cover more topics in our standards, curriculum, and classroom lessons than do higher performing nations—and, consequently, we cover them in less depth. For example, 38 topics were covered in U.S. eighth grade mathematics texts, while the average for other countries was 23.

Curricula in U.S. classrooms also tend to focus on reviewing content, address fewer complex problems than other curricula in other nations, and place less emphasis on making connections among mathematical facts, procedures, and concepts than do classrooms in higher performing nations. U.S. teachers spend more than twice the time reviewing content than do teachers of the highest performing nations, who spend more time introducing and practicing new content. Japanese classes, for example, also feature a greater number of problems that researchers categorize as high in procedural complexity and use the problems to make mathematical connections. Lower-level content is taught in American eighth grade classrooms than is the case in similar classrooms in Japan or Germany. Researchers rated 87% of American lessons low in mathematical content, and no lessons were rated high. In Japan, by contrast, 13% of lessons were rated low, 57% medium, and 30% high.

The middle school curriculum, in particular, “is replete with repetitious and non-challenging material.” For example, the American middle school curriculum covers the same concepts taught year after year—math topics stay in the U.S. curriculum an average of two years longer than the international average—but apparently with little or no added depth.

In terms of instruction, we also have room for improvement—yet we also have research on effective instructional strategies that can drive that improvement. Not surprisingly, some
teaching strategies are more effective than others. Students' exposure to mathematical concepts and skills directly impacts student achievement; the greater the opportunity, the more students learn.17 “Central to this relationship,” according to Nelson, “is the time devoted to instruction on specific subtopics, especially when the instruction reflects demanding expectations for students.”18

To be of high quality, mathematics instruction needs to be rigorous. It needs to focus on meaningfully developing important mathematical concepts, helping students learn both concepts and skills by solving problems; giving students an opportunity to discover and invent new knowledge and opportunities to practice what they have learned; using small group, as well as whole-class, instruction, and developing "number sense."19 In addition, research has shown that the use of concrete materials, or "manipulatives," increases student achievement. Manipulatives are objects such as blocks, tiles, or sticks that are used to help students physically see the workings of a mathematical formula. Fractions can be better understood, for example, if a student has circles of the same size, but divided into different numbers of pieces to manipulate. With manipulatives, a student can then easily see that 5/5, 3/3 and 2/2 are all equal one, for example, and that 2/5 is smaller than 1/2. Using manipulatives helps students understand the ideas or the "why" of mathematics, in addition to teaching them how to use symbols and formulas.

Finally, the thoughtful use of calculators and other educational technologies also improves mathematics achievement. Contrary to what many believe, researchers have found that using calculators as part of mathematics instruction does not diminish students' computational skills20 and, indeed, can "enhance conceptual understanding, greater ability to choose the correct operations, and greater skill in estimation and mental arithmetic without a loss of basic computational skills."21
How Will We Know If We Are Succeeding? Student Assessments

While statewide content and performance standards articulate the expectations for what students should learn, assessments of student performance against those standards can help to influence instruction and learning. The kinds of assessments used and the way their results are presented make a very real difference to the teaching and learning of mathematics:

The choices policymakers make when selecting assessments and designing accountability systems are critical, because they have a major impact on school practice. Assessments that demand a lot of writing, for example, tend to result in more writing assignments in schools.22

There are two primary benefits to high quality, comprehensive assessment systems. First, they can tell us whether we are moving student performance toward our goals and second, they can help educators to improve instruction. At the district and state levels, valid and reliable assessments that are aligned with standards are designed to measure overall student achievement and achievement trends. Typically, such assessments are “summative” in nature. That is, they are generally given to gauge student knowledge and abilities at the end of a course of study. Summative assessments are the cornerstone of many state and federal policies, such as the federal No Child Left Behind Act, which requires states to track improvements in student performance over time. Summative assessments are important components of meaningful accountability systems. In particular, when results of these assessments are grouped by factors such as race, ethnicity, language, poverty, and special learning needs, we can better see where interventions are most needed to ensure that all

All Means All: The Crucial Issue of Equity

National and international research have made abundantly clear that the days when our nation could afford to have only some students know math are gone. If all middle and high school students are to meet the ambitious math standards of today—and they must—addressing issues of equity is of paramount importance.

The equity issues that must be addressed fall into four categories:

1. Equity of Resources—so that all students have the funding, curriculum, materials, and quality teachers they need;
2. Equity of Preparation—so that all students are prepared to take the higher level courses they need;
3. Equity of Access to Courses—so that all students have the opportunity to take the mathematics courses they need; and
4. Equity of Access to the Curriculum—so that the needs of diverse learners and learning styles are addressed in classrooms so that all students can learn what is taught.

Unfortunately, many people are uncertain whether all students are capable of taking higher-level mathematics courses, and, therefore, many students are not given the opportunity. For example, half or fewer African American, Latino and Native American students took Algebra II in 1998, compared to nearly two-thirds of their White and Asian counterparts.1 All students in other countries are exposed to rigorous content, however, and according to the Education Trust, “a growing body of evidence shows that, however well-meaning, these views [that all students cannot perform to high levels] are dead wrong. The fact is, all students benefit from taking high-level courses regardless of their academic record prior to enrollment.”2

students are progressing, rather than only some.

As statewide summative assessments have become more important, some fear educators may feel pressure to “teach to the test.” Some defenders of summative assessments argue that teaching to the test is not a problem if the test is a good one! Others acknowledge the concern and stress that summative assessments should be only one small component of an effective system of assessments for instructional improvement. While summative assessments can be helpful as education leaders make decisions about curriculum, school organizations, and staffing assignments, more helpful are assessments designed to be “formative” in nature. Formative assessments are diagnostic tests that give teachers rapid feedback on the individual progress of students and immediately inform instruction. Formative assessments tend to be given at the classroom level and are key tools that teachers use to make decisions about their day-to-day lesson plans.

Knowing how to use data generated from both formative and summative assessments to improve instruction is not a straightforward process. Teachers need relatively immediate results from their assessments in order to make decisions about day-to-day lesson plans. Teachers and administrators need time to discuss and analyze work, data, and results from individual lessons and from teaching over time. They need professional development to know how to use their analyses to make decisions about curriculum and instruction, about structures related to time spent on task, and about the assignment of teachers, materials, and supplemental services. With access to high quality formative and summative assessment tools and the right kinds of preparation and professional development to understand how to use the results of such assessments, teachers and school leaders will be better equipped to improve and individualize instruction.

Conclusion

An agenda for comprehensive school improvement that will ensure students become mathematically literate in the 21st century should include standards, curriculum, instruction, and assessments based on research about

- the economic, civic, and societal demands that will be facing us in the future;
- the way students learn and use mathematics to meet those demands;
- the means by which teachers and schools deliver effective instruction so that students can learn and use mathematics proficiently; and
- the effect assessment systems have on teaching and learning.

We have much of this research at our fingertips. As we bring high quality standards, curriculum, instruction, and assessments to each of our classrooms and schools, we must use evidence-based research to continue to track the effects of these standards-based interventions on student performance.

All told, shoring up our system of standards, curriculum, instruction, and assessments must advance mathematics education throughout the nation.
Endnotes


8 Abreu, Julio. (2000, Fall).


Acknowledgements

In an effort to explore new ways of improving mathematics education in middle and high schools, the Council of Chief State School Officers and Texas Instruments formed a Technology Research and Development Advisory Committee (“R&D Committee”) in the spring of 2004. This R&D Committee, consisting of state deputy superintendents or commissioners, district superintendents, and CCSSO staff, met in April 2004 to examine ways in which business and education can work together to build models that will enhance mathematical literacy. The idea for a series of short briefing papers on the core components of mathematics education was born out of that first R&D Committee meeting.

While the analysis and suggestions in this briefing paper are informed by the best collective thinking of that group of teachers, administrators, policy makers, product developers, and researchers, the primary authors of this paper are Circe Stumbo and Susan Follett Lusi. Jamie Poolos is the paper’s editor. The authors wish to thank the many reviewers of the original drafts of this paper, including Lisa Brady-Gill and Richard Schaar of Texas Instruments Incorporated Michael DiMaggio and Rolf Blank of the Council of Chief State School Officers, and Patricia I. Wright.

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The Council of Chief State School Officers (CCSSO) is a nonpartisan, nationwide, nonprofit organization of public officials who head departments of elementary and secondary education in the states, the District of Columbia, the Department of Defense Education Activity, and five U.S. extra-state jurisdictions. CCSSO provides leadership, advocacy, and technical assistance on major educational issues. The Council seeks member consensus on major educational issues and expresses their views to civic and professional organizations, federal agencies, Congress, and the public.

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