Trigonometric Graphs
TRIGONOMETRIC GRAPHS

Aim
The aim of this unit is to investigate trigonometric functions of the form
\[ y = a \sin(bx) + c \quad \text{and} \quad y = a \cos(bx) + c. \]

Objectives
Mathematical objectives
By the end of this topic you should know
• how to describe and sketch a trig graphs by observation of its function.
• how to identify a trigonometric function, given a sketch.
• Know the transformation effects of varying the values of a, b and c.

Calculator objectives
By the end of this session you should be able to
• Graph functions via [Y=].
• Draw graphs of trigonometric functions using appropriate [WINDOW] settings.
• Know how to execute (run) a program stored on the TI-83.
• Know how to [TRACE] along a graph to obtain important points

STUDENT TASK
1. Read the Calculator Skills Sheet (page 3) carefully before you start, it may prevent you encountering difficulties with your TI-83.

2. On the worksheets (except page 8)), for each of the given equations you must:
i. sketch the graph obtained using the TI-83,
ii. find the maximum and minimum value of the function,
iii. find the period,
iv. sketch the graph of the “general case” NOT using the TI-83.
v. complete the statements based on your observations.
TRIGONOMETRIC GRAPHS

Calculator Skills Sheet

Before we can start on this unit of work we must first ensure that your TI-83 is in the correct MODE, and is going to operate as we want it to.

This is how we do this

1. Press the **MODE** button.
   The display should look exactly like this.
   If it does not look like this, then using the cursor keys highlight the correct item in each line and press **ENTER** to change the selection.
   Notice: There can only be one item in each line highlighted.

2. Now press the **ZOOM** and **7**.
   This sets the window range to a built in setting which will accommodate all the work in this unit.

3. Press the **2nd** and **FORMAT**.
   This takes you to the WINDOW FORMAT screen.
   It should look like this.
   If it does not then using the cursor keys highlight the correct item in each line and press **ENTER**.
   Once the screen looks like this press **CLEAR**.
   Notice: There can only be one item in each line highlighted.

4. **ALTERNATIVE WINDOW RANGE**
   If you prefer the graphs on the screen to be similar to those you must sketch set the Window Range to this rather than ZTrig.
   Press the **WINDOW** button.
   Enter the Window Range shown.
   Press **ENTER** after each line.
<table>
<thead>
<tr>
<th>Function</th>
<th>Graph</th>
<th>Max</th>
<th>Min</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $y = \sin(x)$</td>
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<tr>
<td>2. $y = \sin(x) + 1$</td>
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<tr>
<td>3. $y = \sin(x) + 2.5$</td>
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<tr>
<td>4. $y = \sin(x) - 1$</td>
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<tr>
<td>5. $y = \sin(x) - 2$</td>
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<tr>
<td>6. $y = \sin(x) + c$</td>
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</tbody>
</table>

General Case

$y = \sin(x) + c$
Complete these statements
In equations of the form $y = \sin(x) + c$ or $y = \cos(x) + c$ changing the value of $c$ moves the graph ____________________________________________
If $c$ is positive then ____________________________________________
If $c$ is negative then ____________________________________________

<table>
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<tr>
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<tbody>
<tr>
<td>7. $y = \cos(x)$</td>
<td><img src="image1.png" alt="Graph" /></td>
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</tr>
<tr>
<td>8. $y = \cos(x) + 1$</td>
<td><img src="image2.png" alt="Graph" /></td>
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</tr>
<tr>
<td>9. $y = \cos(x) + 2.5$</td>
<td><img src="image3.png" alt="Graph" /></td>
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<tr>
<td>10. $y = \cos(x) - 1$</td>
<td><img src="image4.png" alt="Graph" /></td>
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<tr>
<td>General Case</td>
<td><img src="image5.png" alt="Graph" /></td>
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<tr>
<td>11. $y = \cos(x) + c$</td>
<td><img src="image6.png" alt="Graph" /></td>
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<tr>
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<tr>
<td>12. ( y = \sin (x) )</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Max" /></td>
<td><img src="image3" alt="Min" /></td>
<td><img src="image4" alt="Period" /></td>
</tr>
<tr>
<td>13. ( y = 3 \sin (x) )</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Max" /></td>
<td><img src="image7" alt="Min" /></td>
<td><img src="image8" alt="Period" /></td>
</tr>
<tr>
<td>14. ( y = 5 \sin (x) )</td>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Max" /></td>
<td><img src="image11" alt="Min" /></td>
<td><img src="image12" alt="Period" /></td>
</tr>
<tr>
<td>15. ( y = \frac{1}{2} \sin (x) )</td>
<td><img src="image13" alt="Graph" /></td>
<td><img src="image14" alt="Max" /></td>
<td><img src="image15" alt="Min" /></td>
<td><img src="image16" alt="Period" /></td>
</tr>
<tr>
<td>16. ( y = \frac{1}{4} \sin (x) )</td>
<td><img src="image17" alt="Graph" /></td>
<td><img src="image18" alt="Max" /></td>
<td><img src="image19" alt="Min" /></td>
<td><img src="image20" alt="Period" /></td>
</tr>
</tbody>
</table>

**General Case**

17. \( y = a \sin (x) \) | ![Graph](image21) | ![Max](image22) | ![Min](image23) | ![Period](image24) |
Complete these statements.

In equations of the form $y = a \sin(x)$ or $y = a \cos(x)$ changing the value of $a$ changes the ____________________________.

$a$ gives the ___________________ of the function.
<table>
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<tr>
<td>$y = \sin (x)$</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Max" /></td>
<td><img src="image3" alt="Min" /></td>
<td><img src="image4" alt="Period" /></td>
</tr>
<tr>
<td>$y = \sin (2x)$</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Max" /></td>
<td><img src="image3" alt="Min" /></td>
<td><img src="image4" alt="Period" /></td>
</tr>
<tr>
<td>$y = \sin (3x)$</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Max" /></td>
<td><img src="image3" alt="Min" /></td>
<td><img src="image4" alt="Period" /></td>
</tr>
<tr>
<td>$y = \sin (4x)$</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Max" /></td>
<td><img src="image3" alt="Min" /></td>
<td><img src="image4" alt="Period" /></td>
</tr>
<tr>
<td>$y = \sin (\frac{1}{2}x)$</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Max" /></td>
<td><img src="image3" alt="Min" /></td>
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<td><img src="image3" alt="Min" /></td>
<td><img src="image4" alt="Period" /></td>
</tr>
<tr>
<td>$y = \sin (bx)$</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Max" /></td>
<td><img src="image3" alt="Min" /></td>
<td><img src="image4" alt="Period" /></td>
</tr>
</tbody>
</table>
Complete these statements

In equations of the form $y = \sin(bx)$ or $y = \cos(bx)$ changing the value of $b$ changes the ________________.

$b$ gives the ________________.

The ________________ of the graph is always $\frac{\circ}{b}$.
<table>
<thead>
<tr>
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<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>34. ( y = \sin(x) )</td>
<td><img src="image" alt="Graph" /></td>
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<tr>
<td>35. ( y = 3\sin(2x) + 1 )</td>
<td><img src="image" alt="Graph" /></td>
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<tr>
<td>36. ( y = 2\sin(3x) - 2 )</td>
<td><img src="image" alt="Graph" /></td>
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<tr>
<td>37. ( y = 2\frac{1}{2}\sin(3x) + 1 )</td>
<td><img src="image" alt="Graph" /></td>
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<tr>
<td>38. ( y = 2\sin\left(\frac{1}{2}x\right) - \frac{1}{2} )</td>
<td><img src="image" alt="Graph" /></td>
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<tr>
<td>General Case</td>
<td><img src="image" alt="Graph" /></td>
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<tr>
<td>39. ( y = a\sin(bx) + c )</td>
<td><img src="image" alt="Graph" /></td>
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</tbody>
</table>
Complete these statements.

In equations of the form \( y = \sin(bx) + c \) or \( y = \cos(bx) + c \)

- \( a \) changes the _______________.
- \( b \) changes the _______________.
- \( c \) changes the _______________.

The **Maximum** is always _______________. The **Minimum** is always _______________.

The **Period** is always _______________.

---

**General Case**

44. \( y = \cos(bx) + c \)
Can you name each of these functions?

**Scale:**
Vertical: Each dot is 1 unit.
Horizontal: -360° to 360°, each dot is 90°

45. 46. 47. 48.

\[ y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \]

49. 50. 51. 52.

\[ y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \]

**Scale:**
Vertical: Each dot is 1 unit.
Horizontal: -90° to 720°, each dot is 90°

53. 54. 55. 56.

\[ y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \]

57. 58. 59. 60.

\[ y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \quad y = \_\_\_\_\_\_ \]
61. \( y = \sin(x) \)

62. \( y = \sin(x + 45) \)

63. \( y = \sin(x - 30) \)

64. \( y = \cos(x + 30) \)

65. \( y = \cos(x - 45) \)

66. \( y = \sin(x + \alpha) \)
Now run the program **MADTRIG**.

This program will generate a trig. graph of the form
\[ y = c \sin(dx) \quad \text{or} \quad y = c \cos(dx). \]
You have look at the graph and identify the values of \( c \) and \( d \).
The program will prompt you.
Good Luck