STEM ACTIVITIES WITH TI-NSPRIE

USING REAL WORLD DATA

INTRODUCTION

Why use TI-Nspire with real world data?

The data handling features of TI-Nspire and the emphasis in the new secondary curriculum on real-life contexts, curriculum opportunities and cross-curricular dimensions, all combine to give a strong rationale for introducing activities using TI-Nspire which use real data and link STEM subjects.

There is now a wealth of data available on the Internet in a variety of forms. TI-Nspire can make use of this data as it has the facility to copy and paste information from tables and spreadsheets into a Lists & Spreadsheet page. It is then possible to manipulate, display and analyse this data in a variety of ways using facilities from 'Data & Statistics'. It is also possible to further analyse or attempt to model the data by inserting movable lines, plotting functions or fitting a variety of functions using different forms of regression.

Mathematics and statistical education

In 2005 there was a review of the Mathematics curriculum and wide-ranging research was undertaken into the part that statistical education should play in this. The results of this were contained in a report from the Royal Statistical Society’s Centre for Statistical Education in association with QCA (now QCDA).

The key features of this report are that though it was felt that statistical education should be set in contexts from other curriculum areas especially Science, it should still be delivered by mathematics teachers. The mathematics teachers would however need to be supported by appropriate resources and background information.

http://www.rsscse.org.uk/qca/recommendations.htm

Sets of materials for teachers for 8 activities were produced jointly by the RSS and QCA. One of these is 'What’s in your bowl?' which looks at the nutritional content of breakfast cereals and is linked to one of the activities in this booklet.
Curriculum opportunities

The new curriculum in England gives considerable importance to curriculum opportunities in both mathematics and science and also to cross-curricular dimensions which include the following:
- healthy lifestyles
- global dimension and sustainable development
- technology and the media
- creativity and critical thinking

Real world data activities

The QCA/RSS report conclusions combined with the availability of data over the internet and the range of facilities of Ti-Inspire provide a wealth of opportunities for STEM activities using real world data. The examples of data chosen for use in the activities in this booklet fit into two main headings:

Modelling
Some activities offer opportunities to try to find formulas or to fit functions to scientific data to model it mathematically.

Handling data cycle
Other activities offer opportunities to examine real-world data to
- See what the data show and find representations to illustrate these findings more clearly.
- Make and test hypotheses using a student-centred problem solving approach.

This might include
- looking for trends in data and predicting future results
- comparing sets of data, presenting and analysing results and justifying conclusions
**About the activities**

All the activities in this booklet contain some background scientific or other information together with links to appropriate websites for further information and other resources. Many of the activities are suitable for a range of ages and aptitudes with more challenging ideas being suggested as extension activities. The introduction to the activities gives some idea of the subject content and the Ti-Nspire features that could be used. Some activities have additional information such as teachers’ notes, further background information or additional advice for students including getting started with Ti-Nspire. Information to help get started with using Ti-Nspire features is also included in the booklet ‘STEM activities with Ti-Nspire: Introduction’. Further information and Ti-Nspire files for the activities can be found at [www.nspiringlearning.org.uk](http://www.nspiringlearning.org.uk)

## CONTENTS

<table>
<thead>
<tr>
<th>Page</th>
<th>Activity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1 Investigating the nutritional content of breakfast cereals</td>
<td>Learners can make and test hypotheses comparing nutritional contents of breakfast cereals, including interpretation and using inequalities.</td>
</tr>
<tr>
<td>13</td>
<td>2 Hydrocarbons</td>
<td>This activity looks at the structure of different groups of hydrocarbons and investigates mathematical patterns and relationships between their chemical structures and formulas.</td>
</tr>
<tr>
<td>18</td>
<td>3 Perfect pitch</td>
<td>This activity looks for connections between frequencies of musical note sequences and considers wavelengths. It also considers geometric sequences and looks at historical and cultural links.</td>
</tr>
<tr>
<td>23</td>
<td>4 Compare the weather</td>
<td>This activity investigates different aspects of world weather data for a range of different climate extremes. Students are invited to read and interpret a large table of data and to find effective ways to display and analyse subsets of this data in order to illustrate comparisons.</td>
</tr>
<tr>
<td>29</td>
<td>5 Hurricane force</td>
<td>This activity looks at interpreting weather station data, considers wind speed and can be extended to more advanced mathematics, by fitting functions to the Beaufort and Torro scales.</td>
</tr>
<tr>
<td>38</td>
<td>6 Carbon dating</td>
<td>This activity looks at Carbon Dating and Radioactive decay, interpreting graphs and comparing functions and can be extended to include inverse and exponential functions, logarithms, differential equations and scientific theory.</td>
</tr>
<tr>
<td>44</td>
<td>7 Reaction times</td>
<td>Learners can make hypotheses about reaction times and set up an experiment to test hypotheses, then make use of Ti-Nspire to illustrate and analyse their data. This activity is suitable for project work and could be extended using more advanced statistical techniques.</td>
</tr>
</tbody>
</table>
Chapter 1

Investigating the nutritional content of breakfast cereals
Introduction

This activity makes use of TI-Nspire data-handling facilities to investigate the nutritional contents of breakfast cereals. This activity could provide an introduction to data entry and other features of Lists & Spreadsheets and encourage the use of ICT. The introduction to the activity using the ‘traffic light’ food labelling system enables students to meet and work with inequalities in a real context.

The main activity suggests possible lines of enquiry and provides a small database of information on a selection of breakfast cereals in a TI-Nspire file, with a few examples of how such data might be explored and analysed. Ideally students should pose their own questions and select or collect their own data to analyse using the Handling data cycle. Suggestions for where to obtain further data are given.

Background information

There is a wide range of information on the Internet about the nutritional content of foods including on manufacturers and fast food restaurant chain websites. Some examples of these, together with other sources of nutritional information are given later in the activity.

Sources of further ideas:

One school in the QCDA ‘Engaging mathematics for all learners’ project http://www.qcda.gov.uk/22221.aspx, set their students the task of posing their own questions comparing different types of meals from a well know fast food restaurant chain.

‘What’s in your bowl?’

The Royal Statistical Society in association with QCDA have produced and trialled data-handling lesson materials; http://www.rsscse.org.uk/qca/resources0.htm. There are eight activities one of which is ‘What’s in your bowl’. The activity is aimed at 11-14 year olds. Lesson suggestions, background information, teachers’ notes, pupil worksheets and a PowerPoint presentation can be found at http://www.rsscse.org.uk/qca/resource7.htm.

The emphasis is on pupils suggesting their own questions, planning which data to collect and how to collect it and planning how to organise and analyse the data and communicate their findings.

Using TI-Nspire

The TI-Nspire activity gives students a sample data file and instructions for how to manipulate it. They can then go on to plan their own investigation and collect and analyse their own data. Some sources of internet data are in table or spreadsheet form. These can be copied and pasted into a List & Spreadsheet page of TI-Nspire. It may be easier to paste larger data sets into a spreadsheet such as Microsoft Excel first, and then to copy and paste extracts from this into the TI-Nspire. Examples of particular TI-Nspire features which might be useful are also given in the STEM Activities Introduction booklet.
The Activity

The RSS QCA activity ‘What’s in your bowl?’ has discussion suggestions and lesson ideas and recommends a number of websites for further information. Two possible starting points for discussion are suggested below, the first of which could link into the second.

Recommended Daily Allowances (RDA)
The guideline daily amounts for adults are usually quoted as

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>2000Kcal</td>
</tr>
<tr>
<td>Protein</td>
<td>45g</td>
</tr>
<tr>
<td>Carbohydrate (of which sugars)</td>
<td>230g (90g)</td>
</tr>
<tr>
<td>Fat (of which saturates)</td>
<td>70g (20g)</td>
</tr>
<tr>
<td>Fibre</td>
<td>24g</td>
</tr>
<tr>
<td>Salt (of which sodium)</td>
<td>6g (2.4g)</td>
</tr>
</tbody>
</table>

Possible Discussion Points:
• What does this mean?
• Which types of food contain carbohydrate, sugar, fat, fibre etc?
• Which of these should you only eat in moderation and the figure shown is the maximum recommendation?
• Which of these are important and you should aim to eat at least as much as the amount shown?
• How well do you think your food intake for an average day compares with this?
• How could you find out?
• How is food labelled to help you find out?

More information on the traffic light system for labelling is given below.

The TI-Nspire file gives information obtained from a number of breakfast cereal packets. The RSS QCA activity has information from a number of breakfast cereals. It would also be useful to have some examples of cereal packets for students to view.

Some points to note:
• Men, women and children: The above figures are the most common ones quoted. The figures shown are sometimes given as the figures for women with those for men being slightly higher and those for children being slightly lower. These figures could be increased for people with a very active lifestyle. A factsheet of food labels is available from http://www.bda.uk.com/Downloads/foodlabels.pdf
• Portion size: The figures used in the TI-Nspire file for Breakfast cereals are for 100 grams of cereal not an average portion with milk. Portion sizes for different cereals vary and the size suggested by the manufacturer is shown in a column on the spreadsheet. However, research has shown that the average portion size actually used tends to be considerably greater than that the manufacturers’ suggestions.
• Salt and sodium: The TI-Nspire spreadsheet contains the figure for salt. Most packets now have both sodium and salt mentioned, but some manufacturers’ websites only give the sodium content. The salt content would be 2.5 times the figure given for sodium.
• **Maximum or minimum:** For some of the contents such as salt, sugar and fat the figure is a recommended daily maximum while for others such as fibre and protein it is a minimum. This is an important discussion point.

**Traffic light food labelling**

To simplify checking the nutritional content of food, the Food Standards Agency has introduced a ‘Traffic light’ food labelling system on the front of some food packets. **Green** means low (good)  **Amber** means medium (fair)  **Red** means high (bad)

<table>
<thead>
<tr>
<th></th>
<th>Low (Green) Per 100g</th>
<th>Medium (Amber) Per 100g</th>
<th>High (Red) Per 100g</th>
<th>Foods are also labelled as High (red) if one portion contains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat</td>
<td>≤ 3.0g</td>
<td>&gt; 3.0g ≤ 20.0g</td>
<td>&gt; 20.0g</td>
<td></td>
</tr>
<tr>
<td>Saturated fat</td>
<td>≤ 1.5g</td>
<td>&gt; 1.5g ≤ 5.0g</td>
<td>&gt; 5.0g</td>
<td></td>
</tr>
<tr>
<td>Sugars</td>
<td>≤ 5.0g</td>
<td>&gt; 5.0g ≤ 12.5g</td>
<td>&gt; 12.5g</td>
<td></td>
</tr>
<tr>
<td>Salt</td>
<td>≤ 0.30g</td>
<td>&gt; 0.30g ≤ 1.50g</td>
<td>&gt; 1.50g</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Most manufacturers only include the total figure for sugars. Some sugars are naturally occurring (e.g. in fruit added to breakfast cereals) and some sugar may be added in the processing. There are some variations in the traffic light labelling for the two types but this has not been included here.

**Initial activity**

Students look at the contents of the breakfast cereals and try to apply the ‘Traffic light’ labelling system to the salt, sugar, fat and saturated fat content of them. The RSS QCA activity has a work sheet for this.

**How can you quickly identify which cereals are green, amber or red for fat, sugars etc.?**  fig_02

One way to do help with this is to
- display the data in a ‘dot plot’ on a Data & Statistics page and then to
- add two ‘movable lines’ (menu 4) and to
- grab and drag these lines to the appropriate places getting the figure displayed as close as possible (this could be practice at interpreting inequalities and decimal numbers)
- line 1: to the boundary between green and amber (5.0g for sugar, 0.3g for salt)
- line 2: between amber and red (12.5g for sugar, 1.5g for salt)

**Which cereals would have green, amber, red labels?**  fig_03

This can give a feel for the boundaries and a quick picture of the results but is not very accurate.
- If the cereals were numbered, the data could be displayed as a scatter plot from menu 1 on a Data & Statistics page.
- Boundary lines could be entered as functions using ‘Plot function on menu 4. (Note: Y window setting was increased)
- A Graphs page could also be used.
Main Activity

Investigating the nutritional content of breakfast cereals

1. Pose questions
After the initial discussions and activity hopefully students have some ideas of the sort of questions they might consider about the nutritional content of breakfast cereals.

Students may wish to group cereals under certain headings for comparison such as
• main cereal content – wheat/ oats or wholegrain/ others
• specific additions – honey ; chocolate ; nuts; fruit

Teachers may wish to guide this discussion if they want to encourage specific statistical concepts and processes such as comparing two distributions or looking for possible correlation. For example:-
• Do chocolate cereals contain more sugar than honey cereals?
• Do cereals with more sugar contain less protein?

2. Collect data
Having decided on the question they wish to investigate, students could use the given spreadsheet on the website at www.nspiringlearning.org.uk and make their own additions or set up their own spreadsheet using instructions in the STEM Activities Introduction booklet. Possible sources of data are:-
• The spreadsheet in the TI-Nspire file which has a mixture of figures from different sources
• Information on cereal packets
• Information on manufacturers’ web sites.
• Data from the RSS QCDA activity ‘What’s in your bowl?’
• The activity does not need to be comparing breakfast cereals. Fast food restaurants also have comprehensive data on their web sites about the contents of their food.

Breakfast cereals
http://www.kelloggs.co.uk/products/
http://caloriecount.about.com/quaker-oats-nutrition-m2
http://www.cerealpartners.co.uk/
http://www.weetabix.co.uk/nutrition/

Others
http://nutrition.mcdonalds.com/nutritionexchange/nutrition_facts.html#download
http://www.burgerking.co.uk/nutrition
http://www.wimpy.uk.com/menu-bgr.html

Analyse and present the data & Interpret and Discuss

There are further ideas for doing this in the activity ‘Reaction Times’ and in the STEM Activities Introduction booklet. Students should already have collected appropriate data to answer their questions.
Example A – Looking for correlation fig_04 / 05

– Do cereals with more sugar contain less protein?

- Insert a ‘Data & Statistics’ page
- From menu 2 select ‘Add X variable’ and select protein (or sugar)
- Then select ‘Add Y variable’ and choose sugar (or protein)
- What does the diagram show?
- Does it look like there is reasonable correlation between these figures?
  What type of correlation?
- Would a line of best fit be appropriate?
- What would this show?
- What else could you use? (More advanced students)

Example B – Comparing distributions fig_06

– Do fruit cereals contain more fibre than non-fruit cereals?

Note that the spreadsheet of data in the TI-Nspire sample file contains a mixture of cereals and has not been set up for a specific question such as the one posed here, so this is used as an example of the use of TI-Nspire and not of a good investigation.

- Open a New Document and Copy and paste the two sets of data for the different types of cereals into separate Lists & Spreadsheet pages in the same document. fig_07
- You will need to manually insert the column headings and should ensure that you use different names for the two pages (note that the second page has kcal, proteinf etc)
A Box plot is a quick way to compare and analyse these two sets of data

- First insert a Data & Statistics page and set the caption to the appropriate data set e.g. cereal.
- Then ‘Add X variable’ (menu 2) or move to the centre of lower edge of the screen and select variable. Choose the variable from the list e.g. fibre for cereal or fibref for fruitcereal.
- This will give you a dot chart. Then from menu 1 select Box plot. Repeat for the second set of data.
- The two diagrams above fig_08 / 09 were produced automatically using the default scale settings. Menu 5 and ‘Window settings’ could be used to set them to the same window for comparison. They could also be shown side by side by changing the page layout.

What questions would you ask about these two diagrams and what they showed? The mean, median, quartiles and maximum and minimum values can all be found from these box plots or by using the spreadsheet and catalogue features to do the calculations.
Introduction

Mathematically this activity is similar to many others which involve spotting patterns, finding term to term and nth term formulas. The main difference in the activity is that the problem itself comes from Chemistry and the formulas are actual Chemical formulas, so that a genuine connection can be made between subjects and it gives a real world reason for looking for general formulas. The availability of information on the internet also means that students can make predictions using mathematics and then find out more about hydrocarbons and see whether their predictions are correct using Internet research. Some students will inevitably find the general chemical formulas from internet research, so the challenge then is to use the patterns in the chemical structure diagrams to explain why these general rules work.

This activity can make use of a number of features of TI-Nspire without much prior experience so not only does TI-Nspire give the user the facility to link together different aspects of the activity, but the activity itself could also provide an introduction to some of the features of the handheld or software.

Hydrocarbons activity | Alkanes

Atoms of carbon combine with atoms of hydrogen to form hydrocarbons. Carbon atoms have valency 4. This means that each carbon atom has 4 bonds which can link to other atoms. Hydrogen atoms only have a single bond. Saturated hydrocarbons or alkanes are the simplest type of hydrocarbon. The atoms are linked in a straight chain. They are composed entirely of single bonds and are saturated with hydrogen i.e. every bond is used.

The first 3 alkanes are shown. They are methane CH4 fi g_10; ethane C2H6 fi g_11; propane C3H8 fi g_12

- Can you predict what alkanes with more carbon atoms will look like? You could use pencil and paper to do this or you could try a Geometry page to draw these. fi g_12 shows Propane using small circles for carbon, points for hydrogen and line segments for the bonds.

- Can you suggest any connections between the number of carbon atoms and
  - the number of hydrogen atoms
  - the number of bonds
  - the chemical formulas

- Can you complete more rows and columns of the table below?
  You could use a ‘Lists & Spreadsheet’ page to help you do this.

<table>
<thead>
<tr>
<th>Name</th>
<th>Carbon atoms</th>
<th>Hydrogen atoms</th>
<th>Number of bonds</th>
<th>Chemical formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methane</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>CH4</td>
</tr>
<tr>
<td>Ethane</td>
<td>2</td>
<td>6</td>
<td>7</td>
<td>C2H6</td>
</tr>
<tr>
<td>Propane</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Butane</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pentane</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hexane</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Could you predict what happens for other saturated hydrocarbons such as Decane which has 10 carbon atoms?

Could you suggest any general formulas and predict other hydrocarbons and their formulas?

Can you explain the methods that you used to find patterns and formulas? You could use a 'Notes' page for your explanation, but you might also have used formulas in a spreadsheet page or fitted functions to a graph. (There is a helpsheet for students with more information about doing this.)

Can you explain why your rules work? How do the patterns in the diagrams of the chemicals help to explain this?

What else can you find out about hydrocarbons? (Possible internet research)

**Extension activity | Alkenes and alkynes**

Unsaturated hydrocarbons have one or more double or triple bonds between carbon atoms.
Those with one double bond are called **alkenes**.
Those with a triple bond are called **alkynes**

**Alkenes**
The first two alkenes are Ethene $C_2H_4$ and Propene $C_3H_6$.
- Can you sketch some more alkenes and predict possible connections between the number of carbon atoms and the number of hydrogen atoms, bonds and the chemical formula? You might find a table useful like the one used for alkanes.

The first two alkenes (2 and 3 carbon atoms) are shown in fig_14 and fig_15 and have a unique structure. The diagram for Propene has been drawn with the double bond on the left, but the structure would be the same if the mirror image was drawn i.e. the double bond on the right.

The structure for Propene could be broken down as $\text{CH}_2=\text{CH}-\text{CH}_3$

**Alkenes with 4 or more carbon atoms can have more than one structure.**
Butene (4 carbon atoms) for example can have the double bond either in the centre or at one side of the chain of carbon atoms.
- Does the drawing and structure make any difference to the number of hydrogen atoms, bonds or the chemical formula?
- Can you predict the number of different structures for alkenes depending on the number of carbon atoms and use the diagrams to explain how your rules work?

**Alkynes**
What rules and formulas can you predict for alkynes which have one triple bond between carbon atoms?
The first alkyne is Ethyne $C_2H_2$. fig_16
Can you find any other types of hydrocarbons?
Possible further extension activity

Alkanes may be gases, liquids or solids at room temperature. Their boiling points and solidification points vary. The more carbon atoms there are the lower the boiling point so a further more advanced extension activity could be to search for a connection between the boiling point or solidification point and the number of carbon atoms. This has important practical considerations which might be relevant to students, for example which to use as fuel for camping stoves in different climatic conditions. http://library.thinkquest.org/3659/orgchem/alkanes.html

Sources of further information

An internet search will give a lot more information about alkanes, alkenes and alkynes. Some sites such as Wikipedia give rather too much information which can be very daunting and complex. Some sites go straight to the chemical formula. Searches under the three individual headings can prove more useful than a general search for ‘hydrocarbons’. These are some examples:

http://www.chemguide.co.uk/organicprops/alkanes/background.html#top
http://www.chemguide.co.uk/organicprops/alkenes/background.html#top
http://hyperphysics.phy-astr.gsu.edu/HBASE/Organic/alkane.html
http://hyperphysics.phy-astr.gsu.edu/hbase/Organic/alkene.html
http://library.thinkquest.org/3659/orgchem/alkanes.html
http://www.gcsescience.com/ihydrocarbons.htm
http://library.thinkquest.org/3659/orgchem/alkenes-alkynes.html

Further help for students

1. Using formulas in a spreadsheet

One way to test out formulas is to open a ‘Lists & Spreadsheet’ page and enter the information in the table. You could then test out your formulas by entering them into the spreadsheet.

To enter a formula for a particular cell type “=” followed by the cell label e.g. “B5” and the operation e.g. “+1”. Such formulas can be copied and pasted down a row. Press enter to see if you get the value you want.

If you have a general formula you can enter this in the cell at the top of a column. For example if you thought that the number of hydrogen atoms is equal to one more than three times the number of carbon atoms you could type in

- “=3*carbon + 1” in the shaded cell at the top of a new column or
- “=3*b[] + 1” since column B has the carbon figures in. Square brackets [] indicate a column.

Press enter and see if the values in the new column agree with those in the hydrogen column.

- If they don’t match why do you think this is?
- Can you find a rule that does work?
- What about formulas for the number of bonds or for the general chemical formula?
2. **Fitting a formula to a Scatter Plot**

Another way to look for patterns and rules is to open a Graphs page and plot points where the x-coordinate is one of the values (say the number of carbon atoms) and the y coordinate is the value that goes with it from another column (say the number of hydrogen atoms).

- Open a Graphs & Geometry page.
- From menu 4 select ‘zoom quadrant 1’ then from menu 3 select ‘Scatter Plot’
- Use the arrow keys in the boxes to set x and y to the variables that you want from the spreadsheet columns - in this case ‘carbon’ for x and ‘hydrogen’ for y. You can type these in or use the ‘vars’ menu. This will give you a scatterplot for all the values you have entered in these columns.
- The next step is to try to fit the graph of a function, so from menu 3 select ‘function’ and type in your rule after ‘f1(x) =’ in the entry line at the foot of the screen.

For example if you think that hydrogen atoms = 3* carbon atoms + 1, type in $f1(x) = 3x + 1$

**Some questions to think about**

- Does the graph fit the points on the scatterplot?
- Can you find a rule which does fit?
- What are the connections between the points in your table, the function for the graph and your rule?
- What other rules can you find connecting the number of carbon atoms to other features of hydrocarbons such as the number of bonds or the chemical formula?
- Can you explain how the rules you have found can be obtained by looking at the diagrams for the chemical structure?

**Extension suggestions**

Try to set up a new document with a ‘Lists & Spreadsheet’ page so that you can investigate ‘Alkenes’ or ‘Alkynes’

Find out more about alkanes, alkenes and alkynes by doing some internet research. How well do your formulas match? What else could you investigate?
Chapter 3

Perfect pitch
Introduction

This activity looks at connections and patterns in the frequencies and wavelengths of musical notes, and links mathematics to music and science. The mathematics starts by looking at rules to get from one term to the next in a sequence and can lead into reciprocal functions and geometric sequences. Using TI-Nspire can make both the mathematics and the science more accessible.

Background information

Have you ever thought about how musical notes are produced? What happens when you pluck a guitar string or press a key on a keyboard? What makes high or low notes? Vibrations of strings or columns of air produce sound waves. Sound waves are longitudinal waves. The vibration is in the direction that the wave travels. The number of complete vibrations or cycles per second that produce a particular note is called the frequency and is measured in Hertz (Hz). In recent centuries the frequencies of notes played by different instruments of an orchestra have been standardised using a ‘well-tempered scale’ so that they sound in tune with each other despite being in different keys. The instruments of an orchestra all tune up before playing to a particular note – the A above middle C on a piano often referred to as A4 with a standardised frequency of 440Hz. This is shown in the diagram \textit{fig.} \textit{21}. Different systems of labelling notes can be found but just two systems will be referred to in this activity.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig_21}
\caption{Diagram showing the frequency of notes on a piano keyboard.}
\end{figure}

1. Using conventional names for notes

One system is to start at C0 which is below the usual lowest note on a full-sized piano keyboard and each octave after that will be C1, C2, C3, etc as shown in the image \textit{fig.} \textit{21}.

Between the white notes shown on the keyboard the black notes are either flats or sharps. They are named after the adjacent white notes. If they are slightly below the note then they are flats (b) and slightly above then they are sharps (#). For example the black key between an A and a B could be labelled either A\textsuperscript{#} or Bb.
2. The MIDI system

The MIDI system for electronic musical instruments and computers uses a straight count starting with note 0 for C-1 at 8.1758 Hz up to note 127 for G9 at 12,544 Hz.

C-1 and G9 are outside the range of the piano keyboard shown. C-1 is one octave below C0 and 2 octaves below C1 which is the first C to be shown on the keyboard above. At the other end G9 is almost 2 octaves above the highest note shown.

The table below shows some examples of notes that are on the piano keyboard using both labelling systems.

<table>
<thead>
<tr>
<th>note</th>
<th>note</th>
<th>note</th>
<th>note</th>
<th>note</th>
<th>note</th>
<th>note</th>
<th>note</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td>A#0/</td>
<td>B0</td>
<td>C1</td>
<td>C#1/</td>
<td>D1</td>
<td>D#1/</td>
<td>E1</td>
</tr>
<tr>
<td>Bb0</td>
<td>Bb0</td>
<td>C1</td>
<td>C#1/</td>
<td>D1</td>
<td>E1</td>
<td>F1</td>
<td>F#1/</td>
</tr>
<tr>
<td>B0</td>
<td>C1</td>
<td>C#1/</td>
<td>D1</td>
<td>E1</td>
<td>F1</td>
<td>F#1/</td>
<td>Gb1</td>
</tr>
</tbody>
</table>

The activity

At the end of this activity is an extract from a set of data relating note numbers, names, frequencies and wavelengths.

- Do you notice any patterns or possible relationships in the table?
- Can you spot any patterns in the note frequencies?
- What sort of relationship do you think there is between the frequency and the wavelength?
- How can you use the facilities of TI-Nspire to help you with this?

Some ideas and hints to get started

Here are two possibilities that you could think about investigating.

Note frequencies

Looking for sequences, patterns, predictions and rules

- You could look first at the A notes since A4 is the one used for orchestral tuning.
- What do you notice about the frequencies of the A notes starting with A1?
- Can you find a rule to get from the frequency of one A note to the frequency of the one above?
- Can you find a rule connecting the note number to the frequency?

There is a tns file on the website at www.nspiringlearning.org.uk with this information partly set up to help you. You could try out your rule by inserting a ‘Data & Statistics’ page and graphing ‘anote’ (the A note number) on the x-axis against ‘afreq’ (the corresponding frequency) on the y-axis. Use ‘plot function’ to try out your rules.

- You could then go on to look at sequences of other notes such as C and so on. Can you find any general patterns or rules?
- Can you find a general rule to match all the data in the table? You could try using a ‘Data & Statistics’ page and graph the note numbers against their frequency. Then try different types of regression on menu 4 to see if you can find a function that fits the graph.
- An interval of one note is called a semi-tone; two notes would be a tone for example from C1 to D1. The Pythagoreans (followers of Pythagoras) were intrigued by musical harmony and tried to describe it mathematically. They reckoned that to raise a note by one tone you needed to increase
the frequency by multiplying by $\frac{3}{2}$. How well does this rule fit the data in the table?

**Frequency and wavelength**

**What is the relationship?**

- Look at the data. What happens to the wavelength as the frequency increases?
- What sort of relationship is this? Why?
- What would a graph of this relationship look like? Can you find an equation?
- What do you think wavelength means? How could you work it out if you knew the frequency of the note?

**Hints**

- You could try inserting an extra column in the spreadsheet and putting in a formula to multiply the frequency by the wavelength. What do you notice? How could this information help you to find an equation for the graph?
- You could try using a ‘Data & Statistics’ page and ‘Plot function’ on menu 4 to help you. What do you notice?
- The speed that sound travels is approximately 344 metres per second. How is this related to frequency and wavelength?

**Additional notes for teachers including possible solutions.**

**A notes fig. 22 / 23**

Separating out the A notes makes it much easier to spot that the frequency doubles when the note goes up one octave (from A0 to A1 etc) A ‘Data & Statistics’ page could be used with ‘plot function’ to test out a possible rule. Students could then go on to try to predict and test rules for sequences of other notes such as C.

**All notes fig. 24 / 25 / 26**

If doubling the frequency takes the note up one octave or 12 semitones, what would you need to multiply the frequency by to raise the note by just one semitone?

There are several possible approaches.

- One would be to use ‘exponential regression’ to fit an equation to the graph and then to look at the significance of the numbers in the graph equation.
- Another would be to try to calculate the common ratio ($r$) between successive terms and then use this to ‘fit a function’ to the graph.
  Note that $r = \frac{\sqrt{2}}{12}$

**Wavelength and frequency fig. 27**

Hopefully students will recognise at least some of these features

- As frequency increases wavelength decreases
- One is inversely proportional to the other
- The graph will be similar to that of the reciprocal function $y = \frac{1}{x}$
- An alternative form of the equation of the graph would be $xy = \text{constant}$
- Frequency * wavelength = speed of sound If students insert a column in the spreadsheet and use this to multiply the frequency by the wavelength this will give the constant in the equation which is the speed of sound.

‘Plot function’ can then be used to test this out.

34432 is the speed of sound in cm/sec
Further ideas.  fig_28
Students could also investigate the graphs of note number against wavelength. See the Calculator page in the previous example for further information on this. The Pythagoreans used $\frac{8}{5}$ as the ratio between successive tones (2 semi-tones or notes).
This is a good approximation to $\sqrt[5]{2}$. Various calculations could be done to show this on a calculator page.

Further information
http://www.phy.mtu.edu/~suits/notefreqs.html
http://en.wikipedia.org/wiki/Equal_temperament
http://www.phy.mtu.edu/~suits/NoteFreqCalcs.html

<table>
<thead>
<tr>
<th>Midi note number</th>
<th>Note</th>
<th>Frequency (Hz)</th>
<th>Wavelength (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>C0</td>
<td>16.35</td>
<td>2100</td>
</tr>
<tr>
<td>13</td>
<td>C#0/Db0</td>
<td>17.32</td>
<td>1990</td>
</tr>
<tr>
<td>14</td>
<td>D0</td>
<td>18.35</td>
<td>1870</td>
</tr>
<tr>
<td>15</td>
<td>D#0/Eb0</td>
<td>19.45</td>
<td>1770</td>
</tr>
<tr>
<td>16</td>
<td>E0</td>
<td>20.6</td>
<td>1670</td>
</tr>
<tr>
<td>17</td>
<td>F0</td>
<td>21.83</td>
<td>1580</td>
</tr>
<tr>
<td>18</td>
<td>F#0/Gb0</td>
<td>23.12</td>
<td>1490</td>
</tr>
<tr>
<td>19</td>
<td>G0</td>
<td>24.5</td>
<td>1400</td>
</tr>
<tr>
<td>20</td>
<td>G#0/Ab0</td>
<td>25.96</td>
<td>1320</td>
</tr>
<tr>
<td>21</td>
<td>A0</td>
<td>27.5</td>
<td>1250</td>
</tr>
<tr>
<td>22</td>
<td>A#0/Bb0</td>
<td>29.14</td>
<td>1180</td>
</tr>
<tr>
<td>23</td>
<td>B0</td>
<td>30.87</td>
<td>1110</td>
</tr>
<tr>
<td>24</td>
<td>C1</td>
<td>32.7</td>
<td>1050</td>
</tr>
<tr>
<td>25</td>
<td>C#1/Db1</td>
<td>34.65</td>
<td>996</td>
</tr>
<tr>
<td>26</td>
<td>D1</td>
<td>36.71</td>
<td>940</td>
</tr>
<tr>
<td>27</td>
<td>D#1/Eb1</td>
<td>38.89</td>
<td>887</td>
</tr>
<tr>
<td>28</td>
<td>E1</td>
<td>41.2</td>
<td>837</td>
</tr>
<tr>
<td>29</td>
<td>F1</td>
<td>43.65</td>
<td>790</td>
</tr>
<tr>
<td>30</td>
<td>F#1/Gb1</td>
<td>46.25</td>
<td>746</td>
</tr>
<tr>
<td>31</td>
<td>G1</td>
<td>49</td>
<td>704</td>
</tr>
<tr>
<td>32</td>
<td>G#1/Ab1</td>
<td>51.91</td>
<td>665</td>
</tr>
<tr>
<td>33</td>
<td>A1</td>
<td>55</td>
<td>627</td>
</tr>
<tr>
<td>34</td>
<td>A#1/Bb1</td>
<td>58.27</td>
<td>592</td>
</tr>
<tr>
<td>35</td>
<td>B1</td>
<td>61.74</td>
<td>559</td>
</tr>
<tr>
<td>36</td>
<td>C2</td>
<td>65.41</td>
<td>527</td>
</tr>
<tr>
<td>37</td>
<td>C#2/Db2</td>
<td>69.3</td>
<td>498</td>
</tr>
<tr>
<td>38</td>
<td>D2</td>
<td>73.42</td>
<td>470</td>
</tr>
<tr>
<td>39</td>
<td>D#2/Eb2</td>
<td>77.78</td>
<td>444</td>
</tr>
<tr>
<td>40</td>
<td>E2</td>
<td>82.41</td>
<td>419</td>
</tr>
<tr>
<td>41</td>
<td>F2</td>
<td>87.31</td>
<td>395</td>
</tr>
<tr>
<td>42</td>
<td>F#2/Gb2</td>
<td>92.5</td>
<td>373</td>
</tr>
<tr>
<td>43</td>
<td>G2</td>
<td>98</td>
<td>352</td>
</tr>
<tr>
<td>44</td>
<td>G#2/Ab2</td>
<td>103.83</td>
<td>332</td>
</tr>
<tr>
<td>45</td>
<td>A2</td>
<td>110</td>
<td>314</td>
</tr>
<tr>
<td>46</td>
<td>A#2/Bb2</td>
<td>116.54</td>
<td>296</td>
</tr>
<tr>
<td>47</td>
<td>B2</td>
<td>123.47</td>
<td>279</td>
</tr>
<tr>
<td>48</td>
<td>C3</td>
<td>130.81</td>
<td>264</td>
</tr>
<tr>
<td>49</td>
<td>C#3/Db3</td>
<td>138.59</td>
<td>249</td>
</tr>
<tr>
<td>50</td>
<td>D3</td>
<td>146.83</td>
<td>235</td>
</tr>
<tr>
<td>51</td>
<td>D#3/Eb3</td>
<td>155.56</td>
<td>222</td>
</tr>
<tr>
<td>52</td>
<td>E3</td>
<td>164.81</td>
<td>209</td>
</tr>
<tr>
<td>53</td>
<td>F3</td>
<td>174.61</td>
<td>198</td>
</tr>
<tr>
<td>54</td>
<td>F#3/Gb3</td>
<td>185</td>
<td>186</td>
</tr>
<tr>
<td>55</td>
<td>G3</td>
<td>196</td>
<td>176</td>
</tr>
<tr>
<td>56</td>
<td>G#3/Ab3</td>
<td>207.65</td>
<td>166</td>
</tr>
<tr>
<td>57</td>
<td>A3</td>
<td>220</td>
<td>157</td>
</tr>
<tr>
<td>58</td>
<td>A#3/Bb3</td>
<td>233.08</td>
<td>148</td>
</tr>
<tr>
<td>59</td>
<td>B3</td>
<td>246.94</td>
<td>140</td>
</tr>
<tr>
<td>60</td>
<td>C4</td>
<td>261.63</td>
<td>132</td>
</tr>
<tr>
<td>61</td>
<td>C#4/Db4</td>
<td>277.18</td>
<td>124</td>
</tr>
<tr>
<td>62</td>
<td>D4</td>
<td>293.66</td>
<td>117</td>
</tr>
<tr>
<td>63</td>
<td>D#4/Eb4</td>
<td>311.13</td>
<td>111</td>
</tr>
<tr>
<td>64</td>
<td>E4</td>
<td>329.63</td>
<td>105</td>
</tr>
<tr>
<td>65</td>
<td>F4</td>
<td>349.23</td>
<td>98.8</td>
</tr>
<tr>
<td>66</td>
<td>F#4/Gb4</td>
<td>369.99</td>
<td>93.2</td>
</tr>
<tr>
<td>67</td>
<td>G4</td>
<td>392</td>
<td>88</td>
</tr>
<tr>
<td>68</td>
<td>G#4/Ab4</td>
<td>415.3</td>
<td>83.1</td>
</tr>
<tr>
<td>69</td>
<td>A4</td>
<td>440</td>
<td>78.4</td>
</tr>
<tr>
<td>70</td>
<td>A#4/Bb4</td>
<td>466.16</td>
<td>74</td>
</tr>
<tr>
<td>71</td>
<td>B4</td>
<td>493.88</td>
<td>69.9</td>
</tr>
<tr>
<td>72</td>
<td>C5</td>
<td>523.25</td>
<td>65.9</td>
</tr>
<tr>
<td>73</td>
<td>C#5/Db5</td>
<td>554.37</td>
<td>62.2</td>
</tr>
<tr>
<td>74</td>
<td>D5</td>
<td>587.33</td>
<td>58.7</td>
</tr>
<tr>
<td>75</td>
<td>D#5/Eb5</td>
<td>622.25</td>
<td>55.4</td>
</tr>
<tr>
<td>76</td>
<td>E5</td>
<td>659.26</td>
<td>52.3</td>
</tr>
</tbody>
</table>
Introduction

Being able to interpret data from a large data set such as in a spreadsheet table and to choose appropriate methods to display and analyse this data is an important functional skill and is useful across the school curriculum. Conventional spreadsheet software often offers a bewildering selection of display features many of which could be at best inappropriate for displaying data effectively. TI-Nspire offers a good range of clear display options which can help students to make decisions about appropriate forms of display.

This activity gives students a large table of weather data facts and figures chosen from places around the world that display very different weather patterns. They are asked to consider the data and to discuss and make comments and notes about what they notice. They can then go on to use the facilities of TI-Nspire to display the data to illustrate their findings. They can also analyse the data using a variety of statistical calculations.

The activity

Discussion points

The activity could begin with some class discussion about weather around the world.

What figures do you think might be on record?
Why would you want to know about the climate?
• What would you like your holiday or school trip weather to be like?
  How does this vary with the time of year?
  Are some months in some places best avoided?
• Who else might want to know about the weather and why?
  For example which locations might be most suitable for solar energy;
  where is home insulation important; where and when would de-icing measures for roads be needed etc?

Students could then be asked to consider some data and to discuss and make comments and notes about what they notice. For example:
• What do you notice about the monthly rainfall in the places shown in the table in the tns file?
• Which places have very different monthly rainfall?
• How could you display this information so that you can see these differences at a glance?
• What other information could you find from the table that would help you compare rainfall over the year? (Further details in the Teachers notes later)

Sources of data for discussion

• There is a five page spreadsheet with weather data from 20 different world locations (source of the information: Hutchinson World Weather guide. Helicon publishing Ltd). The file contains
  • mm of rain: average number of millimetres of rain falling per month
  • sunshine hours: the average number of hours of sunshine in each month of the year
  • rainy days: the average number of days per month when rain fell
  • min temp: the average daily minimum temperature
  • max temp: the average daily maximum temperature
There is also a prepared .tns file containing the first page of the spreadsheet data on average number of millimetres of rain falling per month. Further pages could be copied and pasted into new .tns files.

Further details of the places used in these files are shown below.

<table>
<thead>
<tr>
<th>Place</th>
<th>Country</th>
<th>Latitude (nearest degree)</th>
<th>Place</th>
<th>Country</th>
<th>Latitude (nearest degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antofagasta</td>
<td>Chile</td>
<td>23 deg South</td>
<td>Las Vegas</td>
<td>USA</td>
<td>36 deg North</td>
</tr>
<tr>
<td>Auckland</td>
<td>New Zealand</td>
<td>37 deg South</td>
<td>London</td>
<td>England</td>
<td>51 deg North</td>
</tr>
<tr>
<td>Bangkok</td>
<td>Thailand</td>
<td>14 deg North</td>
<td>Moscow</td>
<td>Russia</td>
<td>56 deg North</td>
</tr>
<tr>
<td>Beijing</td>
<td>China</td>
<td>40 deg North</td>
<td>Mumbai</td>
<td>India</td>
<td>19 deg North</td>
</tr>
<tr>
<td>Bergen</td>
<td>Norway</td>
<td>60 deg North</td>
<td>Oban</td>
<td>Scotland</td>
<td>56 deg North</td>
</tr>
<tr>
<td>Darwin</td>
<td>Australia</td>
<td>12 deg South</td>
<td>San Francisco</td>
<td>USA</td>
<td>38 deg North</td>
</tr>
<tr>
<td>Death Valley</td>
<td>USA</td>
<td>36 deg North</td>
<td>Seville</td>
<td>Spain</td>
<td>37 deg North</td>
</tr>
<tr>
<td>Innsbruck</td>
<td>Austria</td>
<td>47 deg North</td>
<td>Tokyo</td>
<td>Japan</td>
<td>36 deg North</td>
</tr>
<tr>
<td>Irkutsk</td>
<td>Russia</td>
<td>52 deg North</td>
<td>Ulanbator</td>
<td>Mongolia</td>
<td>48 deg North</td>
</tr>
<tr>
<td>Islamabad</td>
<td>Pakistan</td>
<td>34 deg North</td>
<td>Urumqi</td>
<td>China</td>
<td>44 deg North</td>
</tr>
</tbody>
</table>

Sites in the UK can be compared using the LGFL weather station site [http://weather.lgfl.org.uk/](http://weather.lgfl.org.uk/)

There is more information about what this shows in the activity ‘Hurricane Force’.

The Table view of the LGFL site (shown below) gives all the currently available live readings. This site also has historical data. The number of readings and the lack of great variation in the UK data on this site makes comparisons more difficult to see quickly.
• The Meteorological Office also has sets of data http://www.metoffice.gov.uk/weather/

Using TI-Nspire

Given the prepared tns file containing the rain data, students could start by considering this first. Then for later comparisons they or the teacher could prepare further files by copy and pasting data from the other pages of the Excel spreadsheet. Students could be asked to:
• Write about what they think the data shows in a notes page.
• There are some notes to help later in this activity and also more detailed instructions for producing different charts in the STEM Activities Introduction booklet.

Examples

These are just a few examples of the sort of display that students could use. Many are possible and there are other examples in the STEM Activities Introduction booklet and in the activity ‘Reaction times’. Ideally students will try out a variety and discuss which ones work well and which are not so clear. Teachers can then decide whether to share these examples with their students and at what stage to do this. This might be
• after students have had some opportunity to experiment for themselves with different forms of display or
• teachers may prefer to show them an example first and then let students choose which cities and data would produce interesting looking results.

Both ‘Quick graph’ on the ‘Lists & Spreadsheet’ menu or the default graph on inserting a ‘Data & Statistics’ page allow multiple entries for variables on they axis. Select ‘add variable’ to do this. Note that ‘connect data points’ has been selected.

fig_30 shows the number of rainy days in different months of the year in four world cities. The graph legend fig_31 shows which symbols represent which places.

This type of chart can show the monthly rainfall pattern.

In fig_32 a ‘box plot’ has been used to compare the monthly rainfall in two cities.
• What does this show? (See STEM Activities Introduction booklet)
• What information does this give that the chart above doesn’t show?
• What information is shown in the first example that cannot be seen here?
Statistical calculations
The main aim of this activity is to encourage students to think about effective ways of displaying data, but calculations could also be used for comparison. (Further details are in the STEM Activities Introduction booklet and in the activity ‘Reaction times’.) Data could be read off from the box plot but in fig_33 the mean annual rainfall has been calculated by inserting an extra row at the foot of the table and using ‘mean’ from the catalogue.

![fig_33](image)

**Millimetres of rain falling on average in each month of the year**

![fig_34](image)

**Average number of hours of sunshine per day for each month of the year**

![fig_35](image)

Additional information  |  Weather data
All the data is in a 5 page Microsoft Excel spreadsheet ‘Weather data’ Data from individual pages, columns or rows can be copied and pasted into a TI-Nspire file or individual items of data could be entered manually.

This is what the five pages look like.
Average number of rainy days in each month of the year  fig. 36

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average daily minimum temperature for each month of the year  fig. 37

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average daily maximum temperature for each month of the year in degrees Celsius  fig. 38

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 5

Hurricane force
Introduction

Working with and converting between compound units such as metres per second and kilometres per hour is an unexciting but necessary task in both mathematics and science. TI-Nspire can make the process easier to understand by the use of formulas in spreadsheets and conversion graphs. Hopefully the context that this activity is set in using live weather station data and wind speed scales including those for hurricanes and tornadoes can make the task more relevant and interesting.

This activity is in two parts initially with a further extension (part 3) for more advanced work.

Part 1 | Weather station information
This gives an introduction to weather information by looking at live weather station data and offering an opportunity to discuss the information shown and the units used. Teachers can decide the extent to which they wish to use the weather station and other background information as well as the depth of the discussion and follow up activities. There is a supplementary sheet of background information.

Part 2 | Using TI-Nspire
Weather forecasters need to be able to collect weather data from around the world where it may be displayed in a variety of different units varying from miles per hour, metres per second, knots and using the Beaufort scale (such as Hurricane Force) in shipping forecasts. Students are given a task to ‘set up TI-Nspire so that a weather forecaster could use it to quickly convert the wind speeds here in m/s into other units including the Beaufort scale’.

Part 3 | More about the Beaufort scale
Interesting results can be obtained by trying to fit a function to the mean wind speeds for each wind force on the Beaufort scale. More information about this is given later in ‘additional notes for teachers’

The activity

Part 1 | Weather station information
The London Grid for Learning Networked Weather Station http://weather.lgfl.org.uk/ displays live weather data updated every minute from a number of weather stations in England and Wales. The default view shows a number of dials to read and interpret. Reading these dials and interpreting the information is a valuable functional skill for both mathematics and science. The data collected also provides a starting point for discussions about weather- barometric pressure; humidity; rainfall; ultraviolet index; solar radiation; wind speed. The data gives a real context for exploring the scientific units used and methods for converting from one compound unit such as metres per second to others such as miles per hour or knots.

Information given on the weather station
• Temperature in degrees Celsius and wind chill temperature
• Barometric pressure in hectoPascals (equivalent to millibars)
• Rain rate in mm/hr
• Days with no rain
• Wind direction and average speed in metres per second
• Outside humidity as a percentage
• Solar radiation in Watts per square metre
• Ultra violet index on a scale from 1 -10

Introduction
Find a weather station on the LGfl site that has live data. This could be the nearest one to where you live. Note that not all the stations are recording all the time. Look at the dials and see what you can find out about the weather. Try weather stations in different parts of the country. How does the weather compare?

Some possible questions to ask
• What do you notice?
• Which dials are easy to read? Which need a bit more thought? What do they mean?
• What do you already know about weather from listening to weather forecasts? Terms? Units of measurement?
• What do you know about pressure – what would a high or low figure be? What effect is this likely to have on the weather?
• What do you think the solar radiation dial shows? How could this be useful?
• What does UV index mean? Why would you want to know this?
• What about relative humidity?
• What about wind speed? These are given in metres per second. How are they usually given on weather forecasts? What about shipping forecasts? Which would mean the most to you?
• How would you describe the information given here to someone else so that they understood the weather in this location? Which units would you use? For example:- Wind speeds can be given in a variety of ways. The LGfl weather station gives the speed in metres per second. Is this the usual way it is shown on weather maps? How is it given on television forecasts? Sometimes wind speeds could be in miles per hour, kilometres per hour, knots (nautical miles per hour) and also using the Beaufort scale (Force 1- 12) or higher for hurricanes using the Torro scale.
Part 2 | Using TI-Nspire – Instructions for students

The task
Your task is to set up TI-Nspire so that a weather forecaster could use it to quickly convert the wind speeds here in m/s into other units including the Beaufort scale? You could use Spreadsheets and formulae, conversion graphs or conversion algorithms.

What instructions would you give to the weather forecaster so that he could use the TI-Nspire?

Getting started

Entering data into a spreadsheet
Wind speeds from a variety of locations could be entered into the first column of a ‘Lists & Spreadsheet’ page and then further columns set up to convert these into different units.

The following example fig_40 has taken values for wind speeds in metres per second from the Beaufort scale. For example Force 5 on the Beaufort scale (described as fresh breeze) varies between 8.0 and 10.7 metres per second. 8.0 is the lower speed in mps entered into column B and 10.7 is in column C.

More information at http://www.metoffice.gov.uk/weather/marine/guide/beaufortscale.html (Note that TI-Nspire will only accept single words in spreadsheet cells. Hence violentstorm!)

Converting the wind speed data into other units
Could you set up a table or charts that others could use to convert this information into other units for example change metres per second into km per hour? These are some possibilities:-

• You could insert a calculator page and try out a calculation for one of the speeds shown in the table. Does the answer seem reasonable? Can you find an alternative method that you could use to check? Can you express this as a general rule?

• You could set up extra columns on the spreadsheet and put formulas into each column. There are instructions for doing this in the STEM Activities Introduction booklet. There is also an example for this below.

• You could set up conversion graphs to change from one unit to another, or to convert a speed to the Beaufort scale. There are two examples for this below. Fig_41 uses a ‘Data & Statistics’ page to obtain a graph and fig_41a uses a Graphs page.

Conversion formulas | Using a Spreadsheet page fig_42

• Open the tns file or set up one of your own by entering data into a ‘Lists & Spreadsheet’ page like the one shown earlier.

• Add names in the white cells at the top of extra columns for converting the data – such as
  • Kmperhr
  • Mph
  • Knots

• Enter a formula in the grey formula cell below the column title for example to convert the figures in column b to kilometres per hour you could type in b[[3]*3600/100 Why does this work? Could it be written in a simpler way?

• What other columns and formulas could you add?

Type square brackets [[] to indicate a column.
Conversion graphs – Beaufort scale  |  Using a Data & Statistics page.  fig_42
One way to convert speeds to the Beaufort Scale is to use the data in the table.
• From the Home screen (‘Insert’ for software) select ‘5: Data & Statistics’
• Move to the x-axis, select ‘enter variable’ and choose BeaufortScale.
• Move to the y-axis and choose ‘lowermps’ then select CTRL menu
  (right click for software) and ‘add y variable’. Then select ‘uppermps’
• To convert a particular speed say 29mps to the scale select menu 4 and ‘plot
  function’ enter f(x) = 29. This is Force 11 on the Beaufort scale. How can you
  tell from the graph?
• If you have more columns on your table you could do this for other units.
The data in the table could also be used to set up conversion graphs for other
pairs of units, but if formulas have been used in the table then these could be
used as functions in a Graphs page.

Conversion graphs – Graphs of functions  |  Using a Graphs page.  fig_43
• Insert a Graphs page from the Home screen on the handheld or ‘Insert’ on
  the software.
• From menu4 select ‘window settings’: Then set the window to the required
  size depending on the units to be converted. Use the arrow keys to move
down the settings table. The example shown is for converting from km per
hour to knots.
• From menu1 select ‘6: Text’ and label the x-axis as the value that you know
  and the y-axis as the value you want to find out.
• Next enter the formula you have worked out to convert from x to y. Your
  formula should contain an ‘x’. In this case from km per hour(x) to knots(y).
  One possible formula for y would be y = f(x) = x/1.86  Why? Can you find
  other ways? Is this formula accurate enough?
• To use the conversion graph: from menu 7 ‘Points and Lines’ select 2: ‘Point
  On’, move to the graph line and put a point on the graph. Then using the
  hand tool (for software- select the pointer from menu 1), grab the point
  and drag it to an appropriate point on the graph to read off the figures.
In the example shown 60 km per hour is approximately 32.3 knots.

Part 3  | Hurricanes, tornadoes and fitting functions
You can find out more about the Beaufort scale at
http://en.wikipedia.org/wiki/Beaufort_scale
The foot of this web page has links to hurricane scales such as the Saffir-Simpson
hurricane scale and the Torro scale for classifying tornado wind speeds.
These scales could be used to extend the Beaufort scale to higher numbers.

The task
If you use a ‘Data & Statistics’ page to plot the mean speeds (average of the lower
and upper speeds) in metres per second for each point on the Beaufort scale,
can you find a function that will fit this data? Some possibilities are:-
• use the plot function option on menu 4 to try to find a function that will fit
  the Beaufort scale data.
• use one of the regression options to find a suitable function.
The page on the Torro scale may give you some ideas to check out.
You could try this for different units.
Additional notes for teachers

Further information about the terms and the units used together with useful web links is in the document ‘Weather data facts and figures’.

Accuracy

Teachers may feel it is useful to discuss with students issues relating to the degree of accuracy required and how and whether to fix the document settings. This could also apply to upper and lower bounds such as those used for the Beaufort scale e.g. 5.5-7.9 and 8.0 – 10.7 and whether to plot these as 5.45-7.95 and 7.95 -10.75 etc (Note that the tns file has been set to one decimal place to fit the Beaufort scale figures but this may not be accurate enough for other calculations.) Alternatively you may prefer just to fix the number of decimal places to 1 and to leave the Beaufort scale figures as they are without further discussion.

To fix the document settings using the software go to ‘File’; then ‘settings’; then ‘document settings’; then ‘display digits’ fig_45. On the handheld Home screen, select 5: ‘Settings and Status’ then 2: ‘Settings’ and 1: ‘General’ then ‘Display Digits’. fig 45 and fig 45a

Calculations

There are many different ways to go about the calculations and encouraging students to look for alternatives or simplifications will help self checking. There are a variety of different possibilities in the information on the Weather data facts and figures sheet. The simplest forms will also be easier to insert into the spreadsheet or as functions for conversion graphs.

Extension ideas

Graph fitting for the mean speed in mps on the Beaufort scale
http://en.wikipedia.org/wiki/TORRO_scale
The function shown plotted in fig_46 which comes from the Torro web page, gives a good fit to the mean speed in metres per second.

Weather data facts and figures

Atmospheric pressure

Atmospheric pressure is measured using a barometer, in units of millibars (mb), hectoPascals (hPa), or centimetres or inches of mercury. Variations in atmospheric pressure can affect the weather as air flows from regions of high pressure to those with low pressure. Atmospheric pressure also depends on altitude as the higher you are above sea level the less air is above you pressing down. Pressures from weather stations are usually adjusted so they give figures for sea level.

On a weather chart, lines joining places with equal sea-level pressures are called isobars. Weather charts showing isobars indicate anticyclones (areas of high pressure) and depressions (areas of low pressure). More information is available from http://www.bbc.co.uk/weather/features/weatherbasics/airpressure.shtml

This page also has links to high pressure and low pressure.
http://en.wikipedia.org/wiki/Atmosphere_(unit)
www.globe.gov/trr-ppt/pressure.ppt has a PowerPoint presentation to download about barometric pressure.

The National Physical Laboratory has an on-line barograph
http://resource.npl.co.uk/pressure/pressure.html
Units of pressure

**Pascal** (SI unit of pressure) = 1 Newton per sq metre

1 HectoPascal (HPa) = 100 Pascals

1 Newton = Force needed to give an mass of 1 kg an acceleration of 1 ms\(^{-2}\)

**Bar** – unit of pressure = 10\(^5\) Pascals

= 0.987 atmospheres (average atmospheric pressure at sea level).

Typical atmospheric pressure at sea level ranges from 970-1040 millibars

**millibar** (mb) = one thousandth of a bar or 100 Pascals equivalent to 1 HectoPascal

http://kids.earth.nasa.gov/archive/air_pressure/

Humidity

http://www.bbc.co.uk/weather/features/weatherbasics/humidity.shtml
Humidity can be measured in several ways, but relative humidity is the most common. In order to understand relative humidity, it is helpful to first understand absolute humidity.

**Absolute humidity** is the mass of water vapour divided by the mass of dry air in a volume of air at a given temperature. The hotter the air is, the more water it can hold.

**Relative humidity** is the ratio of the current absolute humidity to the highest possible absolute humidity. It is expressed as a percentage. A reading of 100 percent relative humidity means that the air is totally saturated with water vapour and cannot hold any more, making rain likely. Relative humidity near the ground does not have to be 100% for rain to fall.

Humans are very sensitive to humidity. If the air is at 100-percent relative humidity, sweat will not evaporate into the air. As a result, we feel much hotter when the relative humidity is high. If the air temperature is 24 degrees Celsius with a relative humidity of 0%, the air temperature feels more like 21 degrees C whereas with 100% relative humidity it would feel more like 27 degrees C.

Wind

A wind is named for the direction from which it blows. A wind blowing from southeast to northwest is a southeast wind. The direction from which wind blows is called windward. The direction toward which it blows is leeward. An object is upwind from a person when it is between the person and the source of the wind. The person is downwind from the object. Wind direction is shown by a weather vane.

Wind speed can vary greatly. Near the earth's surface, it is measured by an instrument called an anemometer. In the upper atmosphere, wind speed is determined by tracking a balloon.

Wind speed is measured in knots (nautical miles per hour), statute (land) miles per hour, or in metres or feet per second.

**The Beaufort scale** is based on a system devised by Admiral Sir Francis Beaufort in 1806. The scale uses numbers from 0 to 12 for wind speed—the higher the number the greater the speed. http://en.wikipedia.org/wiki/Beaufort_scale
The Torro scale is an extension of the Beaufort scale for tornadoes and hurricanes. $T_0$ is equivalent to force 8 on the Beaufort scale, $T_1$ to force 10, $T_2$ to force 12 and so on. [http://en.wikipedia.org/wiki/TORRO_scale](http://en.wikipedia.org/wiki/TORRO_scale)

The World Meteorological Organization has adopted the knot as the international unit for measuring wind speed. [http://science.howstuffworks.com/wind-info.htm](http://science.howstuffworks.com/wind-info.htm)

Further information for converting wind speeds into different forms.

For all these conversions a suitable degree of accuracy can be agreed by discussion.

- 1 hour = 60 minutes; 1 minute = 60 seconds
- 1 mile = 1760 yards; 1 yard = 3 feet; 1 foot = 12 inches
- 1 mile is approximately 1609.34 metres (In athletics a metric mile is taken as 1600 metres)

How would you convert one mile to metres if the only fact that you knew was that one inch is approximately 2.54cm?

Nautical miles

1 nautical mile is approximately 1852 metres or 6076 feet or 1.15 miles or use the facts below.

- One nautical mile makes an angle of one minute (60 minutes = 1 degree) at the centre of the earth.
- You can estimate how long a nautical mile is if you know the radius of the earth (3963 miles or 6378 km)

Solar radiation

Outside the earth’s atmosphere, solar radiation has an intensity of approximately 1370 watts/metre$^2$. This is the value at the top of the atmosphere and is called the Solar Constant.

On the surface of the earth on a clear day, at noon, the direct radiation from the sun will be approximately 1000 watts/metre$^2$ in many places. The energy from the sun can be captured using solar panels. The availability of energy is affected by the location and factors such as latitude, height above sea level, season, and time of day. The biggest factors affecting the available energy in a particular location are cloud cover and other weather conditions.

Units

The watt (symbol: W) is a unit of power in the International System of Units (SI). One watt is equivalent to 1 joule (J) of energy per second. For mechanical energy, one watt is the rate at which work is done when an object is moved at a speed of one metre per second against a force of one Newton.

$1W = 1Js^{-1} = 1kgm2s^{-3} = 1Nms^{-1}$

For electrical energy: Work is done at a rate of one watt when one ampere flows through a potential difference of one volt.

Solar radiation is measured in watts per square metre W/m$^2$. 

---

The diagram illustrates the conversion of hurricane force.
Ultra violet index

The UV index is an international standard measurement of how strong the ultraviolet (UV) radiation from the sun is at a particular place on a particular day. Its purpose is to help people to effectively protect themselves from UV light. Excessive exposure causes sunburns, eye damage, skin aging, and skin cancer. Public-health organizations recommend that people protect themselves when the UV index is 3 or higher.

The UV index is a linear scale. An index of 0 corresponds to zero UV radiation such as at night. An index of 10 corresponds roughly to mid-day sun and a clear sky. Indices greater than 11 are quite common in the southern hemisphere where the Ozone layer is depleted. The numbers are related to the amount of UV radiation reaching the surface of the earth, measured in W/m², but the relationship is not simple and cannot be expressed in physical units. The UV index is however designed to give a good indication of likely skin damage.

General information

http://science.howstuffworks.com/meteorological-terms-channel.htm
http://www.metoffice.gov.uk/education/teachers/indepth_understanding.html
http://www.metoffice.gov.uk/education/index.html

Weather for sailors: This web site shows a world map which can display a variety of weather observations such as barometric pressure. http://www.sailwx.info/index.html
Chapter 6

Carbon dating
**Introduction**

This activity looks at radioactive decay and how it is used in particular for carbon dating archaeological remains.

The use of TI-Nspire and the very quick and straightforward way that appropriate data can be inserted into a ‘Lists & Spreadsheet’ page and a suitable curve can be fitted to the data, could help to make this topic much more accessible to younger students. They can make use of the graph and ‘Graph trace’ to find useful information without needing to worry about the meaning of ‘exponential’ or interpreting the equation shown.

There are possibilities for extending the activity to look at more advanced mathematics such as the form of the exponential regression function and the theory behind exponential decay, including topics such as integration, exponential functions and logarithms.

**Background information**

The ‘How stuff works’ website at [science.howstuffworks.com/carbon-14.htm](http://science.howstuffworks.com/carbon-14.htm) has several pages of useful information about carbon dating and also a video clip which could be used as an introduction to class discussion. There is a summary here and further information from this and other websites in the Additional information section at the end of the activity. Also included in the additional information is a section containing further mathematical details and some solutions which could be used depending on the age and background of the class.

Cosmic rays enter the earth’s atmosphere in large numbers every day. It is not uncommon for a cosmic ray to collide with an atom in the atmosphere, creating a secondary cosmic ray in the form of an energetic neutron, and for these energetic neutrons to collide with nitrogen atoms. When the neutron collides, a nitrogen-14 (seven protons, seven neutrons) atom turns into a carbon-14 atom (six protons, eight neutrons) and a hydrogen atom (one proton, zero neutrons).

Carbon-14 is radioactive, with a half-life of about 5,700 years.

The carbon-14 atoms that cosmic rays create combine with oxygen to form carbon dioxide, which plants absorb naturally and incorporate into plant fibres by photosynthesis. Animals and people eat plants and take in carbon-14 as well. The ratio of normal carbon (carbon-12) to carbon-14 in the air and in all living things at any given time is nearly constant. Maybe one in a trillion carbon atoms are carbon-14. The carbon-14 atoms are always decaying, but they are being replaced by new carbon-14 atoms at a constant rate. At this moment, your body has a certain percentage of carbon-14 atoms in it, and all living plants and animals have the same percentage.
As soon as a living organism dies, it stops taking in new carbon. The ratio of carbon-12 to carbon-14 at the moment of death is the same as every other living thing, but the carbon-14 decays and is not replaced. The carbon-14 decays with its half-life of 5,700 years, while the amount of carbon-12 remains constant in the sample. By looking at the ratio of carbon-12 to carbon-14 in the sample and comparing it to the ratio in a living organism, it is possible to determine the age of a formerly living thing fairly precisely.

**Important note** Because the half-life of carbon-14 is 5,700 years, it is only reliable for dating objects up to about 60,000 years old. However, the principle of carbon-14 dating applies to other isotopes as well. Potassium-40 is another radioactive element naturally found in your body and has a half-life of 1.3 billion years. Other useful radioisotopes for radioactive dating include Uranium -235 (half-life = 704 million years), Uranium -238 (half-life = 4.5 billion years), Thorium-232 (half-life = 14 billion years) and Rubidium-87 (half-life = 49 billion years).

### The Activity

**Half life** is defined as the time taken for the activity of a given amount of a radioactive substance to decay to half of its initial value. The lifetime of an individual radioactive atom is unpredictable but for a large number of similar atoms the average lifetime is quite predictable.

For carbon-14 with a half life of approximately 5700 years this means that if something had been dead for 5700 years, you would expect the proportion of carbon-14 atoms to be 50% of that found in any living animal or plant specimen. This information can be used to date archaeological remains such as animal and human skeletons or wooden objects such as boats.

For really old remains such as some fossils, this method cannot be used because the number of carbon-14 atoms remaining after this time is too small. There are other radioactive isotopes in the human body which could be used instead such as Potassium-40 which has a half-life of 1.3 billion years.

### The Task

How could you set up a TI-Nspire so that you could use it to:

- find the percentage of carbon-14 atoms remaining in specimens of different ages or
- find the age of a specimen if you knew the percentage of carbon-14 atoms remaining

When you have set your graph up and can trace values from it, try setting your own questions and using the graph to answer your questions or those of others. For example:

- How many years will it be before approximately one third of the carbon-14 atoms have decayed?
- What percentage of carbon-14 atoms are likely to remain after 60 000 years?
Getting started
Try to put more information into the columns in this table for carbon 14
(Half life 5700 years)

<table>
<thead>
<tr>
<th>year</th>
<th>0</th>
<th>?</th>
<th>?</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage of carbon-14</td>
<td>100%</td>
<td>50%</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

Now try to enter this information into a ‘Lists & Spreadsheet’ page of TI-Nspire.

**Set up the spreadsheet** \textit{fig. 48}
Instructions to set up a spreadsheet page are given in the STEM Activities Introduction booklet. There is also a sample file already set up for this activity.

**Insert a ‘Data & Statistics’ page** \textit{fig. 49}
- from menu 2 select ‘Add X variable’ and select ‘time’
- from menu 2 select ‘Add Y variable’ and select percent
- This will give you a Scatterplot.

**Fit a curve to the data** \textit{fig. 50}
- From menu 4 choose ‘regression’ and select ‘show exponential’
Don’t be put off by the term ‘exponential’ or the equation shown. What you want is a curve that is a good fit to the points.
- You may prefer to drag the equation out of the way.

**Reading from the graph** \textit{fig. 51}
To read off values from the graph go to menu 4 and ‘Graph Trace’. You can grab the point and move it along the graph so that you can read values from the graph.

**Changing the graph scales** \textit{fig. 52}
If you wish to go beyond the values shown you can change the window setting by going to menu 5; select ‘window settings’ and change maximum x value for example to 70 000
- You could also change the window settings (menu 5) to zoom in for greater accuracy or to zoom out.

**Extension Activities**

**Radioactive decay**
Students could use a similar method to set up tables and graphs and look at the half life for other radioactive substances. They could compare the graphs that they get and look for similarities and patterns in the functions for the exponential regression curves.

**Form of the regression function**
TI-Nspire gives exponential regression functions in the form $k \cdot a^x$
For this activity the regression function gives the percentage of the isotope remaining as $100 \cdot a^x$ where a is a constant which varies with the half life of the isotope.
Students could investigate the relationship between the value of the constant and the half life.

- Choose a suitable range of half lives to work with (Note: Half lives of 1 year or half a year give interesting results). There are a large number of websites giving information about the half lives of radioactive isotopes such as http://www.buzzle.com/articles/list-of-radioactive-elements.html or http://www.iem-inc.com/toolhalf.html and some examples are listed below.

- Repeat the steps 1-3 above with different half lives. For each one note the constant (a) in the regression function 100*a^x

- Complete a second spreadsheet with columns for the half life and the constant and look for possible patterns and rules.

- Look for possible functions that might fit the data. This time you will need to use ‘plot function’ on menu 4.

Can you derive this relationship by considering the theory behind exponential decay?

These are a few examples of half lives

<table>
<thead>
<tr>
<th>isotope</th>
<th>Half-life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uranium 238</td>
<td>4.5 billion years</td>
</tr>
<tr>
<td>Uranium 235</td>
<td>700 million years</td>
</tr>
<tr>
<td>Uranium 234</td>
<td>245 000 years</td>
</tr>
<tr>
<td>Plutonium 239</td>
<td>24 300 years</td>
</tr>
<tr>
<td>Caesium 137</td>
<td>30 years</td>
</tr>
<tr>
<td>Thorium 232</td>
<td>14 years</td>
</tr>
<tr>
<td>Strontium 90</td>
<td>28 years</td>
</tr>
<tr>
<td>Cobalt 60</td>
<td>5 years</td>
</tr>
<tr>
<td>Iodine 125 and 131</td>
<td>8 days</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>isotope</th>
<th>Half-life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Americum 241</td>
<td>450 years</td>
</tr>
<tr>
<td>Ruthenium 106</td>
<td>1 year</td>
</tr>
<tr>
<td>Xenon 137</td>
<td>4 minutes</td>
</tr>
<tr>
<td>Tritium (Hydrogen 3)</td>
<td>12 years</td>
</tr>
<tr>
<td>Curium 242</td>
<td>163 days (appx. ½ year)</td>
</tr>
<tr>
<td>Curium 243</td>
<td>35 years</td>
</tr>
<tr>
<td>Curium 244</td>
<td>18 years</td>
</tr>
<tr>
<td>Antimony 125</td>
<td>2 years</td>
</tr>
</tbody>
</table>

### Notes for teachers

#### Possible solutions

These are some examples of the constants in the regression equation for different half lives. They have been entered into a new spreadsheet `fig_53`

- The inclusion of half lives of 0.5, 1 and 2 makes it easier to spot a possible connection as shown in the screen shot `fig_54`.

- Possible formulas could be tested by inserting into the formula cell in the next spreadsheet column.
Further theory

If \( N \) is the number of atoms at time \( t \), the rate of decrease of \( N \) is proportional to the number of atoms remaining. \[
\frac{dN}{dt} = -\lambda N
\]

where \( \lambda \) is a constant called the radioactive decay constant.

\[
\int \frac{1}{N} \, dN = \int -\lambda \, dt
\]

\[
\ln N = -\lambda t + C
\]

\[
N = e^{\lambda t + C} = e^C \cdot e^{\lambda t}
\]

If the number of atoms at time zero is \( N_0 \) then \( N_0 = e^C \) and \( N = N_0 e^{\lambda t} \)

The fraction of atoms remaining will be \( \frac{N}{N_0} = e^{\lambda t} \)

For half life where \( t = t_{1/2} \)

\[
\frac{N}{N_0} = \frac{1}{2} = e^{\lambda t_{1/2}} = \frac{1}{2} = e^{\lambda t_{1/2}} = 2 = \lambda t_{1/2} = \ln 2 = \frac{\ln 2}{\lambda}
\]

(1)

In the TI-Nspire example the percentage remaining is of the form \( 100^a a^x \) so the fraction remaining is \( a \) where \( a \) represents time.

If \( a = e^{\lambda t} \) then \( t_{1/2} \cdot \ln a = -\lambda t \) and \( \lambda = -\ln a \)

(2)

(1) and (2) give

\[
\lambda t_{1/2} = \ln 2 \quad \text{and} \quad \lambda t_{1/2} = -t_{1/2} \ln a \quad \text{thus} \quad \ln a = \frac{1}{t_{1/2}} \ln 2 \quad \Rightarrow \quad a = 2^{ \frac{1}{t_{1/2}} }
\]

This gives \( a \) in the TI-Nspire regression formula as \( a = 2^{ \frac{1}{t_{1/2}} } \) which matches the graph shown.

Radioactive Decay Constant

The rate of radioactive decay is typically expressed in terms of either the radioactive half-life, or the radioactive decay constant. They are related as follows: fig_55

The decay constant is also sometimes called the disintegration constant. The half-life and the decay constant give the same information, so either may be used to characterize decay. Another useful concept in radioactive decay is the average lifetime. The average lifetime is the reciprocal of the decay constant as defined here.

http://hyperphysics.phy-astr.gsu.edu/Hbase/Nuclear/halfl2.html#c4

A number of websites have information including the theoretical background for example:

http://hyperphysics.phy-astr.gsu.edu/Hbase/Nuclear/halfl2.html#c3


http://www.gcsescience.com/prad16-half-life.htm

http://hyperphysics.phy-astr.gsu.edu/Hbase/Nuclear/halfl2.html#c1


http://library.thinkquest.org/27948/decay.html

http://www.buzzle.com/articles/list-of-radioactive-elements.html
Chapter 7

Reaction times
Introduction

Experiments to test and compare reaction times offer opportunities for hypothesis testing and devising and setting up experiments in a scientific way to control as many variables as possible. A wide range of data handling skills at varying levels can be used to display and analyse the data and consider the significance of the results. TI-Nspire will display results in a variety of ways and do a wide range of statistical calculations. This means that less experienced students can access statistical displays and measures to analyse in order to test their theories that might otherwise have been too complex or time consuming.

The stages in the data handling cycle will require the students to:-
1. Pose a question about 'Reaction times' and plan how they will collect data to try to answer it.
2. Collect the data
3. Represent and analyse their data
4. Interpret and discuss their results

Possible questions for students to think about.

- What factors could influence reaction times? Which one(s) will you consider? What question will you ask?

At the planning stage students need to consider all the factors that might influence the results of their tests and what precautions they need to take to ensure that the test is the same for everyone taking it.

- What data will you record, how will you record it?
- How can you set up the experiment to collect the data and ensure they are reliable and not influenced by other factors?
- Have you collected all the data you need? Were there any problems which might influence the results?
- How will you enter the data into TI-Nspire and which features will you use to display and analyse your results?
- What conclusions did you reach? Can you explain your conclusions to others using your display to back up your conclusions? Would you expect to get similar results if you repeated the experiment? Did you answer your original question or do you need to collect more data or repeat the experiment?

Advanced level students could also consider the significance of their results.

Activity

Posing the question

There needs to be some initial whole group discussion about reaction times and the factors that might affect them. Ideally the class will suggest the factors which might influence reaction times, and then decide on a question that they would like their group to answer, bearing in mind the constraints of the individual classroom.

These are some possible discussion points:-

- gender differences
- preferred/ non-preferred hand
- distractions e.g. noise, conversations,
- alertness – time of day; before or after food, lack of sleep, after caffeine
- Do you get better if you practice with a particular test?
- Do different tests produce similar results?
• Do you need to have particularly good reactions to play some sports in which case do players of these sports perform better in reaction time tests?

Some examples
• The BBC programme 'Bang goes the theory' [http://www.bbc.co.uk/bang/] has launched an experiment to collect data to see whether brain training exercises do really improve performance over a period of time. Amongst the tests they are trying are tests for 'Reaction times'.
• The BBC Science pages also have a reaction time test which is linked to a page about sleep and how sleep deprivation and also caffeine can affect reaction time. [http://www.bbc.co.uk/science/humanbody/sleep/sheep/ reaction_version5.swf]
• Recent evidence suggests that driver's reaction times are affected if they are having a mobile phone conversation even if the phone is a hands-free one.
• Some internet reaction timers (see below) use different shapes or colours – do these influence the results? Most tests are initiated by the computer but the Nrich website test is initiated by the person being tested – does this make a difference? How do the ruler experiments compare?

Setting up the experiment and collecting the data.
These are some things that you might wish your students to consider when setting up their experiments.

• Reproducibility – What rules do you need to set to ensure the experiment is the same for everyone?
• Reliability – How reliable are your results? Would the same thing happen if the experiment was repeated?
• Interaction – How can you ensure that later people tested will not be influenced by what was done before?
• Representative groups – If a larger scale experiment is involved, how can you ensure that the sample you choose is really representative of the whole population?

Practical experiments using a ruler
The tester holds a ruler (30cm or longer) against a wall with the zero mark at the bottom. The person being tested puts a finger at the zero mark just clear of the ruler. The ruler is then dropped and has to be stopped by the outstretched finger (there are pictures on the websites in the two example below). The distance the ruler drops before it is stopped is then recorded. Repeat several times. What precautions do you need to take to ensure the results are fair? (See the notes above.)
This experiment measures reaction times using distances. These can be converted into times using an approximate rule that in t seconds the distance travelled \( d = 5t^2 \) metres. How will this help you to convert distances in centimetres into times in seconds? (See teacher’s notes)
The Royal Statistical Society’s Centre for Statistical education (RSSCSE) has links in their resource section to activities from a Schools council project one of which is ‘Reaction Times’ under a general heading of ‘Practice makes perfect’. http://old.rsscse.co.uk/pose/level1/book6/sectionb.htm

Internet Reaction Timers
There are several websites that will collect data on Reaction Times. These are quick to use so students working in pairs could collect their data one pair at a time using one computer in a classroom, while another activity was going on. They will need to make decisions on how many samples to take, whether to write each individual result down or whether to put the average of five, which some websites give. These are some examples:-

1 Sheep dash from BBC Science http://www.bbc.co.uk/science/humanbody/sleep/sheep/reaction_version5.swf
   On this website, which is part of the BBC websites Science section about sleep, you have to click the mouse when you see a sheep break for freedom. The average of five attempts is given. The times are given to one thousandth of a second which is a greater degree of accuracy than other web sites. [You may want to reach a collective decision about what to do with penalty results (3 seconds) for false starts as these can distort the data, but may be worth investigating separately. Sets of five which included penalty scores were ignored in the sample file shown below.] The idea on this Science section of the website is to see how factors such as drinking caffeinated drinks or lack of sleep affect reaction times.

2 http://getyourwebsitehere.com/jswb/rttest01.html
   This reaction timer is a set of traffic lights set to red. Click when they change to green. Average of sets of five results is given.

3 http://www.mathsisfun.com/games/reaction-time.html
   For this reaction timer you click when a small circle changes from yellow to red. Average of sets of five results.

   For the NRich reaction timer you click the mouse to make a star disappear and then click again when it reappears. As you effectively choose the start time yourself for this reaction timer, does this produce different results from other timers?

5 Experiments at school http://www.experimentsatschool.ntu.ac.uk/experiments/reaction/
   has experiments to collect data but you can also download data sets from other schools or from different parts of the world.

Entering and displaying the data.
Ideally in this activity students will pose their own questions and collect appropriate data. How they choose to analyse and display their data will then depend on the question that they posed initially. There are notes in the STEM Activities Introduction booklet about entering data and some forms of display. The notes for students at the end of this activity show examples of different forms of analysis and display with questions about these to help them get some ideas, but it is important to state that the examples of data collected for these examples is from too small a sample to provide valid conclusions.
Interpreting and discussing results.
Students need to analyse their results with regards to the question that they posed initially.
• Can they backup their findings with statistical displays and calculations?
• Would they expect to get similar results with a different group of people?
• How do their findings compare with others?
• Do they need to repeat the experiment under different or stricter conditions?

Additional notes for teachers

Distance to time conversion for ruler experiment.
If using the practical experiment with rulers the distances can be converted into times. Depending on the students you may want to do one or more of:-
• Explain why the approximate rule \( d = 5t^2 \) works (NB \( d \) is in metres, \( t \) is in seconds and the acceleration due to gravity \( g \) is taken as 10ms\(^{-2}\))
• Ask them to rearrange the rule into one to find \( t \).
• Show them how to insert an extra column into the spreadsheet with a formula to change \( d \) into \( t \).

Notes on displays featured in ‘Further help for Students’
Quick graphs: The line shown has a very small negative gradient which would suggest improvement over time but this needs discussing as pupils need to realise the dangers of extrapolating in this way.
Box plots: The outlier in the lower box plot- the person being tested using ‘Sheep dash’ reckoned that this was a fluke and that he had actually jumped the gun and made a false start for this score. He thought it was just luck as it was so close to the actual sheep leaving that with the mouse delay the computer hadn’t picked it up. It should have given him a 3 second penalty.
Histograms: For more advanced students if more data are collected – how well does the data fit a normal distribution?

Analysis: It is a useful skill for students to be able to interpret the data table and select appropriate information such as \( \bar{x} \) from the large array of other items that they may not understand.

Further help for students

Entering the data | An example
You will need to record data manually for most of the experiments given above. This could be done by entering the data directly into a ‘Lists & Spreadsheet’ page of a TI-Nspire.
The example fig_56 shows data from 20 tries by two people both right handed, one female (1) and one male (2) using right and left hands with the BBC ‘Sheep Dash’ test.

Important: This example has been chosen to illustrate the use of a variety of TI-Nspire features and it is important to realise that it is not a good example of collecting data to answer a specific question and that there is insufficient data here to reach any valid general conclusions. It would be much better to concentrate on one question at a time rather than 3 questions as here which could be:
Does the male have faster reactions than the female?
Do the two people have faster reactions with their preferred hand?
Do the two people improve their reaction times during the tests that they do?

Displaying the data | ‘Quick graphs’ example
There is an option to display a graph directly from the spreadsheet page.
Click on a column you wish to illustrate. From menu 3 select ‘Quick Graph’
Select the axes in turn and set each axis to the data you wish to display (use tab on the handheld)
The graph below shows right hand against trial number. Menu 2 and ‘Connect data points’ was used to join the points. Do you think this person is getting better as time goes on?
With the graph selected, from menu 4 choose ‘Regression’ and ‘Show linear’. This adds a straight line to best fit the points.
Why might you choose to display your data in this way?
What question could it answer?
In which direction is this line going?
What does this tell you?
What other information can you obtain from the equation of the graph?

Example using a ‘Data & Statistics’ page | Using two y variables
Insert a ‘Data & Statistics’ page. Set the axes as before. CTRL menu will give you the option to add another y variable and to get a second set of data for the second person. This means you can display two graphs simultaneously.
The example graph shown compares the right hand performance of two people.
Is this a good way to display the results?
Box plot
Another way to compare two sets of data is to use a Box plot. This shows the extreme values, the median value and the upper and lower quartiles which are explained in the diagram below. Instructions to do this are in the STEM Activities Introduction booklet.
- What does the box plot show about the two sets of data? For example half of person 1’s scores are less than 0.33, whereas three quarters of person 2’s scores are below this figure.

Histograms
The screen shot shows a split page with two ‘histograms’ which are produced in a similar way to the box plots described above.
- What does this show?
- Which are good ways to display this type of data? Why?
- If you had a lot more data what shape would you expect the graph to take?
**Analysing the data**

A lot of analysis can be done from statistical diagrams such as box plots which show min, max, median and lower and upper quartiles.

Data analysis: From the original spreadsheet other information can be found by selecting menu ‘Stat calculations’ and ‘one variable statistics’. Select the number of lists you want to do this for. Four lists were used in this example. Then select the titles for each list and select the first column to put the results in – in this case column I was used.

You can use this to compare the mean scores ($\bar{x}$) and median scores as well as other figures such as the minimum and maximum values and the lower and upper quartiles ($Q_1$ and $Q_2$). Don’t be put off by the large number of figures in the table and just concentrate on those that might be useful.

How significant are your results? (For Advanced level students) - Ti-Nspire has the facility for performing significance tests and finding confidence intervals. How could these be used?

---

This gives

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Right 1</td>
<td>Right 2</td>
<td>Left 1</td>
<td>Left 2</td>
</tr>
</tbody>
</table>

---

![One-Variable Statistics](image.png)

![fig_63](image.png)

![fig_64](image.png)
STEM activities with TI-Nspire

The coming decade will see an increasing need for a flexible work force possessing a wide range of skills in Science, Technology, Engineering and Mathematics (STEM) in order to meet the needs of the new high level industries and to be able to produce technologically complex products. The booklets in this series were developed with this need in mind. They aim to provide stimulating activities which link aspects of STEM and at the same time encourage the use of technology and an awareness of its potential.

Why use TI-Nspire for STEM subjects?

TI-Nspire provides a learning platform which dynamically links a variety of ICT applications including documents, graphs, geometry, statistics, spreadsheets, data logging and a calculator. This dynamic linking assists students in making connections not only between different areas of mathematics but also with other areas of the curriculum and STEM subjects in particular.

There is now a wealth of data available on the Internet in a variety of forms which can be copied directly into TI-Nspire. Data can also be captured by linking to a variety of data logging devices such as motion sensors or by recording manually or using video analysis. It is then possible to manipulate, display and analyse this data in a variety of ways or to try to model the data using a variety of data handling and function plotting tools.

About the booklets

All the activities contain some background scientific or other information together with links to appropriate websites. Many of the activities are suitable for a range of ages and aptitudes with more challenging ideas being suggested as extension activities. Further information and TI-Nspire files for the activities can be found at www.nspiringlearning.org.uk.

There are five booklets in the series:

Introduction - contains a brief introduction and instructions for getting started using some of the features of TI-Nspire that are used frequently in the other booklets.

Capturing data: Modelling and Interpretation - contains activities which use a variety of data logging probes to collect real data and analyse it further.

Using real world data - contains activities which investigate and analyse data in a variety of ways from ready-made or easily generated data sets in a variety of contexts.

Mathematics in motion - contains investigations into modelling motion based on different forms of data collection: manual, video and data logging.

Computer graphics - brings together a number of different mathematical approaches which are used by computer programmers to create and manipulate images; techniques that have now found their way into “virtual reality”. These activities make use of TI-Nspire applications especially Graphs and Geometry.