Activities

with

Handheld Technology

Classroom resources for teaching and learning mathematics at 11-16, with the TI-83 or TI-84 family of graphics calculators
Contents

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About the activities

These ten activities have been written for teachers by teachers. Each one suggests an activity that uses handheld technology to teach topics that are central to the National Curriculum. The activities are in no particular order and the National Curriculum Levels suggested above are for guidance only.

For each activity there are detailed notes for the teacher describing ways in which the activity has been used successfully in real classrooms. These notes link the activity directly to the Framework for teaching mathematics: Years 7, 8 and 9. There are also student worksheets that may be photocopied freely.

The activities are designed for use with the TI-83 or TI-84 families of calculators. The assumption is that you will have access to a class set of calculators and also a calculator that can be used for whole-class teaching. This demo calculator can take various forms including the TI-SmartView™ emulator or a Viewscreen™. It is possible for schools to borrow from Texas Instruments the very latest technology, both for the class and the teacher - see http://education.ti.com/uk.

The 10 activities illustrate using the graphics calculator to teach a wide range of mathematical topics. Some require very little previous experience with using the calculator whereas others are for students and teachers who have already acquired some skill and confidence with using the handheld technology. They have been chosen to illustrate a range of things the calculator can do effectively, including simple calculation and algebra on the home screen, graphing and drawing on the graphing screen, using lists to deal with large data sets and using one of the commonly available software applications. One activity also uses the TI motion sensor known as the CBR™ together with its built-in software.

Five of the activities have been adapted from those published previously in 30 Calculator Lessons for Key Stage 3, a photocopiable resource published by A+B books. (www.AplusB.co.uk). Details of this and other resources supporting the use of handheld technology in the teaching of mathematics are listed at the back of this book.
Activity 1: There and back again

Framework reference: Page 85

Strand: Calculations
Topic: Number operations and the relationships between them
Pupils should be taught to: Consolidate understanding of the operations of multiplication and division, their relationship to each other and to addition and subtraction.

Year group: 8
Objectives: Use inverse operations
Key Vocabulary: Inverse, order of operations
Resources required: Class set of calculators plus Viewscreen, or TI-SmartView emulator.

Summary
For this activity students need only simple calculator skills. Using the machine for the actual calculations shifts attention onto the operations and their inverses.

Instructions for the teacher
(1) Remind students of the need for brackets when dealing with some compound operations such as “add 7 then divide by 3”. This can be done by asking them to choose any starting number and then produce screens such as these: the compound expression must give the same answer as the two simple operations.

Add 7 then divide by 3

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 + 7</td>
<td>12</td>
</tr>
<tr>
<td>Ans/3</td>
<td>4</td>
</tr>
<tr>
<td>(5+7)/3</td>
<td>4</td>
</tr>
</tbody>
</table>

Subtract 12 then multiply by 2

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>X - 12</td>
<td>9</td>
</tr>
<tr>
<td>Ans*2</td>
<td>18</td>
</tr>
<tr>
<td>(X-12)*2</td>
<td>18</td>
</tr>
</tbody>
</table>

In the second screen the starting number has been previously stored in X. In this lesson it may be best always to use brackets around the first binary operation.

(2) Remind students of the meaning of inverse and that addition and subtraction are inverses, as are multiplication and division. Do this using the demo calculator as shown in the following example.

- Choose any number and press [ENTER]
- Press × 3 [ENTER] (Now we are “there”.)
- Press ÷ 3 [ENTER] (Now we are “back again”.)

Repeat, but this time divide first and then multiply.

(3) Students work through Handout 1. You may wish to use the last question only with more able students. (The problem referred to in Activity 4 shows up if you choose a negative value for X.)

(4) Class-teach the inverse of a multiple operation, perhaps beginning as shown on the right and then moving on to the shorter version below. Keep using the phrase “there and back again”.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>64</td>
</tr>
<tr>
<td>Ans+2</td>
<td>66</td>
</tr>
<tr>
<td>Ans/3</td>
<td>22</td>
</tr>
<tr>
<td>Ans*3</td>
<td>66</td>
</tr>
<tr>
<td>Ans-2</td>
<td>64</td>
</tr>
</tbody>
</table>

You will need to use 2nd [ANS] here and repeated use of 2nd [ENTRY] will also save some time.

(5) Move on to Handout 2.

(6) Finally, draw together results from Activities 5-7 and work towards a statement of a general rule for finding the inverse of multiple operations.

This activity was first published in 30 Calculator Lessons for Key Stage 3 (A+B Books).
Activity 1: There and back again

1) Screensnap

Make your calculator look exactly like these. 

\[
\begin{array}{c|c}
\hline
\text{X} & 21 \\
\text{Ans} \times 3 & 63 \\
\text{Ans} \div 3 & 21 \\
\hline
\end{array}
\]

Hint: To store a number in X use \( \text{STO}\) and \( \text{X,1,0,n} \).

This illustrates that multiply by 3 is the inverse of divide by 3.

But what if X were a really big number, or a fraction, or a negative number...?

2) Pick your own

Store any number you like in X and repeat Activity 1. Make sure you end up with the same number you started with. Is multiply by 3 the inverse of divide by 3, whatever number you start with?

3) Other inverses

(a) Produce two screens to show that add 3 is the inverse of subtract 3.

(b) Write the inverse of subtract 0.125 here ____________

(c) Write the inverse of divide by 0.125 here ____________

4) There-and-back with a square

These screens seem to show that squaring and square rooting are inverses. Try this with your own nasty X numbers.

To enter \( \sqrt{\text{Ans}} \) press 2nd \( \sqrt{} \) 2nd [ANS] [1].

If you make X nasty enough you’ll find that this won’t work. What’s the problem?
Activity 1: There and back again

5) Screensnap back again

Make your calculator look exactly like these.

Hints:
• To enter Ans press [2nd] [ANS].
• Save yourself time on the second screen by using [2nd] [ENTRY] several times.

This illustrates that divide by 7 then subtract 5 is the inverse of add five then multiply by 7.

6) Find the inverse

In each case draw two screens to illustrate your answers.

(a) What is the inverse of add six then multiply by 17?
________________________________________

(b) What is the inverse of divide by 19 then subtract 51?
________________________________________

(c) What is the inverse of add 0.25 then multiply by 0.005?
________________________________________

7) Longer journeys there and back

(a) What is the inverse of... add 2 then divide by 3 then subtract 4?
________________________________________

Draw two screens to illustrate your answer.

(b) Make up some more long there-and-back journeys.
Activity 2: Round and round

Framework reference: Page 218

Strand: Shape, space and measures
Topic: Coordinates
Pupils should be taught to: use coordinates in all four quadrants
Year group: 7
Objectives: Use conventions and notation for 2D coordinates in all 4 quadrants; find coordinates of points determined by geometric information.
Key Vocabulary: Centre, diameter, radius
Resources required: Class set of calculators plus Viewscreen, or TI-SmartView emulator

Summary
This activity allows students to explore coordinates by dealing with the centre point and radius of circles

Instructions for the teacher
(1) First set up a window with friendly coordinates.

- Press [WINDOW]
- Type the settings shown here. To navigate around the screen you can use the cursor keys:
  ▲ ▼ ◀ ▶
- Be sure to use ◀ the negative key.
- Press [2nd] [QUIT] to return to the home screen.

(2) Circles can be constructed using a command from the DRAW menu on the home screen.

- Press [2nd] [DRAW] to select the DRAW menu.
- Scroll down with ◀ to see the bottom of this menu.
- Choose option 9 (Circle). You can either press 9 or scroll down the menu and press [ENTER].
- Press:

<table>
<thead>
<tr>
<th>Circle</th>
<th>Centre</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0, 0)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(0, 0)</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>(0, 0)</td>
<td>3</td>
</tr>
</tbody>
</table>

2(b)

<table>
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<tr>
<th>Circle</th>
<th>Centre</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1, 1)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(1, 1)</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>(1, 1)</td>
<td>5</td>
</tr>
</tbody>
</table>

2(c)

<table>
<thead>
<tr>
<th>Circle</th>
<th>Centre</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(2, 1)</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>(2, 1)</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>(2, 1)</td>
<td>4</td>
</tr>
</tbody>
</table>

2(d)

<table>
<thead>
<tr>
<th>Circle</th>
<th>Centre</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1, 1)</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>(0, 1)</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>(−1, 1)</td>
<td>4</td>
</tr>
</tbody>
</table>

Teacher’s notes

1. [1, 1, 3] (The commas are important, but the calculator will understand if you forget to close the bracket.)

- Press [ENTER]

You will have drawn a circle centre (1, 1) with radius 3 units.

Press [2nd] [QUIT] to return to the home screen and enter the command for the next circle.

An easy way to do this is to use [2nd] [ENTRY] to recall the last entry. Then use ◀ to move to a digit and overtype it. If you need less or more digits use [DEL] or [2nd] [INS].

The worksheet gives students the chance to explore sets of circles drawn like this.

Answers

The teacher’s notes include additional instructions and diagrams for the activity. The framework reference is provided for context. The text is organized with headings, bullet points, and tables to clearly outline the instructions and answers.
Activity 2: Round and round

First:

Reset your calculator...
- Press 2nd [MEM] to get the MEMORY menu
- Choose 7 (Reset...)
- Choose 1 (All RAM...)
- Choose 2 (Reset)

Next:

Set up a window with friendly coordinates...
- Press WINDOW
- Type in the settings shown here
  (be sure to use the negative [-] key).
- Press 2nd [QUIT] to return to the home screen

Now, check that you can draw a circle with centre (1, 0) and radius 4:

- Press 2nd [DRAW] to select the DRAW menu.
- Choose 9: Circle
- Type 1 0 4
- Press ENTER

Important!

To clear any drawings:
- Press 2nd [DRAW] to select the DRAW menu.
- Choose 1: ClrDraw

To draw another circle
press 2nd [QUIT] to return to the home screen and enter
the next command.
Activity 2: Round and round

Make your calculator draw each of these screens

Draw and complete a table like this for each one:

<table>
<thead>
<tr>
<th>Circle</th>
<th>Centre</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(..., ...)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(..., ...)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(..., ...)</td>
<td></td>
</tr>
</tbody>
</table>

(a)

(b)

(c)

(d)

Make up more puzzles like this.
Activity 3: Prime time

Framework reference: Page 53

Strand: Numbers and the number system.
Topic: Integers, powers and roots.
Pupils should be taught to: Recognise and use multiples, factors and primes; use tests of divisibility.
Year group: 8
Objectives: Use a calculator to explore divisibility.
Key Vocabulary: Factor, prime factor.

Resources required: Class set of calculators plus Viewscreenor TI-SmartView emulator.

Summary

In this activity students investigate prime numbers, using the calculator to check for divisibility. Shortcuts are proposed, enabling the checking process to be automated.

Instructions for the teacher

Lead a discussion exploring primeness.

E.g. Is 223 prime? Using the demonstration calculator, divide 223 by 2, 3, 4... etc.

Discuss with the class how far it is necessary to go on – as far as 221, or 110, or what?
Establish that there is no point in going on beyond, say 15 (since 15 squared = 225).

Also, was it necessary to try every number? There was no need to try 2, since 221 is odd. In fact there was no need to try any even divisor. Also there was no need to try 5 since only numbers ending in 0 or 5 are divisible by five.
In fact it is only really necessary to try consecutive primes.
Confirm whether or not 223 is prime.

Is 209 prime? Do this more efficiently by dividing only by known smaller primes and stopping at a suitable point – roughly the square root of 209 (15² = 225, so 15 would do). Students may find it difficult to understand this last point so it might better be delayed until later in the lesson.

Distribute the handouts and get students to tackle the first question.

Demonstrate a semi-programming method for testing for primes as shown in section 2 of the handout. An advantage of this approach is that the calculations are set up quickly. A disadvantage is that the method is highly inefficient as it divides by every consecutive whole number, not just every consecutive prime number. Does this matter?

Teacher’s notes

Students continue working through the handouts.

The follow-up discussion could pick up the following themes:
• How exactly does the ‘semi-program’ in Activity 2 work?
• What other tests for divisibility do they know?
(See page 52 of the Framework.)

You could also discuss writing a program for testing primes. One issue is how the calculator can register whether the division is exact. This can be done using the = command in the TEST menu. So, iPart(A/B)=A/B will be true if B divides A exactly.

Answers

1)

<table>
<thead>
<tr>
<th>Number</th>
<th>Largest prime factor needed</th>
<th>Prime?</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>209</td>
<td>11</td>
<td>no</td>
<td>11, 19</td>
</tr>
<tr>
<td>337</td>
<td>17</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>411</td>
<td>3</td>
<td>no</td>
<td>3, 137</td>
</tr>
<tr>
<td>2003</td>
<td>43</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>43</td>
<td>yes</td>
<td></td>
</tr>
</tbody>
</table>

2) (a) Each time [ENTER] is pressed X is increased by 1. You can stop pressing [ENTER] when X divides exactly into 209 or when X exceeds the square root of 209.
(b) 223 is prime.
(c) 293, 443, 461, 367, are all prime.
207 has the factors 3, 9, 23 and 69.
341 has the factors 11 and 31.
459 has the factors 3, 9, 17, 27, 51 and 153,
3(a) 2011 2017 2027 2029 2039
2053 2063 2069 2081 2083
2087 2089
(b) 25, 21, 14
(c) There are no known patterns in the occurrence of prime numbers.

This activity was first published in 30 Calculator Lessons for Key Stage 3 (A+B Books).

There is more on prime numbers in Calculator Maths: Number (p.48-51) including an efficient simple program for testing for primeness.
Activity 3: Prime time

1) Is it prime?
(a) Starting with the number 2, write below the first 15 prime numbers.

(b) Use the method of dividing by consecutive prime numbers to test whether the following are prime. Write your answers in the table (one has been done for you).

<table>
<thead>
<tr>
<th>Number</th>
<th>Largest prime factor tested</th>
<th>Prime? (yes/no)</th>
<th>If not prime, the factors are...</th>
</tr>
</thead>
<tbody>
<tr>
<td>209</td>
<td>11</td>
<td>no</td>
<td>11, 19</td>
</tr>
<tr>
<td>337</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>411</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2) A partial program for primes
(a) Enter the following on the home screen of your calculator.

```
1 STO X.T.0.n ENTER
X.T.0.n + 1 STO X.T.0.n ALPHA : 2nd [1] X.T.0.n ; 209 ÷ X.T.0.n 2nd [1] ENTER
```

Keep pressing ENTER and watch what happens.
Then answer the following questions.
• What happens to the X value each time ENTER is pressed?
• How do you decide when to stop pressing ENTER:
  if the number isn’t prime?
  if the number is prime?

(b) To test a different number (say 223) for primeness, edit the key sequences as follows.
Press CLEAR and then 2nd [ENTRY] 2nd [ENTRY] ENTER (this enters the command 1→X)

Press 2nd [ENTRY] 2nd [ENTRY] and edit the value 209 to read 223.
As before, repeatedly press ENTER and watch the screen. Is 223 prime?

(c) Use the method of part (b) to test whether the following numbers are prime.

293 443 461 367 207 341 459
If a number is not prime, write down two of its factors.
3) **Prime concerns**

(a) The year 1999 was prime. What are the next twelve prime years? Why will it be unusual to be alive in the 2080s?

(b) How many primes are there:

- between 1 and 100,
- between 100 and 200,
- between 900 and 1000.

(c) How many primes end in 1, 2, 3, 4, ... Is there a pattern? The table below may help.

### Primes to 1000

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>11</th>
<th>13</th>
<th>17</th>
<th>19</th>
<th>23</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
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<td>37</td>
<td>41</td>
<td>43</td>
<td>47</td>
<td>53</td>
<td>59</td>
<td>61</td>
<td>67</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td>73</td>
<td>79</td>
<td>83</td>
<td>89</td>
<td>97</td>
<td>101</td>
<td>103</td>
<td>107</td>
<td>109</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>127</td>
<td>131</td>
<td>137</td>
<td>139</td>
<td>149</td>
<td>151</td>
<td>157</td>
<td>163</td>
<td>167</td>
<td>173</td>
<td></td>
</tr>
<tr>
<td>179</td>
<td>181</td>
<td>191</td>
<td>193</td>
<td>197</td>
<td>199</td>
<td>211</td>
<td>223</td>
<td>227</td>
<td>229</td>
<td></td>
</tr>
<tr>
<td>233</td>
<td>239</td>
<td>241</td>
<td>251</td>
<td>257</td>
<td>263</td>
<td>269</td>
<td>271</td>
<td>277</td>
<td>281</td>
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<td>283</td>
<td>293</td>
<td>307</td>
<td>311</td>
<td>313</td>
<td>317</td>
<td>331</td>
<td>337</td>
<td>347</td>
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<td>379</td>
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<td>983</td>
<td>991</td>
<td>997</td>
<td></td>
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</tr>
</tbody>
</table>
Activity 4: Drawing shapes

Framework reference: Page 12

Strand: Using and applying mathematics to solve problems.
Topic: Applying mathematics and solving problems
Pupils should be taught to: Solve word problems and investigate in a range of contexts.
Year group: 7
Objectives: Use coordinates to draw shapes.
Key Vocabulary: Coordinates, quadrilateral, parallelogram, cube.
Resources required: Class set of calculators plus Viewscreen or TI-SmartView emulator. Link cables (to share pictures) and TI-Graph Link to print them.

Summary

This activity introduces the two different ways of drawing straight lines on the calculator. Both methods offer opportunities for stressing the importance of coordinates.

Instructions for the teacher

Before the lesson, create a friendly graphing window on the demonstration calculator. Press [WINDOW] and enter the values shown here.

![Graphing Window](Image)

Also prepare a drawing to display, which will attract attention as students arrive for the lesson.

Show students how to enter on their calculators the friendly window settings shown above. Explain how to carry out direct drawing:

- Press [GRAPH] (you must start on the graphing screen) and then [2nd] [DRAW]. 2 to choose Line( from the menu.
- Use the cursor keys to move to the point where the line will start and press [ENTER]

![Direct Drawing](Image)

Teacher’s notes

- Move to the point where the line will end and press [ENTER] again. Use the coordinates at the bottom of the screen to make sure the end points are where you want them to be.
- Continue moving and pressing [ENTER] until the shape is complete. Explain how to clear the drawings: press [2nd] [DRAW] 1 to choose ClrDraw from the menu.

![ClrDraw](Image)

Explain indirect drawing, which will be used later. Here you must start on the home screen, so press [2nd] [QUIT].

Press [2nd] [DRAW] 2 to choose Line( and then complete the command by entering the coordinates of the end points, separated by commas.

To enter another similar command you can save time by using [2nd] [ENTRY] and editing.

Students can then be asked to create their own drawings using both direct and indirect drawing. Direct drawing is more fun but the difficulty is then to ensure that students are using the coordinates at the bottom of the screen, rather than just producing the drawing by eye.

There are activities on the handouts that retain the fun aspect, while shifting attention to the coordinates.

In a final plenary, you may want to display some of the students’ drawings. They can store their pictures and then transfer them to the demo calculator using the link cable.

There may be other important issues to discuss which the activities may have highlighted, e.g. the use of coordinates, the properties of shapes and representing 3-D shapes.

This activity was first published in 30 Calculator Lessons for Key Stage 3 (A+B Books).

Another way of drawing shapes is to specify coordinates as two lists and then drawing a scatterplot or line graph. This is a particularly powerful idea and opens up the possibility of transforming the shape. See Calculator Maths: Shape (pages 36-39)
Activity 4: Drawing shapes

Handout 1

1) Initial activity

Use direct drawing to produce your initials on the screen.
Make sure they are exactly right by keeping an eye on the coordinates at the bottom of the screen.

2) Follow the star

What are the coordinates of the corners of this six-pointed star?
Use direct drawing to produce it on your calculator.
Now try making a similar eight-pointed star.

3) Triangle tangle

This shape is made using three identical triangles that only touch at the corners.
Can you make a better one using lots of identical triangles?

4) Heading north-east

What shape is produced by the commands shown here?
Enter a fourth line to complete a symmetrical shape.

5) Line pattern

Can you spot the pattern of numbers in these commands?
Can you predict the pattern of lines they will draw?
Enter the commands to check your prediction.

What should the next Line command be and what line in the drawing will be produced?
And the next? Why does the line pattern go wrong?
Activity 4: Drawing shapes

6) A sound base

Enter the command shown here.
Now imagine that this is the base of a square
Draw the rest of the square using indirect drawing.

Clear the screen and enter \texttt{Line(3,1,6,1)} again.
Now imagine that the line is the base of a parallelogram that is
twice as high as the square.
Draw the rest of such a parallelogram using indirect drawing.

7) Name the shape

Each of these four screens will draw the opposite sides of a quadrilateral.

In each case see if you can predict the shape, then enter the commands, draw the other two sides and see if your prediction was right.

8) Skeleton cube

Use your calculator to draw the complete outline of the skeleton cube that has been started here.

9) Taking a different view

Try to imagine the three-dimensional shape represented here.
Now imagine the shape turned round or stood on its end. What would it look like?

Try to draw it on your calculator.
KS 3 Framework ref: Pages 165-7

Strand: Algebra
Topic: Graphs and Functions
Pupils should be taught to: Generate points and plot graphs of functions
Year group: 8
Objectives: Plot the graphs of linear functions. Recognise that equations of the form \( y = mx + c \) correspond to straight line graphs
Key Vocabulary: Linear, function, graph
Resources required: Class set of calculators plus Viewscreen or TI-SmartView emulator

Summary

In this activity students are asked to engage with different functions of the form \( y = mx + c \) and to look for the variability generated with changes in \( m \) and \( c \). Also, a link is made to the characteristics of the table of values for each function.

Instructions for the teacher

First reset the demo calculator and enter the friendly window settings shown on Handout 1.

To enter the function \( Y = 2X + 1 \), press \( Y= \) to see this screen.

Press 2 \( X,T,\theta,n \) + 1

Now look at the table of values:

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

You can display the function by moving the cursor to highlight the header row for \( Y_1 \):

Press (right cursor) 
Press (up cursor)

Ask questions such as: “What are the numbers in the function?”.

Teacher’s notes

“What do the 2 and the 1 represent in the table?”. Take some suggestions, but do not formalise them; students will be able to formalise their ideas in the class activity.

Now look at the graph:

Press \( \text{GRAPH} \)

To display the name of the function and show values of points on the line, press \( \text{TRACE} \)

Use the left \( \text{<} \) and right \( \text{>} \) cursor keys to move along the line.

Ask questions such as:

“What are the numbers in the function?”
“What do the 2 and the 1 represent in the graph?”. Take suggestions as before.

Comments on handouts

In questions 5-7 students can enter four functions together. All four functions can be viewed on the table or the graph. Then use all four cursor keys as below.

(If you are using TI-Smartview, the use of the View\(^3\) function is really helpful here.)

A function of the form \( Y = MX + C \) represents a straight line. In the table of values the \( M \) means the step or difference between the \( Y \) values and the \( C \) means the value for \( X=0 \).

In the graph, the \( M \) means the steepness or gradient and the \( C \) means the value of \( Y \) where the line crosses the axis.
Activity 5: \( Y=MX+C \)

First:

Reset your calculator…

- Press \(^2\text{nd} \) [MEM] to get the MEMORY menu
- Choose 7 (Reset…)
- Choose 1 (All RAM…)
- Choose 2 (Reset)

Set up a window with friendly coordinates…

- Press \( \text{WINDOW} \)
- Type in the settings shown below
  (be sure to use the negative \( \text{-} \) key).

Make sure that you can draw a graph, and look at its table of values:

To enter the function \( Y = 2X + 1 \)

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
</tr>
</tbody>
</table>

Now to look at the table of values

press \(^2\text{nd} \) [TABLE]

And to explore the graph

Press \GRAPH
And then \TRACE

Now try this

1. (a) The function \( Y = 2X + 1 \) has the numbers 2 and 1 in it.
   Look at the table of values.
   What effect do you think these two numbers have in the table of values?

   (b) Look at the graph.
   What effect do you think these two numbers have in the graph?
Activity 5: Y=MX+C

In Question 1 you should have written down your first ideas about what effect the numbers in the function have on the table and the graph. Now check your ideas as you look at some more functions.

Some more functions

2. Start again and look at the table and graph for $Y = 3X + 1$
   [Note: use the CLEAR key to get rid of the $2X + 1$]

3. Start again and look at the table and graph for $Y = X + 1$

4. Start again and look at the table and graph for $Y = 3X - 1$

5. Now look at these four functions together:
   - $Y = 2X + 2$
   - $Y = 2X - 1$
   - $Y = 2X + 3$
   - $Y = 2X - 3$

   **Four together**
   You don't need to CLEAR each one of these. Use the down cursor † and use Y1, Y2, Y3 and Y4

6. Now look at these functions together:
   - $Y = X - 1$
   - $Y = 3X - 1$
   - $Y = 4X - 1$
   - $Y = -X - 1$

   **Key care**
   Use □ to enter a negative and □ for subtract.
   So to enter $-X - 1$ you must press □ X,T,0,n □ 1

7. Now look at these functions:
   - $Y = -2X + 2$
   - $Y = -3X + 2$
   - $Y = -X + 2$
   - $Y = 2X + 2$

8. The letter $M$ is normally used for number in front of the $X$.
   The letter $C$ is normally used for number after the $X$.
   Update your report about the meaning of the $M$ and the $C$. 
Activity 6: Trig ratios

Framework reference: Page 245

<table>
<thead>
<tr>
<th>Strand: Shape, space and measures.</th>
<th>Topic: Measures and mensuration.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pupils should be taught to:</strong> Begin to use sine, cosine and tangent to solve problems.</td>
<td><strong>Year:</strong> 9.</td>
</tr>
<tr>
<td><strong>Objective:</strong> Consider sine, cosine and tangent as ratios.</td>
<td><strong>Key Vocabulary:</strong> Adjacent, opposite, hypotenuse, sine (sin), cosine (cos), tangent (tan), trigonometric ratios.</td>
</tr>
<tr>
<td><strong>Resources required:</strong> Rulers and protractors. Class set of calculators plus Viewscreen TI-SmartView emulator.</td>
<td></td>
</tr>
</tbody>
</table>

Summary

This activity introduces the concept of trigonometric ratio. It asks students to hypothesize about the size of the ratios and to examine them graphically. It is likely to take much more than one lesson to cover these aspects adequately. The activity assumes some basic calculator familiarity.

Instructions for the teacher

Discuss the definition of the hypotenuse, opposite and adjacent sides of a right-angled triangle.

Ask the students to imagine a right-angled triangle with one angle of 30º (“See it in your mind’s eye.”). Imagine the triangle growing but still retaining the sizes of its angles. Ask the students to comment on the ratios of the sides to each other. Encourage students to discuss their ideas with each other and make some hypotheses.

Give out the handout sheets. Students need to measure six angles and the lengths of the corresponding opposite, hypotenuse and adjacent sides. Encourage careful measurement as the accuracy (or lack of it!) will be revealed later.

The handouts give the instructions to produce scatterplots of the different ratios plotted against the angles in degrees. This activity encourages students from the outset to consider the trigonometric ratios as functions of size of the angle.

They then superimpose the graphs of each trigonometric ratio onto their scatterplots, providing feedback on errors and accuracy of measurements.

Teacher’s notes

The students could initially work in pairs, examining one ratio each. Subsequently three pairs of students could be grouped to compare and comment on their individual results.

In a plenary, the teacher will need to focus on each of the ratios in turn, using the demo calculator to show each of the graphs of the trigonometric functions for 0 < x < 90º.

Explain how the graphs can be used to identify the value of the sine, cosine or tangent of any angle.

You can press TRACE (to move from the scatterplot onto the graph) and either move along the graph using the cursor keys or go directly to the required value by keying it in.

Alternatively, pressing 2nd [TABLE] reveals a table of sine values and you can scroll down to find a required value. (You may need to adjust the settings in TBLSET.)

This activity was first published in 30 Calculator Lessons for Key Stage 3 (A+B Books).

For a different approach to trig. ratios, see Calculator Maths: Shape, pp 20-23. There, lengths of the sides are taken from similar triangles drawn on the calculator’s screen.
Activity 6: Trig ratios

1) Measuring and calculating

For each of the right-angled triangles on the resource sheet:

- measure the marked angle;
- label the opposite, adjacent and hypotenuse sides;
- use your calculator to work out the three ratios:
  \[
  \frac{\text{Opposite}}{\text{Hypotenuse}}, \quad \frac{\text{Adjacent}}{\text{Hypotenuse}}, \quad \frac{\text{Opposite}}{\text{Adjacent}}
  \]
- record the measurements and calculations in this table.

<table>
<thead>
<tr>
<th>Lengths of sides</th>
<th>Trigonometric ratios (2 d.p.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle (º) L1</td>
<td>Opposite</td>
</tr>
<tr>
<td>(a)</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td></td>
</tr>
<tr>
<td>(e)</td>
<td></td>
</tr>
<tr>
<td>(f)</td>
<td></td>
</tr>
</tbody>
</table>

2) Assessing the ratios

Look carefully at the three ratios. Comment on the size of the numbers.

Thinking carefully about the properties of right-angled triangles, could the ratios \(\frac{\text{Opposite}}{\text{Hypotenuse}}\) or \(\frac{\text{Adjacent}}{\text{Hypotenuse}}\) ever be greater than 1? Explain your thinking.

What about the ratio \(\frac{\text{Opposite}}{\text{Adjacent}}\)? What is the greatest value this ratio could have?

3) Set up your calculator

Now enter some of these data into the calculator’s lists, so you can explore the relationship between the angle and each of the ratios in turn.

First press [MODE] and select the settings shown by using the arrow keys and pressing [ENTER].

Press [WINDOW] and enter the values shown.

Why do you think these values have been chosen?

You will probably want to change these window settings later in the activity.
Activity 6: Trig ratios

4) Finding the sines

Press \textbf{STAT} 1 and enter the angles into L1 and each corresponding ratio \( \sin(\theta) \) into L2.

Once you have entered all the data, set up a scatterplot to plot these points on a graph; press \textbf{2nd} [STAT PLOT] 1 and make your screen look like the one on the right. To select something, highlight it and press \textbf{ENTER}.

Pressing \textbf{GRAPH} should reveal the scatterplot.
Make a sketch of it and comment on its key features.

How could you use it to predict some lengths of a right-angled triangle with an angle of 25°?

The ratio \( \frac{\text{Opposite}}{\text{Hypotenuse}} \) is so useful it has been named the sine of the angle (abbreviation \( \sin \)).

It has its own key on the calculator: \textbf{SIN}

You can superimpose the graph of the function \( Y1 = \sin(X) \) onto your calculator screen to check the accuracy of your measurements and calculations.

Press \textbf{Y=} and type \textbf{SIN} X,T,\theta,X \textbf{;} \textbf{GRAPH}.
How accurate were your measurements and calculations?
Write a comment on the shape of the graph.

5) Finding the Cosines

Now focus on the ratio \( \frac{\text{Adjacent}}{\text{Hypotenuse}} \), the cosine (or \( \cos \)) for each angle.

Enter their values into L3.
In the setup screen for Plot1, change the Ylist setting to L3 before pressing \textbf{GRAPH}.
Superimpose the function \( Y1 = \cos(X) \) onto your graph.
Write a comment about the graph.

6) Off at a tangent

Finally, focus on the ratio \( \frac{\text{Opposite}}{\text{Adjacent}} \), the tangent (or \( \tan \)) of each angle.

Enter their values into L4.
In the setup screen for Plot1, change the Ylist setting to L4 before pressing \textbf{GRAPH}.
Superimpose the function \( Y1 = \tan(X) \) onto your graph.
You might need to change the \textbf{WINDOW} settings to fit all of your points onto the graph.
Write a comment about the graph.

Teachers Teaching with Technology™
Activity 6: Trig ratios

Resource sheet

(a) 

(b) 

(c) 

(d) 

(e) 

(f)
Activity 7: Substitution

Framework reference: Page 138

Strand: Algebra
Topic: Equations, formulae and identities
Pupils should be taught to: Use formulae from mathematics and other subjects
Year group: 7
Objectives: Substitute positive integers into simple linear expressions.
Key Vocabulary: Substitute, expression, variable, statement
Resources required: Class set of calculators plus Viewscreen or TI-SmartView emulator

Summary

This activity uses calculator memories to examine the effect of substituting into algebraic expressions. The activity assumes no previous calculator experience.

Instructions for the teacher

(1) Begin by resetting the calculator as on Handout 1. Note that this has the effect of storing a zero in every memory location.

(2) Notice the alphabetic characters above the corresponding keys on the keyboard. To enter these letters you must first press [ALPHA].

• Type 2
• Type the A by pressing [ALPHA] followed by [A]
• Type 5
• Press [ALPHA] B
• Press [ENTER]

Now give some values for A and B. Ask the class for possibilities. e.g. A = 3 and B = 5.

• Press 3 [STO] [ALPHA] A
• Press [ENTER]

• Press 5 [STO] [ALPHA] B
• Press [ENTER]

Now type 2A+B as before. “We have substituted the values A=3 and B=5 into the expression 2A+B”.

Repeat with other values of A and B. In particular extend to include negative values. Be sure to use the [CLEAR] key for negative.

Notice that using [CLEAR] will clear the screen but has no effect on the content of the memories. Values stay in the memory until they are overwritten or until the calculator is reset.

(3) Now use different expressions and other variables. e.g. Suppose a sequence has Nth term 5N – 2. What is the Nth term? i.e. what is the value of 5N – 2?

• Press 5 [ALPHA] N [2]
• Press [ENTER]

(Why do we get -2?)

What is the 15th term?

• Press 15 [ALPHA] N
• Press [ENTER]

Now instead of typing the 5N – 2 again you can use [2nd] [ENTRY] to recall previous entries. In this case 5N – 2 was the last but one entry.

• Press [2nd] [ENTRY] to recall the last entry.
• Press [2nd] [ENTRY] again to recall the 5N – 2.
• Press [ENTRY] to evaluate the expression.

Now work out the 10th term, the 18th term, etc.

Answers to questions on handouts

1 a) 17 b) 7 c) 37 d) -13 e) 4.5
2 a) 6 b) 0 c) 3 d) -7 e) 15
3 a) 16 b) 18 c) 1 d) 8 e) 27
4 a) 5 b) 2 c) 1 d) 17 e) 2
Activity 7: Substitution

First, reset your calculator...
- Press 2nd MEM to get the MEMORY menu
- Choose 7 (Reset...)
- Choose 1 (All RAM...)
- Choose 2 (Reset)

Entering expressions
You can use the [ALPHA] key to type letters.

Enter the expression 3A + 1 ...
- Type: 3
- Press [ALPHA]
- Type A (It’s written above the [MATH] key)
- Type + 1
- Press [ENTER]

Why did you get 0?
If you substitute the value A = 5 into this expression, what do you think the value of the expression will be?
Write it down.

Enter the value A=5 ...
- Type 5
- Press [STO]
- Press [ALPHA]
- Press A
- Press [ENTER]

Recall the expression 3A + 1 ...
- Press 2nd [ENTRY] to recall the previous entry
- Press 2nd [ENTRY] to recall the 3A + 1
- Press [ENTER]

Explain why you get 16.
Work these out
Write down your own idea first.
Then use the graphics calculator to check.

1. Use the expression $5A + 2$.
What is the value of the expression when:
   a) $A = 3$
   b) $A = 1$
   c) $A = 7$
   d) $A = -3$
   e) $A = 0.5$

2. In the expression $12 - 3B$, what is the value of the expression when:
   a) $B = 2$
   b) $B = 4$
   c) $B = 5$
   d) $B = 7$
   e) $B = -1$

3. Here is an expression: $3C + 2D$.
What is the value of the expression when:
   a) $C = 2$ and $D = 5$
   b) $C = 4$ and $D = 3$
   c) $C = -3$ and $D = 4$
   d) $C = 4$ and $D = -2$
   e) $C = -5$ and $D = -6$

4. Now use the expression $E^2 + 1$
What is the value of the expression when:
   a) $E = 2$
   b) $E = 1$
   c) $E = 0$
   d) $E = -4$
   e) $E = -1$

5. Write a short explanation to tell someone else how to substitute values into expressions. Give examples.
Activity 8: Ball bounce

Framework reference: Page 173

Strand: Algebra
Topic: Graphs of functions
Pupils should be taught to: Construct functions arising from real-life problems and plot their corresponding graphs.
Year group: 9
Objectives: Use a motion detector and graphical calculator to plot the distance-time graph of a bouncing ball.

Key Vocabulary: Function, graph, distance-time graph

Resources required: A Calculator-based ranger (CBR) connected to a demonstration calculator (Viewscreen or TI-SmartView emulator)

Summary
This activity uses the CBR as a distance sensor. The software uses an application called ball bounce to take a series of distance measurements very quickly over a 4-second period. The user can then look at distance-time, velocity-time and acceleration-time graphs of the motion.

The instructions below show how to set up the calculator and CBR unit and how to navigate the software. There are also some suggestions for data to collect. There are no handouts associated with this activity.

Instructions for the teacher
(1) Loading the software

Press [APPS]
Select CBL/CBR – it may not be number 2 on your machine.

When you see this screen press any key to continue.

From the menu below choose 3:RANGER and press [ENTER]

This is the main menu. Choose 3:APPLICATIONS and then choose 1:METERS

From the next menu choose 3:BALL BOUNCE. At this point the calculator will prompt you if the CBR is not connected.

(2) Arranging the kit

Using the longest lead you have, plug the CBR firmly into the calculator. The screen prompts you to prepare the experiment by holding the moving object in a suitable place. It is not necessary to be ready quite yet. Just press [ENTER].

Now get everything ready. It is easiest to use the TRIGGER button on the CBR unit. The unit will start clicking and at every click it will take a measurement. Remember that it will carry on taking readings for 4 seconds, so you will have to act quickly!
Activity 8: Ball bounce

(3) A first look at the data

Press the trigger and drop the object. Hopefully it will fall and bounce, remaining in line with the motion sensor.

When the CBR unit has finished clicking, press ENTER on the calculator. A graph showing distance against time will be shown.

How can you interpret this representation of the motion that occurred? When did the motion start? Where are the bounces shown? What is the straight line for the last two seconds representing?

What do the tick marks on the axes represent? How do you know?

Take your time over this part of the activity—it is essential that the students are very clear in their understanding of how the graph relates to the motion that they saw.

You can use the left [ and right ] cursor keys to trace the points on the graph and the coordinates at the bottom of the screen indicate the time elapsed and the height of the object.

Press ENTER to see the plot menu. If your data has not worked out well, you can choose 5 to repeat the sample.

Choose 2:VEL-TIME to display the corresponding velocity-time graph.

Again it is important to interpret the graph in relation to the motion. Ask questions such as “what does a negative velocity represent?” or “why is the velocity zero in the middle of this bounce?”

(4) Smoothing the data set

Press ENTER to see the plot menu again. Once again look at the distance-time graph. Much of it is not relevant because it shows data before and after the motion you are interested in.

Press ENTER to return to the plot menu again.

This time choose 4:PLOT TOOLS to view your graph differently.

This Plot Tools menu allows you to improve the graph’s appearance with smoothing and you can also choose to select just part of the graph (to get rid of the parts before and after the actual motion took place).

Choose 2: SMOOTH DATA.

There are four options. Choose 2:MEDIUM. This is appropriate for most experiments.

This is the graph lightly smoothed. (Compare it with the actual data plotted previously)

Discuss what smoothing actually does to the data set. What values are no longer shown? Why is this a good idea? What are the disadvantages?

Try light and heavy smoothing and see their effect too.
Activity 8: Ball bounce

(4) Removing irrelevant data

Press [ENTER] and again choose 4:PLOT TOOLS.

This time choose option 1 to select the domain.

Discuss which parts of the graph are showing irrelevant data. The calculator will allow you to select a relevant part of the graph and look at it more closely.

Select the left bound of the domain you want using the left [Left] and right [Right] cursor keys. Keep an eye on the X and Y values at the bottom of the screen too.

When you have decided press [ENTER].

Now in a similar way you need to select the right bound of the domain you want and press [ENTER].

A new re-scaled graph is automatically displayed between the points you chose. (Note: choose your bounds with care, you cannot recover the original graph).

Press [ENTER] to return to the Plot Menu. You can now look at the three different graphs and discuss their interpretation again.

(5) Suggestions about data collection

It can be difficult to control the line of motion of the object. It must remain in line with the sensor throughout its motion.

The simplest situation to consider is an object such as a heavy book falling under gravity. Clamp the sensor to an overhanging shelf or some high place. Hold the book underneath it and let it fall. Remember that the sensor is only reliable for distances more than 50cm and less than 6m.

What difference does it make to the graph if the sensor is placed below the object?

Does a heavy book fall faster than a light one? (Many students will think that it must.)

Investigate the effects of air resistance with a light object (but not so light it will float out of line!)

To record the bouncing of a ball it is best to use a large one, such as a football or basketball with the CBR clamped above it. For more than one bounce it can be difficult to control the line of motion of the ball. It must remain in line with the sensor throughout its motion and, unless the floor is perfectly even the ball will often bounce out of line.

Another possibility for investigation is vertical oscillation on a spring: for example, attach the object to a piece of elastic fixed below or above the CBR. Pull the object gently down and release it.

The Ranger software can also be used to create distance-time graphs of students moving at different speeds towards and away from the CBR, clamped horizontally.

There are teacher notes and handouts to support such an activity in Lesson 28 of 30 Calculator Lessons for Key Stage 3 (A+B Books).
Activity 9: Coins, marbles and dice

KS 3 Framework ref: Page 283-5

<table>
<thead>
<tr>
<th>Toss</th>
<th>Outcome</th>
<th>Fraction of heads</th>
<th>Fraction of tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>1/2</td>
<td>0/1</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>2/2</td>
<td>2/2</td>
</tr>
<tr>
<td>3</td>
<td>T</td>
<td>2/3</td>
<td>1/3</td>
</tr>
<tr>
<td>4</td>
<td>H</td>
<td>3/4</td>
<td>1/4</td>
</tr>
</tbody>
</table>

Continue up to 10 single coin tosses.

This activity uses the software called Prob Sim—a probability simulator that is one of the standard “Apps” supplied with TI calculators.

Summary

The aim of the activity is to estimate the theoretical probability from the experimental probability and to form a view of the extent to which the experimental is a good indicator of the theoretical. Note particularly that, however many trials, the experimental probabilities of the outcomes are very unlikely be exactly equal to the theoretical probabilities!

To make this activity meaningful for students, it is best to start the lesson (or devote the preceding lesson) to practical hands-on experiments with real dice, spinners etc. Students should be encouraged to sketch bar charts showing the outcomes of these experiments, so that they realise the power of being able to quick produce accurate charts from a large number of trials, something that the software does well.

Instructions for the teacher

Using the demo calculator, set up the application and show how it works. You can follow the procedure in Activities 1 and 2 of Handout 1.

Make a table on the board as shown. Repeat single coin tosses and fill in the table.

Increase the number of coin tosses by 10 by pressing ZOOM (below +10) and then by 50 by pressing TRACE (below +50).

Fill in the table with the new values. Notice that the handouts ask students to use approximate fractions, e.g. a bit more than half.) However you may prefer to show students to use the actual fractions.

You can use the cursor key [→] to see the frequencies of tails and heads.

Questions:

“What do you notice about the probabilities of getting a head or a tail” – they get closer to half as we toss the coin more.

“Are there ever exactly half heads or tails?” – Well, the chances of that are very low.

Now repeat this experiment with other types of trials, e.g. dice, pick marbles and spinner.

Press YES (below ESC)
Press YES (below ESC again)
Press YES (below YES)
This returns you to the main menu, where you can choose a different simulation.

In each standard case, the theoretical probabilities of each outcome are equal. However, in the Pick Marbles simulation, it is possible to modify the composition to make the outcomes different, and this is explored in Activity 8 on Handout 3.
Activity 9: Coins, marbles and dice

1) First load the probability simulation:

On your calculator press APPS
Use the cursor \( \Box \) to place the highlight beside the application Prob Sim like this.
Press ENTER

2) Try using the simulation

Press 1 to choose Toss Coins.

Notice the boxes at the bottom of the screen. If you want to enter TOSS you press the key below it marked \( \text{WINDOW} \).

Do this and see what happens.
Try pressing \( \text{WINDOW} \) (below +1) a few times.
Try pressing \( \text{ZOOM} \) (below +10) and \( \text{TRACE} \) (below +50)
To start again press \( \text{GRAPH} \) (below CLEAR)
and press \( \text{Y=} \) (below YES)

3) Draw up a table

Make a copy of this table in your exercise book. It will need about 20 rows.

<table>
<thead>
<tr>
<th>Toss number</th>
<th>Outcome</th>
<th>Fraction of heads</th>
<th>Fraction of tails</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

On the calculator press \( \text{WINDOW} \) (below +1) to toss the coin.
Fill in the fraction of heads and tails you have got so far.
Throw again and fill in your table.
Keep going until you have made 10 coin tosses and filled up 10 rows.
4) **Lots more coin tosses**

Press Zoom (below +10) and watch 10 more coin tosses all in one go!

Fill in the table: remember this is now 20 coin tosses in total.

Now you can probably only write an approximate fraction.

For example, write either: 

- a bit more than ___
- a bit less than ___
- exactly ___

Use +10 again. Fill in your table. Do this 5 more times.

Press Trace (below +50) and watch 50 more coin tosses.

Fill in the table. Remember to add an extra 50 coin tosses to the total.

Repeat with +50 until your whole table is filled up.

Answer these questions in your exercise book:

- What do you notice about the probabilities of getting a head or a tail?
- Are there ever exactly half heads or tails?

5) **A table for dice**

Use ESC to return to the main Prob Sim menu.

Repeat the experiment using option 2: Roll Dice

Use a table like this:

<table>
<thead>
<tr>
<th>Roll number</th>
<th>Outcome</th>
<th>Fraction of 1's</th>
<th>Fraction of 6's</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6) **Spinning**

Use ESC to return to the main Prob Sim menu.

Repeat the experiment using option 4: Spin spinner

Use a table like this:

<table>
<thead>
<tr>
<th>Spin number</th>
<th>Outcome</th>
<th>Fraction of 1's</th>
<th>Fraction of 4's</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Activity 9: Coins, marbles and dice

7) A marbles table
Use ESC to return to the main Prob Sim menu.
Repeat the experiment using option 3: Pick marbles.
Use a table like this:

<table>
<thead>
<tr>
<th>Pick number</th>
<th>Outcome</th>
<th>Fraction of A's</th>
<th>Fraction of B's</th>
</tr>
</thead>
</table>

8) More marbles
Return to the main Prob Sim menu.
Choose option 3: Pick marbles again.
Press ZOOM (below SET).

Press WINDOW (below ADV).
Enter 10 A's, 20 B's and no others as shown here.

Return to the simulation by pressing GRAPH (below OK) twice.

Now repeat the experiment with the same table as above.

(a) What is the probability of drawing an A? (remember there are 30 marbles in the bag).
(b) What is the probability of drawing a B?
(c) What do you notice about the probabilities of getting marble A or B?

9) Even more marbles
Repeat the experiment with different numbers of marbles, for example:
10 A’s, 10 B’s, 30 C’s, 50 D’s, 100 E’s.

Answer the same questions as for Activity 8.
### Activity 10: Cumulative frequency

**Teacher’s notes**

<table>
<thead>
<tr>
<th>Interval</th>
<th>Mid-points</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 9</td>
<td>4.5</td>
<td>1</td>
</tr>
<tr>
<td>10 - 19</td>
<td>14.5</td>
<td>2</td>
</tr>
<tr>
<td>20 - 29</td>
<td>24.5</td>
<td>7</td>
</tr>
<tr>
<td>30 - 39</td>
<td>34.5</td>
<td>12</td>
</tr>
<tr>
<td>40 - 49</td>
<td>44.5</td>
<td>19</td>
</tr>
<tr>
<td>50 - 59</td>
<td>54.5</td>
<td>26</td>
</tr>
<tr>
<td>60 - 69</td>
<td>64.5</td>
<td>23</td>
</tr>
<tr>
<td>70 - 79</td>
<td>74.5</td>
<td>7</td>
</tr>
<tr>
<td>80 - 89</td>
<td>84.5</td>
<td>3</td>
</tr>
<tr>
<td>90 - 99</td>
<td>94.5</td>
<td>0</td>
</tr>
</tbody>
</table>

(3) Discuss with the class their responses to Activity 3(c). The summaries are as follows

<table>
<thead>
<tr>
<th></th>
<th>L1</th>
<th>L2, L3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>6</td>
<td>4.5</td>
</tr>
<tr>
<td>Q1</td>
<td>14.5</td>
<td>44.5</td>
</tr>
<tr>
<td>Med</td>
<td>54</td>
<td>54.5</td>
</tr>
<tr>
<td>Q3</td>
<td>61</td>
<td>64.5</td>
</tr>
<tr>
<td>Max</td>
<td>87</td>
<td>84.5</td>
</tr>
</tbody>
</table>

The answers are different because transforming the original data into a frequency table involved a loss of information. Summaries of frequency data are based on the simplifying assumption that all the values are centred on the interval that contains them. So, scores like 31, 61, 72, 41, etc. will not match the frequency data well, whereas marks like 45, 74, 36, 65 etc. will.

(4) Using the demo calculator, lead the class carefully through Activities 4 and 5, emphasising that this is a ‘less than’ cumulative frequency graph. i.e. it indicates how many students scored less than the value plotted on the X axis. You will probably need to explain quartiles and how they can be read from a cumulative frequency graph. (Note that in Activity 5 taking horizontal values of 25, 50 and 75 will not be exactly correct – for example the median is halfway between the 50th and 51st value – but with a batch as large as this the error is negligible and can be ignored.)

(5) Then ask the class to tackle Activities 4 – 5.

(6) Boxplots provide a simple and intuitive means of dealing with the median and quartiles. Handout 3 provides an extension activity in which a boxplot of the data in L1 is super-imposed on the cumulative frequency graph.

This activity was first published in 30 Calculator Lessons for Key Stage 3 (A+B Books).

**Calculator Maths: Handling Data, p 40** provides activities to introduce students to boxplots.

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**Framework reference: Page 259**

**Strand:** Handling data.

**Topic:** Processing and representing data.

**Pupils should be taught to:** Calculate statistics from the data, using ICT as appropriate.

**Year group:** 9

**Objectives:**
- Estimate the median and interquartile range for a large set of grouped data.

**Key Vocabulary:** Median, interquartile range, cumulative frequency diagram, boxplot.

**Resources required:** Class set of calculators plus Viewscreen or TI-SmartView emulator.

**Summary**

This is very much a teacher-directed session and uses quite advanced calculator skills. It assumes that the students have already come across grouping data into class intervals.

The lesson goes through the following stages:
- Generate data → Frequency table → Cumulative frequency table → Cumulative frequency graph.

Students then use the cumulative frequency graph to read off the values of the quartiles and the median. Optionally, they will see another method of finding quartiles and median – using a ‘boxplot’.

**Instructions for the teacher**

1. Revise grouping data into class intervals, by working through Activities 1 and 2 using the demo calculator. Explain how to calculate 1-Var summaries of data from a single list (list L1) and frequency data from two lists (lists L2 and L3).

The method described in Activity 1 of the handout involves storing 100 randomly chosen integer scores, modelled by a normal distribution. Of course, there is no need for students to understand how or why this works – it is simply a way of getting the same set of fairly realistic data into all the calculators. (An alternative method would be to use link cables.)

2. Distribute the calculators and handouts and ask students to work through Activities 1 – 3.

The completed table in Activity 2 is shown in the next column.
Activity 10: Cumulative frequency

1) Data
You can make your calculator create 100 numbers that could represent
the marks of 100 students.
First, reset the calculator’s random facility. Press:

```
2 [STOP] MATH [×] 1 [ENTER]
```

Now press:

```
MATH [×] 5 MATH [×] 6 50 [+] 15 [+] 100 [)]
```

2) Frequency table
Now you are going to arrange all the L1 data into intervals of 0-9, 10-19, 20-29 etc.

(a) Draw up a table like this:

<table>
<thead>
<tr>
<th>Interval</th>
<th>Mid-points</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–9</td>
<td>4.5</td>
<td>...</td>
</tr>
<tr>
<td>10–19</td>
<td>14.5</td>
<td>...</td>
</tr>
<tr>
<td>20–29</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>20-29</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>30–39</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>40–49</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>50–59</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>60–69</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>70–79</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>80–89</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>90–99</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

(b) In the second column enter all the mid-points of the intervals. (Notice 4.5 is halfway between 0 and 9.)

(c) Sort list L1 into ascending order. To do this press

```
[STAT] [2nd] [L1] [ENTER]
```

Return to the list screen (press [STAT] 1) and scroll down the re-ordered list L1, counting the number of scores in each of the intervals 0-9, 10-19, 20-29, ...
Record these frequencies in the third column of the table.

Use the frequency table to answer the following questions.
(d) How many students scored between 30 and 39 inclusive?
(e) How many students scored between 50 and 69 inclusive?
(f) How many students scored less than 50?
(g) How many students scored less than 70?
(h) On your calculator enter the interval mid-points into list L2 and the frequencies into list L3.

3) Summarising frequency data
(a) Enter the command 1-Var Stats L1 (press [STAT] [×] 1 [2nd] [L1] [ENTER]). Scroll down the screen to find and write down the following summary values for L1.

Minimum, Lower quartile, Median, Upper quartile, Maximum.

(b) Enter the command 1-Var Stats L2,L3 and write down the same five summary values for the frequency data in L2 and L3.

(c) Write a sentence comparing your answers to parts (a) and (b), explaining any similarities or differences.
Activity 10: Cumulative frequency

(4) 'Less than' cumulative frequency table

Now you are going to draw up a table to show the number of students who scored less than the various marks.

For example, from your frequency table you can see:

- 1 student scored less than 10 (i.e. in the range 0-9)
- 3 students (1+2) scored less than 20 (i.e. in the range 0-9, or 10-19), etc.

Enter the values 10, 20, 30, ...100 in L4 (these are the 'less than' scores).

Return to the home screen and apply the cumSum command to L3, storing the results in L5, as follows. Press:

```
y 9 ~ y f € y h Í
```

Return to the list screen and move the right cursor so that you can view lists L4 and L5 together.

(a) How many students scored less than 60?
(b) How many students scored less than 80?
(c) How many students scored between 20 and 49 inclusive?

(5) Cumulative frequency graph

You can now use the data in L4 and L5 to plot a cumulative frequency graph.

Press 2nd STAT PLOT 1 to select Plot1 and select the settings shown.
Choose suitable Window settings and display the cumulative frequency graph by pressing TRACE.

You can use the cumulative frequency graph to read off the median (Med), lower quartile (Q1) and upper quartile (Q3). In this example, the batch size is 100, so Q1, Med and Q3 correspond to scores with horizontal values of roughly 25, 50 and 75.

(a) Return to the home screen and enter Horizontal 25 by pressing:

```
2nd DRAW 3 25 ENTER.
```

See the effect that this has on the graph. Use the free-floating cursor keys (not TRACE) to estimate where this horizontal line crosses the cumulative frequency graph. This is your estimate of the lower quartile: write it down.

(b) With a similar approach, use the lines Horizontal 50 and Horizontal 75 to estimate the median and the upper quartile. Write them down.

(c) Compare these estimated values with the answers you produced in Activity 3.
Activity 10: Cumulative frequency

(6) Boxplots

The calculator provides another way of displaying the median and quartiles—using a boxplot.

(a) Use Plot 2 to display a boxplot of the data in L1 on the same graphing screen. Choose the settings shown here.

Press [TRACE] and the left and right cursor keys to read off the five key positions of the boxplot
(minX, Q1, Med, Q3 and maxX).
Write them down.

(b) Work out and write down the value of the interquartile range, Q3 - Q1.

(c) Compare Q1, Med and Q3 values with the values you estimated from the cumulative frequency graph in Activity 5.
How accurate was the estimate?

(d) The data in L2 and L3 are similar to the data in L1 but they are in a frequency format.

Set up Plot 3 to display a boxplot of the frequency data in L2 and L3 on the same graphing screen. Choose the settings shown here.
Switch off the cumulative frequency graph (Plot1).

Plot the boxplots of L1 and L2, L3 on the same screens.
Are there differences?
Can you explain them? Think back to what you did in Activity 3.
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