



## Boxing It Up

### Activity 11

#### NCTM Standards

- ◆ Problem Solving Standard – Solve problems that arise in mathematics and other contexts.
- ◆ Connections Standard – Recognize and apply mathematics in contexts outside of mathematics.
- ◆ Representation Standard – Use representations to model and interpret physical, social and mathematical phenomena.
- ◆ Algebra Standard – Understand patterns, relations, and functions. Approximate and interpret rates of change from graphical and numerical data. Understand and compare the properties of classes of functions.

#### Materials

- ◆ TI-89
- ◆ Five sheets of standard sized paper ( $8\frac{1}{2} \times 11$ ) for each student

#### Topics in Calculus:

Derivatives, Applications of Derivatives

#### Overview:

In this activity, the students will experiment with creating a box from a standard size piece of paper. Then the students will use the TI-89 graphing calculator to find the maximum volume of a box made from the piece of paper.

Name: \_\_\_\_\_ Date: \_\_\_\_\_

# BOXING IT UP

In this activity, you will find the best dimensions to maximize the volume of a box made from an 8½ by 11 piece of paper. To explore this concept, make five different boxes by varying the height of the boxes. Start with small box heights and work toward larger ones.

**STEP ONE:** Once you have determined a height, use a ruler to measure the height along each edge of the paper.

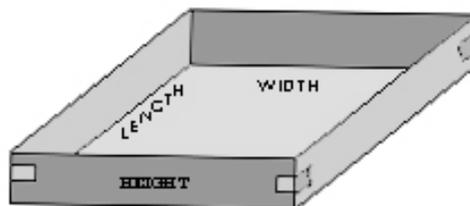
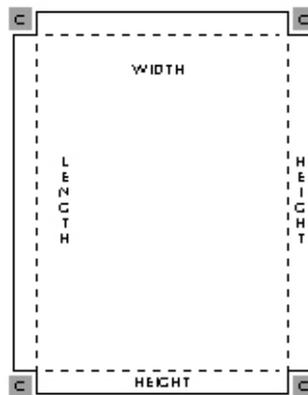
**STEP TWO:** In each corner of the paper, there should be square formed by the lines you drew (represented by the dotted lines to the right). Cut out each of the squares (represented by *c*).

**STEP THREE:** Fold each of the sides along the line you drew. Then tape the sides together. Label the *height (h)*, *width (w)*, and *length (l)* along with each of their measurements.

**STEP FOUR:** Repeat **STEPS ONE** through **THREE** for each of remaining boxes. Remember to gradually increase the height of the box as you continue. Then, fill in the missing information.

Surface Area of a box:  $SA = l \cdot w + 2(l \cdot h) + 2(w \cdot h)$

Volume of a box:  $V = l \cdot w \cdot h$



	Length	Height	Width	Surface Area	Volume
Box 1					
Box 2					
Box 3					
Box 4					
Box 5					

1. What relationship is there between the height and the volume? \_\_\_\_\_  
\_\_\_\_\_
2. How does the height affect the length and width? \_\_\_\_\_  
\_\_\_\_\_
3. What do you notice about the surface area and the volume? \_\_\_\_\_  
\_\_\_\_\_
4. What dimensions give you the most volume? Why? \_\_\_\_\_  
\_\_\_\_\_
5. What dimensions give you the least surface area? Why? \_\_\_\_\_  
\_\_\_\_\_

Using the steps below, verify the solutions from the first page, then answer the questions below.

**STEP ONE:** Convert  $w$  and  $l$  into terms of  $h$ .  $w =$  \_\_\_\_\_  $l =$  \_\_\_\_\_

**STEP TWO:** Substitute  $w$  and  $l$ , from **STEP ONE** in the volume and surface area formulas. Then differentiate each of the formulas. Press  $\boxed{2nd} \boxed{d}$   $expression$   $\boxed{,} \boxed{h} \boxed{)}$   $\boxed{ENTER}$

**STEP THREE:** Then, change all of the variables  $h$  to  $x$  and enter the volume equation into Y1 and the surface area equation into Y2. Press  $\boxed{\blacktriangleright} \boxed{[Y=]} Y1 = expression \boxed{ENTER} \boxed{\ominus} \boxed{ENTER} Y2 = expression \boxed{ENTER}$ .

**STEP FOUR:** Press  $\boxed{F2} \boxed{\alpha} \boxed{A}$  to view the graph of the functions. If the window is not appropriate, press  $\boxed{\blacktriangleright} \boxed{[WINDOW]}$  to set the window yourself and then press  $\boxed{\blacktriangleright} \boxed{[GRAPH]}$ .

**STEP FIVE:** Now, to compute the maximum for the volume, press  $\boxed{F5}$  for the MATH and then select 4:Maximum. Then, select the first equation using the  $\boxed{\ominus} \boxed{\ominus}$  keys. Then, use the  $\boxed{\blacktriangleleft} \boxed{\blacktriangleright}$  keys to select the value to the left of the maximum and press  $\boxed{ENTER}$ . Then use the  $\boxed{\blacktriangleleft} \boxed{\blacktriangleright}$  keys to select the value to the right of the maximum and press  $\boxed{ENTER}$ . The maximum value will appear at the bottom of the screen.

Write the value here: \_\_\_\_\_

**STEP SIX:** Now, to compute the minimum for the surface area, press  $\boxed{F5}$  for the MATH and then select 3:Minimum. Then, select the second equation using the  $\boxed{\ominus} \boxed{\ominus}$  keys. Then, use the  $\boxed{\blacktriangleleft} \boxed{\blacktriangleright}$  keys to select the value to the left of the minimum and press  $\boxed{ENTER}$ . Then use the  $\boxed{\blacktriangleleft} \boxed{\blacktriangleright}$  keys to select the value to the right of the minimum and press  $\boxed{ENTER}$ . The minimum value will appear at the bottom of the screen.

Write the value here: \_\_\_\_\_

1. What is the maximum volume that a box made from an  $8\frac{1}{2}$  by 11 piece of paper can hold? \_\_\_\_\_

2. How does your experiment from the first page differ from the results in STEP FOUR from above?

\_\_\_\_\_

\_\_\_\_\_

3. What does the intersection of the graphs tell you? \_\_\_\_\_

\_\_\_\_\_

4. What are the bounds for the volume? (What are the smallest and largest volumes possible?) \_\_\_\_\_

\_\_\_\_\_

5. What are the dimensions of the box with maximum area? Height? \_\_\_\_\_ Width? \_\_\_\_\_ Length? \_\_\_\_\_

6. What generalizations about the dimensions of a box can be made from this activity?

\_\_\_\_\_

\_\_\_\_\_