



The Spread of Disease with Differential Equations

Activity 8

NCTM Standards

- ◆ Problem Solving Standard – Solve problems that arise in mathematics and other contexts.
- ◆ Connections Standard – Recognize and apply mathematics in contexts outside of mathematics.
- ◆ Representation Standard – Use representations to model and interpret physical, social and mathematical phenomena

Materials

- ◆ TI-89

Topics in Calculus:

Differential Equations

Overview:

This activity will have the students use the logistics equation to model the spread of a disease.

Spread of Disease

After winter break a student infected with a flu virus returns to your dorm. One hundred students live in the dorm. The pattern of the spread of disease in seven days is illustrated in the table below.

Number of Days	Number of Infected Students
1	1
2	2
3	4
4	8
5	13
6	24
7	36

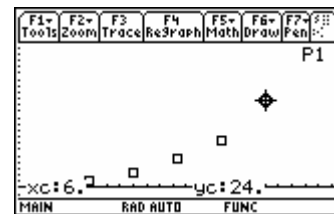
Exercise 1:

Enter the data in the Data Matrix Editor of a TI-89. Draw a scatter plot.

Solution:

F1- Tools	F2- Plot Setup	F3- Cell	F4- Header	F5- Calc	F6- Util	F7- Stat
DATA	days	no. inf				
1	c1	c2	c3			
2	1	1				
3	2	2				
4	3	4				
5	4	8				
r1c1=1						
MAIN RAD AUTO FUNC						

F1- Tools	F2- Plot Setup	F3- Cell	F4- Header	F5- Calc	F6- Util	F7- Stat
DATA	days	no. inf				
4	c1	c2	c3			
5	4	8				
6	5	13				
7	6	24				
8	7	36				
r7c1=7						
MAIN RAD AUTO FUNC						



Exercise 2:

Use the Logistic Population Model to analyze the spread of disease.

Solution:

The logistic population model states that the rate of change of the infected population with respect to time is directly proportional to the product of the number of people infected times the number of people who are not infected. This statement is represented by the differential equation $y' = ky(100-y)$ and initial condition $y(1) = 1$ where

y' = the rate of change of the number of infected people with respect to time;

y = the number of infected people;

t = time in seconds; and

k = the constant of proportionality.

- a) Use the command **deSolve** on the Home screen to solve the differential equation.

The calculator screen shows the following steps:

- Menu: F1 Tools, F2 Lists, F3 Math, F4 Distributions, F5 Prb, F6 1-D, F7 2-D, F8 3-D, F9 Stats, F10 Programs, F11 Settings, F12 Help
- Command: `deSolve(y' = k · y · (100 - y) |`
- Equation: $y = \frac{100 \cdot e^{100 \cdot k \cdot t}}{e^{100 \cdot k \cdot t} + 99}$
- Initial condition: `...*(100-y) and y(0)=1, t, y)`
- Status bar: MAIN, RAD AUTO, SEQ, BATT B/9

- b) Use another data point to obtain an equation with one variable, k.

The calculator screen shows the following steps:

- Equation: $y = \frac{100 \cdot e^{100 \cdot k \cdot t}}{e^{100 \cdot k \cdot t} + 99} | t = 7 \text{ and}$
- Equation: $36 = \frac{100 \cdot e^{700 \cdot k}}{e^{700 \cdot k} + 99}$
- Command: `ans(1) | t=7 and y=36`
- Status bar: MAIN, RAD AUTO, FUNC, 1/3

- c) Solve for k.

The calculator screen shows the following steps:

- Equation: $36 = \frac{100 \cdot e^{700 \cdot k}}{e^{700 \cdot k} + 99}$
- Command: `approx(solve(36 = \frac{100 \cdot e^{700 \cdot k}}{e^{700 \cdot k} + 99}, k))`
- Result: $k = .005743$
- Command: `approx(solve(ans(1), k))`
- Status bar: MAIN, RAD AUTO, FUNC, 1/4

- d) Substitute this value for k in the solution to the differential equation.

The calculator screen shows the following steps:

- Equation: $y = \frac{100 \cdot e^{100 \cdot k \cdot t}}{e^{100 \cdot k \cdot t} + 99} | k = .005$
- Equation: $y = \frac{100 \cdot (1.7758)^t}{(1.7758)^t + 99}$
- Command: `ans(3) | k=.005742508150302...`
- Status bar: MAIN, RAD AUTO, FUNC, 1/5

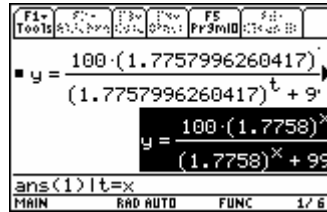
Answer: The mathematical model is highlighted in the screen above.

Exercise 3:

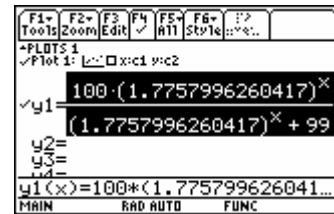
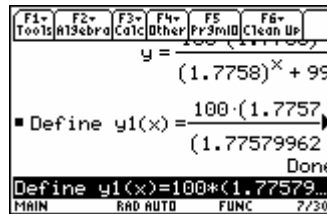
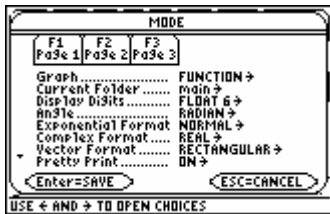
Graph the model and the data.

Solution:

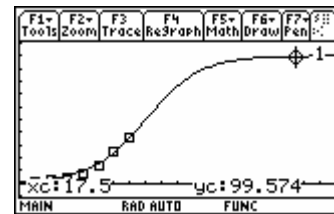
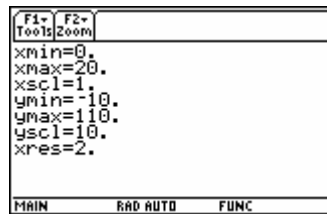
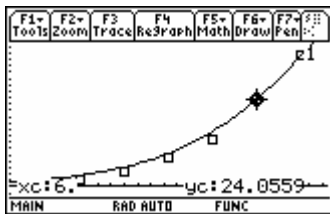
1. Substitute x for t so that you can enter the solution of the differential equation in **Function MODE**.



2. Copy this equation into the **Y=** editor. Make sure the TI-89 is in **FUNCTION MODE**.



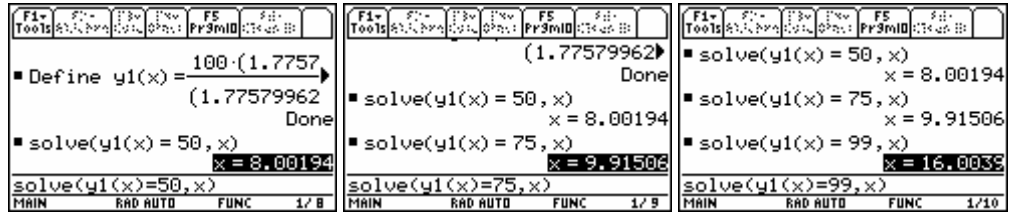
3. Use **F2 Zoom, 9:ZoomData** to graph the scatter plot of the data and the solution of the differential equation, $y_1(x)$ in this case. Change the window.



Exercise 4:

Use the model, $y_1(x)$, to predict when the number of infected students will be: 50, 75, and 99.

Solution:



Answer: The model, $y_1(x)$, predicts that
50 students will be infected with the flu after 8 days..
75 students will be infected with the flu after 9.9 days..
50 students will be infected with the flu after 16 days..

Exercise 5:

Compare the model with the data points.