Bottling Up the Profits

Activity 6

NCTM Standards

- Problem Solving Standard – Solve problems that arise in mathematics and other contexts.
- Connections Standard – Recognize and apply mathematics in contexts outside of mathematics.
- Representation Standard – Use representations to model and interpret physical, social and mathematical phenomena.
- Algebra Standard – Understand patterns, relations, and functions. Approximate and interpret rates of change from graphical and numerical data. Understand and compare the properties of classes of functions.

Materials

- TI-89
- Examples of Solids

Topics in Calculus:

Derivatives, Applications of Derivatives

Overview:

This activity is designed to let students explore one of the most popular applications of derivatives. In this activity, the students will create their own drink and find a cost effective container to bottle the drinks in.

Teacher Directions:

It would be helpful to bring in several examples of each type of solid used in this activity. Although the activity is self-guided, walk around to help the students, especially with the first example.

EXTENSION: Have the students calculate the cost of using each of the types. Also have them find the difference in cost between a solid having minimum surface area and one without minimum surface area. You can also have the students decorate foam solids (of their choice) with their drink's name, logo, radius, height, and volume.

Name: _____ Date: _____

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In this activity, you will determine best shape and size container you should use to bottle your own beverage. The shapes that we will use will be a cylinder, a cone and a sphere. In order to maximize the profit, you need to find the size for each shape for which the surface area is minimized. Each bottle needs to hold 12 fluid ounces or 355mL.

Write the requested formulas for each for each of the shapes below. Use *h = height* and *r = radius.*

	Cylinder	**Cone**	**Sphere**
Volume (*v*)	= _____	= _____	= _____
Surface Area (*s*)	= _____	= _____	= _____

Use the steps listed below to find the minimum value for the radius and the height using the TI-89. Steps three and eight are for the cylinder and cone only. Follow the worksheet on the next page and use the directions below for each shape.

STEP ONE: Enter the expression that represents the volume and save it as *v*. Use the same terms as in the formulas. *expression* **V** (To get π on the TI-89, press .)

STEP TWO: Enter the expression that represents the surface area and save it as *s*. Use the same terms as in the formulas. *expression* **S**

STEP THREE: (For the cylinder and cone only.) Using the solve command. solve the volume equation for *h* in terms of *r*. **1:solve(** *v* **=** *expression* *h*

STEP FOUR: Evaluate the surface area equation (*s*) at the value for *h* from **STEP THREE**.
S ,

STEP FIVE: Now, find the derivative of the solution from **STEP FOUR** and save the solution as *D*.
r **D**

STEP SIX: Set the derivative equal to zero and solve the new equation from **STEP FIVE**. This solution is the radius at which the shape is minimized. **1:solve(** 0 **=** *expression* *r* .

STEP SEVEN: Find the second derivative to test that the solution from **STEP SIX** is a minimum value. Use the equation from **STEP FIVE**, which is the first derivative. Then evaluate the second derivative at the value for *r* in **STEP SIX**. *expression* *r* **D** then to evaluate the equation at *r*: **R** .

STEP EIGHT: (For the cylinder and cone only.) Solve for the height of the shape by using the value for *r* in the volume equation.

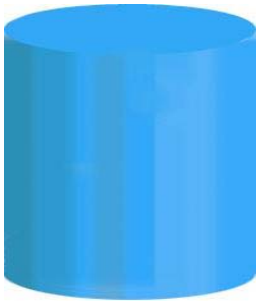
What is the name of your drink? _____

What would it taste like? _____

_____

For each of the shapes listed on the first page, complete the information below.

1. Cylinder



Write the function you used from STEP FOUR: _____

Write the derivative of the function from STEP FIVE: _____

Write the second derivative from STEP SEVEN: _____

What is the radius that minimizes the surface area? _____

What is the height (from STEP EIGHT) that minimizes the surface area? _____

What is the minimal surface area? _____

2. Cone

Write the function you used from STEP FOUR: _____

Write the derivative of the function from STEP FIVE: _____

Write the second derivative from STEP SEVEN: _____

What is the radius that minimizes the surface area? _____

What is the height (from STEP EIGHT) that minimizes the surface area? _____

What is the minimal surface area? _____

3. Sphere

Write the function you used from STEP FOUR: _____

Write the derivative of the function from STEP FIVE: _____

Write the second derivative from STEP SEVEN: _____

What is the radius that minimizes the surface area? _____

What is the minimal surface area? _____

Which shape has the least amount of surface area? Why?

_____

_____

Which has a greater effect on the surface area, radius or height? Why?

_____

_____