Piecewise Functions, Continuity and Differentiability

Activity 2

NCTM Standards

- Connections Standard – Recognize and apply mathematics in contexts outside of mathematics.
- Representation Standard – Select, apply and translate among mathematical representations to solve problems

Materials

- TI-89

Topics in Calculus:

Functions and Equations, Derivatives, Limits and Continuity

Overview:

Students will examine the continuity and differentiability of piecewise functions.

Piecewise Functions, Continuity and Differentiability

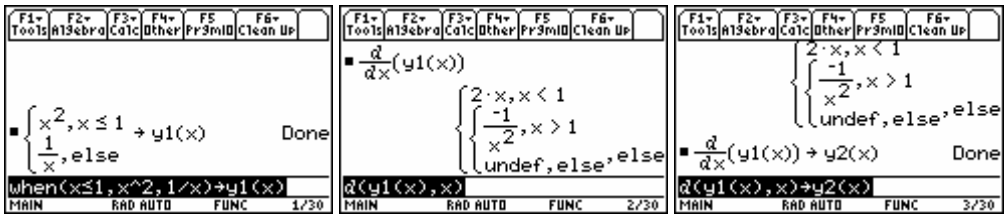
Exercises:

1. Consider the function given by $f(x) = \begin{cases} x^2, x \le 1, \\ 1/x, x > 1 \end{cases}$.

   Graph *f* and its derivative, *f′*.

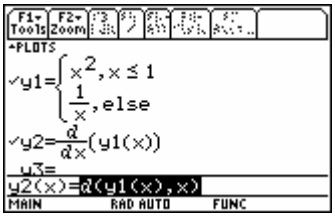
Solution:

Reproduce the following screens on your TI-89.

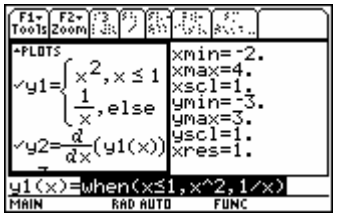


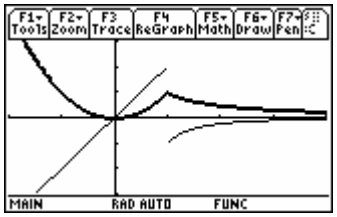
On the home screen use the **when** and **STO>**commands for the first condition of the piecewise function and store it in *y*1(x). Find the derivative of *y*1 and store it in *y*2(x). **Note**: Nested **when** commands define the three-part rule for the derivative.
This derivative could also be entered as:
Define y2(x) = when(x<1,2x,when((x>1,-1/x^2,undef)).

Another method to define *y*1 and *y*2 is portrayed below in the Y= editor.



Since different graphing styles are allowed for each function, use the dot style for the graph of the derivative function with jump discontinuity and thick for the graph of *y*1.



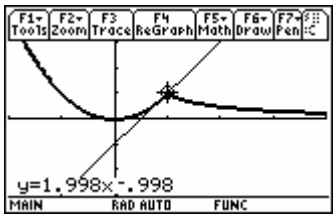


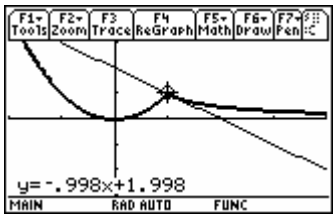
2. Is *y*1 continuous at *x* = 1?

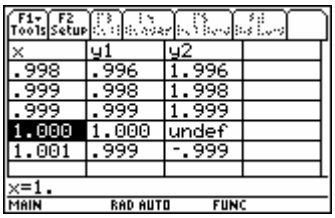
   Answer: Since *y*1(1) = 1 and $lim_{x \to 1}$ *y*1*(x)* = 1, *y*1 is continuous at *x* = 1.

3. Is $y1$ differentiable at $x = 1$?

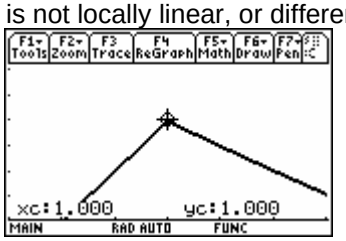
Solution:
Use the graph of the original function to explore the question of differentiability at $x = 1$. Draw the tangent lines at $x = .999$ and $x = 1.001$ to visualize the approximate values of the left and right hand derivatives at $x = 1$. Access the command by selecting **F5 (Math)**, **A: Tangent**. Type in the x value where the tangent line is to be drawn. Notice its equation in the bottom left hand corner of the screen. Look at the **TABLE**.







Alternatively, zoom in three times at the point (1, 1) and notice that a corner appears. This illustrates that the function is not locally linear, or differentiable, at $x = 1$.



Additional Exercise:

Consider the function given by $f(x) = |x|$. Graph f and its derivative, f'.
Is f continuous at $x = 0$? Is f differentiable at $x = 0$?