Fuel for the Fire

TEACHER NOTES

Objectives

Students will

- learn about constant rates of change.
- apply linear functions in a scientific context.
- apply volumes of cylinders in a scientific context.
- learn about relating rates with a linear function.
- learn about the STEM career Fuel Systems Engineer.

Vocabulary

- cryogenic fuel
- liquid hydrogen (LH_2)
- liquid oxygen (LOX)
- function
- rate of change
- ratio
- value of a ratio

About the Lesson

- The lesson tells the story of Jonathan Looser, the propulsion lead for the Space Launch System (SLS).
- Throughout the lesson, students will become familiar with how changes in one characteristic of a cylinder may cause changes in another related characteristic of the cylinder.
- Students will apply knowledge of volume of cylinders, proportional relationships, and linear functions to a STEM context.
- **Teaching time:** one to two 45-minute class period(s).

TI-Nspire™ Navigator™

- Send out the Fuel for the Fire.tns file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Lesson Materials

- Compatible TI Technologies: TI- Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This lesson includes screen captures taken from the TI-Nspire™ CX handheld. It is also appropriate for use with the TI-Nspire family of products, including TI-Nspire™ software and TI-Nspire™ Apps. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the lesson for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials.

Lesson Files:

Student Activity

- Fuel for the Fire_Student.doc
- Fuel for the Fire_Student.pdf

TI-Nspire document

- Fuel for the Fire.tns

Background

STEM CAREER

This lesson focuses on the math involved in fueling NASA's Space Launch System (SLS). Jonathan Looser, propulsion lead for the SLS, oversees the design, development, testing, integration, and operation of the main propulsion systems on the 200-foot long core stage. Looser has a team of specialists that he works with on the project. There are many individuals with varied backgrounds who are involved in every aspect of the project, from engineers and technicians to chemists and physicists. All of the team members are working toward one goal, successful launch and return of each SLS flight.

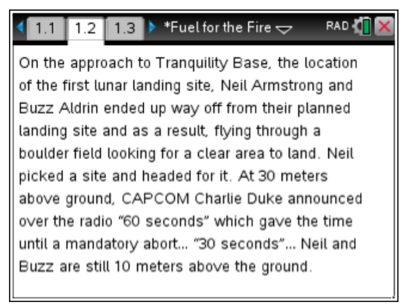
OVERVIEW

Students will be engaged in a game landing the Eagle module at Tranquility Base. They will explore how the shape of a fuel tank is related to a graph of height vs. time as the tank is filled at a constant rate. Students will use the knowledge gained from that exploration to determine a function for computing the volume of fuel given the height in the cylindrical tanks of the SLS. Finally students will use that function to launch the SLS on its first mission around the moon.

This highly interactive lesson immerses students into the world of rocket science, the STEM careers associated with it, and the math and science behind space exploration.

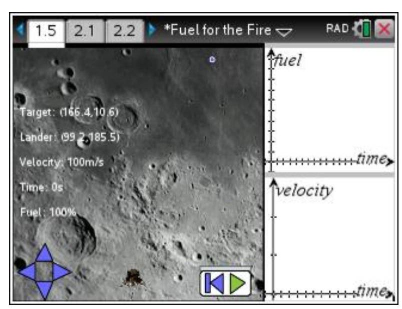
Move to pages 1.2 – 1.4.

1. Pages 1.2 through 1.4 give students the historical perspective on the first lunar landing. Students are given a chance to explore the historical moment when the Eagle Lunar Module touched down at Tranquility Base for the first time. Students should have enough time to read the story and relate to the historical events from July 20, 1969. Focus here should be on the mission's accomplishment, with emphasis on the fact that the Module almost ran out of fuel due to unforeseen complications when landing on the surface. The main idea is that fuel monitoring is very important.

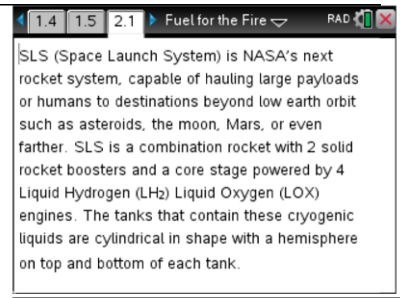


Move to page 1.5.

2. Page 1.5 gives students a chance to land on the moon while monitoring their fuel consumption and speed. This could easily become a game, so there is a menu option for collecting time and fuel consumption data for comparison should you choose to use it. Be sure students understand that the arrows work opposite of natural assumptions. Up slows down, down speeds up, left turns right, and right turns left in an effort to model retrograde rockets.

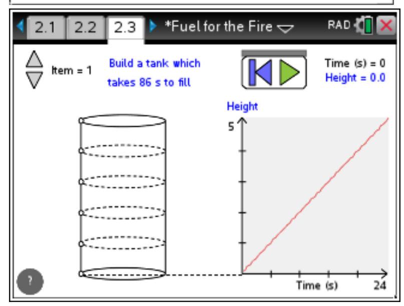


Move to pages 2.1 – 2.2.



3. These pages give students some background information on the SLS. The SLS core stage is powered by 4 engines that burn a mixture of liquid oxygen and liquid hydrogen. The two tanks contained in the core stage are cylindrical with a hemisphere on the top and bottom.

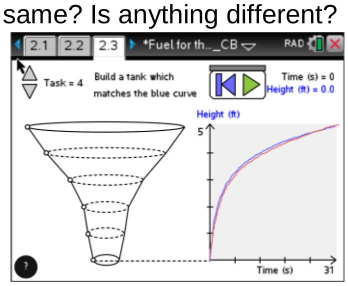
Move to page 2.3.



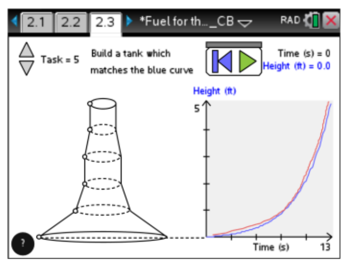
4. Students are presented with a tank that has 6 moveable points connected by lines. They are first asked to construct tanks that fill in a certain amount of time, then they are asked to create tanks that will have a certain height vs. time graph as they are filled at a constant rate of volume. The focus here is on establishing the relationship between the graph of height vs. time and the shape of the tank. Students should begin to develop the idea that the cross sectional area of the tank affects the rate of change of height.

Questions related to page 2.3

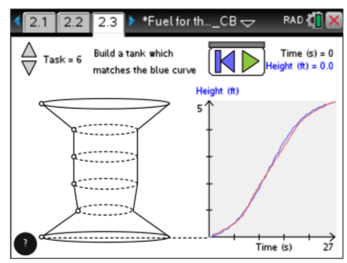
Q1. Sketch your solutions to Task 4, 5 and 6 below. Compare your tanks with a classmate. What is the same? Is anything different?



Task 4



Task 5



Task 6

<u>Answer</u>: Actual solutions will vary but students should notice that the general shape of their solutions will be the same.

Q2. Shift the task back to Task 1. Create a narrow cylinder, select the play button, and record the time it takes to fill the cylinder. Create a wide cylinder and record the time it takes to fill the cylinder. Which cylinder fills the fastest?

<u>Sample Answer</u>: The narrow cylinder only took 6 seconds to fill while the wide cylinder took 87 seconds. The narrow cylinder filled much quicker than the wide cylinder.

Q3. Change the sides of the tank so it is no longer a cylinder. What changes do you observe in the graph as you change the sides of the tank?

<u>Sample Answer</u>: When the tank is a cylinder, the graph appears linear. When the sides are tilted in or out, the graph changes to nonlinear.

More questions related to page 2.3

Q4. Make a cone-shaped tank that goes from wide at the bottom to narrow at the top. Describe the graph of height vs. time. Select play to fill the tank. Is the rate of change of height increasing, decreasing, or remaining the same?

Sample Answer: When the tank goes from wide to narrow, the graph is nonlinear and getting steeper. The rate of change of height over time is increasing.

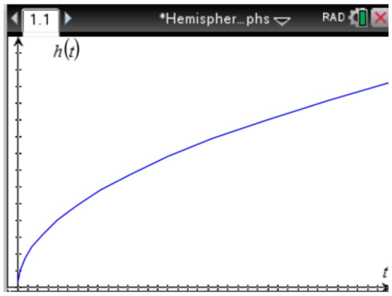
Q5. Make a cone-shaped tank that goes from narrow at the bottom to wide at the top. Describe the graph of height vs. time. Is the rate of change of height increasing, decreasing, or remaining the same?

Sample Answer: When the tank goes from narrow to wide, the graph is nonlinear and getting flatter. The rate of change of height over time is decreasing.

Q6. Make a conjecture about how the cross-sectional area of the tank affects how fast height of fuel in the tank changes over time.

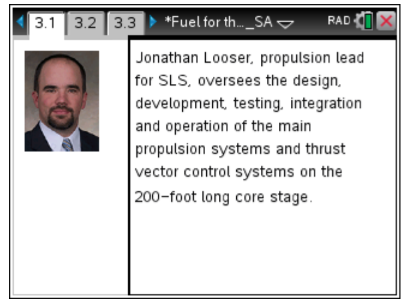
Sample Answer: When the area is large, height increases slowly, and when the area is small, height increases rapidly.

Q7. Now that you've seen how the shape of the tank is related to the shape of the graph of height vs. time when the fuel rate is constant, which of the following graphs shows how the height of the liquid hydrogen tank of the core stage of the SLS will change over the first 45 minutes if the rate of fill is assumed to be constant? (Hint: try using the file to model the shape of the bottom half of the tank as closely as possible to justify your answer.)



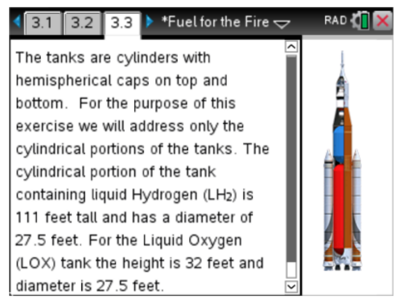
Answer: D. The bottom of the SLS tank is a hemisphere on the bottom of a cylinder so it goes from narrow to wide and then once it reaches the cylindrical portion of the tank, the tank's width remains constant. Therefore, the rate of change of height will be rapid at first and then slow down until it reaches the cylindrical portion of the tank, and then the rate of change would remain constant. The graph for D is nonlinear and the rate of change is decreasing in the first part, and then it remains constant throughout the remainder of the time.

Move to pages 3.1 – 3.2.



5. Pages 3.1 and 3.2 introduce Jonathan Looser, propulsion lead for the SLS. Looser oversees the design, development, testing, integration and operation of the main propulsion systems and thrust vector control systems on the 200-foot long core stage. Prior to his current position, Looser supported the Space Shuttle Program, working on both the external tank and the rocket fuel tank for the space shuttle and shuttle propulsion systems. A native of Huntsville, Alabama Looser graduated in 2002 from the University of Alabama in Huntsville (UAH) with a bachelor's degree in mechanical engineering. He started working for NASA's Marshall Space Flight Center as a student the year earlier as part of Marshall's cooperative education program, then joined Marshall full time following his graduation.

Move to pages 3.3 – 3.5.



6. Students are given some information on the dimensions of the tanks. For this exercise we are only considering the cylindrical portions of the tank. Information on the fueling process is given in these pages and will help guide students in the simulation and questions that follow.

Questions related to pages 3.2 – 3.5

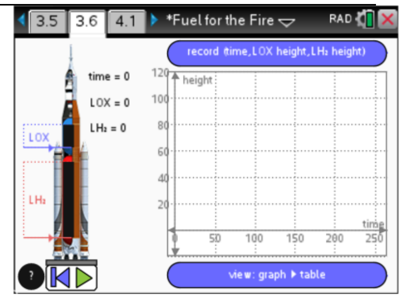
Q8. The area of the base of each tank is the same, what is the area of the base of both tanks to the nearest square foot?

<u>Sample Answer</u>: Approximately 594 square feet

Q9. Excluding the hemispherical caps, what is the approximate volume of each tank to the nearest cubic foot?

<u>Sample Answer</u>: For the LOX tank the volume is 19,008 cubic feet. For the LH_2 tank the volume is 65,934 cubic feet. These are only the cylindrical portions of the tanks and therefore not exact for the total volume of each tank including the caps.

Move to page 3.6



7. On this page students will observe the tanks of SLS being fueled simultaneously with liquid hydrogen and liquid oxygen. Students should observe the graph of height of fuel vs. time for each tank and note that each has four linear parts. (Note: The first two time periods have nearly the same rate of change and so are somewhat indistinguishable visually.) This is because the cryogenic fuels require that the tanks be cooled down sufficiently to limit boil off during the fueling process. Working with cryogenic fuels requires Jonathan and his team to account for boil off since both fuels boil off rapidly at room temperatures. For the purpose of this activity, the boil off is not considered. Students should use the data collection button to collect data to fill the tables for the LOX tank and LH_2 tank with height and time data. They will use this data to calculate the average rate of change, the slope, for each section of the fueling function.

Questions related to 3.6

Q10. The total time to fill each tank is the same. Based on the graph, how long does it take to fill the tanks?

Answer: 255 minutes.

Q11. What do you notice about the rate of change of height over time?

Sample Answer: The slope for each graph appears to fluctuate between a few constant rates. Slow at first, slightly faster, quite fast, and finally, much slower again. Note: students may only notice 3 distinct rates, but there are 4 distinct rates. The first parts of each graph are very similar, making it difficult to distinguish the parts.

Q12. Given that the tanks have straight sides, what do you think is causing the graph to have this shape?

Sample Answer: The fueling rate is being adjusted to account for the cooling of the tanks and the system down to cryogenic temperatures.

More questions related to page 3.6

Q13. Replay the simulation and take two data points for height and time filling for each portion of the graph. Be sure your points are within each of the boundaries in each table. Compute the rate of change in height of the fuel in each tank with respect to time, $\frac{\Delta h}{\Delta t}$, for the distinct portions of the graph. Complete the table below for each time period.

Note: The delta notation may be new to students so work with them on recognizing how it is similar to the familiar slope equation; $slope = \frac{y_2 - y_1}{x_2 - x_1}$. Be sure students' data fall within the prescribed boundaries.

Answer: Student data will vary, but the computed average rate of change should approximately match the following. Due to rounding there will be slight differences in the rates of change computed. Below are sample data.

For LOX:

Time period	t_1	h_1	t_2	h_2	Rate of Change $\frac{\Delta h}{\Delta t} = \frac{h_2 - h_1}{t_2 - t_1}$
0 to 45	0	0	32	1.91	$\frac{\Delta h}{\Delta t} = \frac{1.91 - 0}{32 - 0} = 0.0597$ ft/min
45 to 75	56	3.47	72	4.61	$\frac{\Delta h}{\Delta t} = \frac{4.61 - 3.47}{72 - 56} = 0.0713$ ft/min
75 to 135	88	7.93	128	17.46	$\frac{\Delta h}{\Delta t} = \frac{17.46 - 7.93}{128 - 88} = 0.2383$ ft/min
135 to 255	160	21.81	255	32.01	$\frac{\Delta h}{\Delta t} = \frac{32.01 - 21.81}{255 - 160} = 0.1074$ ft/min

For LH_2:

Time period	t_1	h_1	t_2	h_2	Rate of Change $\frac{\Delta h}{\Delta t} = \frac{h_2 - h_1}{t_2 - t_1}$
0 to 45	0	0	32	0.41	$\frac{\Delta h}{\Delta t} = \frac{0.41 - 0}{32 - 0} = 0.0128$ ft/min
45 to 75	56	1.76	72	3.47	$\frac{\Delta h}{\Delta t} = \frac{3.47 - 1.76}{72 - 56} = 0.1069$ ft/min
75 to 135	88	22.38	128	79.55	$\frac{\Delta h}{\Delta t} = \frac{79.55 - 22.38}{128 - 88} = 1.4293$ ft/min
135 to 255	160	94.03	255	111	$\frac{\Delta h}{\Delta t} = \frac{111 - 94.03}{255 - 160} = 0.1786$ ft/min

Q14. Between what two times in the filling process is the height of fuel in the LOX tank changing the fastest? The slowest?

Sample Answer: Fastest is between 75 and 135 minutes, and slowest is between 0 and 45 minutes.

Q15. In each part of the graph is the rate of change of height over time for the LH_2 tank increasing, decreasing, or remaining constant?

Sample Answer: In each part of the graph, the rate of change is constant. This is because of the cylindrical shape of the tank. The same is true for the LOX tank.

Q16. The fill rate, $\frac{\Delta V}{\Delta t}$, for each portion of the fueling process is given in the table below. Determine the value of the ratio of fill rate to the rate of change of height for each portion to the nearest ft^2.

Sample Answer: Ratio values will vary slightly due to rounding in student data, but all should be very close to 594 ft^2.

LOX:

Time period	$\left(\frac{\Delta V}{\Delta t}\right) \frac{ft^3}{min}$	Value of the ratio $\left(\frac{\Delta V}{\Delta t}\right):\left(\frac{\Delta h}{\Delta t}\right)$
0 to 45 minutes	35.400	$\frac{\frac{\Delta V}{\Delta t}}{\frac{\Delta h}{\Delta t}} = \frac{35.400 \frac{ft^3}{min}}{0.0597 \frac{ft}{min}} = 593\ ft^2$
45 to 75 minutes	42.468	$\frac{\frac{\Delta V}{\Delta t}}{\frac{\Delta h}{\Delta t}} = \frac{42.468 \frac{ft^3}{min}}{0.0713 \frac{ft}{min}} = 596\ ft^2$
75 to 135 minutes	141.600	$\frac{\frac{\Delta V}{\Delta t}}{\frac{\Delta h}{\Delta t}} = \frac{141.600 \frac{ft^3}{min}}{0.2383 \frac{ft}{min}} = 594\ ft^2$
135 to 255 minutes	63.732	$\frac{\frac{\Delta V}{\Delta t}}{\frac{\Delta h}{\Delta t}} = \frac{63.732 \frac{ft^3}{min}}{0.1074 \frac{ft}{min}} = 593\ ft^2$

LH_2:

Time period	$\left(\frac{\Delta V}{\Delta t}\right) \frac{ft^3}{min}$	Value of the ratio $\left(\frac{\Delta V}{\Delta t}\right):\left(\frac{\Delta h}{\Delta t}\right)$
0 to 45 minutes	7.662	$\frac{\frac{\Delta V}{\Delta t}}{\frac{\Delta h}{\Delta t}} = \frac{7.662 \frac{ft^3}{min}}{0.0128 \frac{ft}{min}} = 599\ ft^2$
45 to 75 minutes	63.672	$\frac{\frac{\Delta V}{\Delta t}}{\frac{\Delta h}{\Delta t}} = \frac{63.672 \frac{ft^3}{min}}{0.1069 \frac{ft}{min}} = 596\ ft^2$
75 to 135 minutes	849.003	$\frac{\frac{\Delta V}{\Delta t}}{\frac{\Delta h}{\Delta t}} = \frac{849.003 \frac{ft^3}{min}}{1.4293 \frac{ft}{min}} = 594\ ft^2$
135 to 255 minutes	106.140	$\frac{\frac{\Delta V}{\Delta t}}{\frac{\Delta h}{\Delta t}} = \frac{106.140 \frac{ft^3}{min}}{0.1786 \frac{ft}{min}} = 594\ ft^2$

Q17. Does the value of the ratio of fill rate to rate of change of height for both tanks change or remain approximately the same over the entire filling process?

Sample Answer: The value of the ratio remains approximately the same. Note: Student answers will vary based on rounding.

Q18. How is your response to question 17 related to your response in question 8? Is there a feature of the tanks that is related to the value of the ratio? Explain why this makes sense based on the formula for the volume of a cylinder.

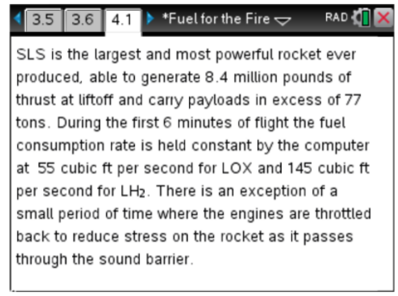
Sample Answer: In both cases I calculated a value close to 594 square feet. It appears that the value of the ratio of fill rate to rate of change of height is the area of the base. This makes sense since the formula for volume of a cylinder is V = Bh.

Q19. Using your answer to question 18, write an equation in the form y = ax relating the rate of change of volume of fuel in the tank, $\frac{\Delta V}{\Delta t}$, in terms of the rate of change in height in the tank, $\frac{\Delta h}{\Delta t}$.

Sample Answer: $\frac{\Delta V}{\Delta t} = 594\left(\frac{\Delta h}{\Delta t}\right)$. The variables here are the rate of change of volume, $\frac{\Delta V}{\Delta t}$, and the rate of change of height, $\frac{\Delta h}{\Delta t}$. Though the notation is unfamiliar this is a directly proportional relationship in the form *y* = *ax*. This closely parallels the formula for volume of a cylinder V = Bh.

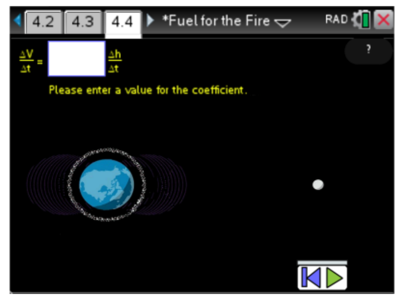
Read pages 4.1 – 4.3

8. These pages give students some more background information on the SLS and its first planned mission, EM-1.



Move to page 4.4.

9. This page allows the students to check their response to question 19. The relationship that they discovered is used to determine the fuel consumption function for the SLS liftoff. If everything is correct, they will have a successful launch of the SLS on EM-1 (the first mission for the SLS). If they have not determined an appropriate relationship between $\frac{\Delta V}{\Delta t}$ and $\frac{\Delta h}{\Delta t}$, their launch will fail.



TI-Nspire™ Navigator Opportunities

Make a student the Live Presenter to demonstrate his or her tank solutions that fill in a given time, or their data collection tables from the tank filling simulation.

Assessment

Students will answer questions throughout the lesson to ensure they understand the concepts of rate of change, linear functions, and how the shape of a tank affects the rate of change of height over time as the tank is filled at a constant rate.

For more information about NASA's Space Launch System and rocket propulsion check out these links:

https://www.nasa.gov/exploration/systems/sls/index.html

http://www.nasa.gov/centers/marshall/propulsion.html