



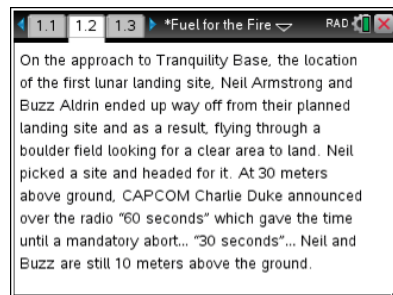
### Open the TI-Nspire document *Fuel for the Fire.tns*.

You've probably heard about Neil Armstrong and his first footstep on the moon but have you ever considered the fuel it took to get there? Fueling NASA's Space Launch System (SLS) is no small task. In this lesson you will apply your knowledge of linear functions and rates of change to rocket science.



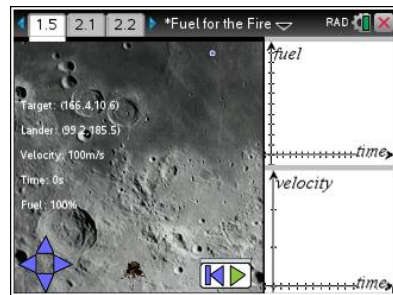
### Move to pages 1.2 – 1.4.

- Pages 1.2 – 1.4 tell the story of perhaps the single best known accomplishment of the space program: the first humans landing on the moon. Read through these pages to become familiar with Neil Armstrong and Buzz Aldrin's experience landing on the lunar surface for the first time in human history.



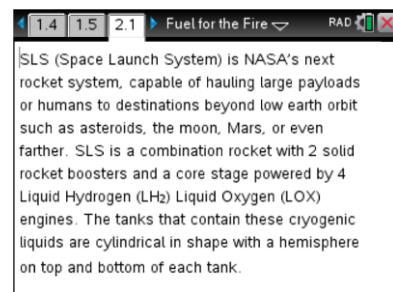
### Move to page 1.5

- On page 1.5 press the play button to launch the Eagle Lunar Module on the page. Use the arrow keys to try to direct the module to the target on the moon's surface. Be sure to get your velocity below 20 m/s before you run out of fuel or fly past the target.



### Move to pages 2.1 – 2.2

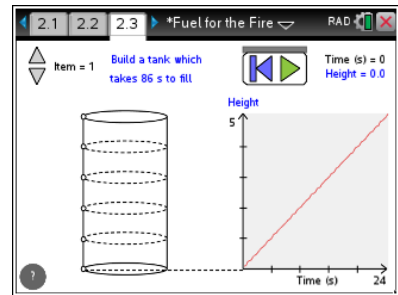
- These pages give some background information on the SLS and its core stage fuel tanks.





### Move to page 2.3.

4. Use the points to manipulate the shape of the tank. Pay attention to how the rate of change of height of fuel in the tank changes as you make the tank different shapes. Complete the six tasks. Tasks 1–3 ask to have a tank filled in certain time, and 4–6 ask for a tank that has a specific height vs. time graph.



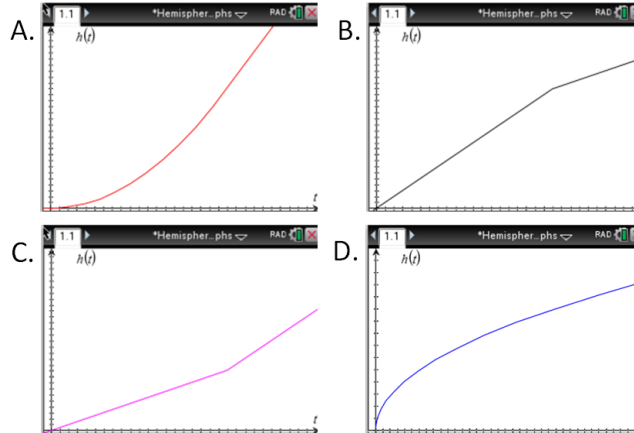
### Questions related to page 2.3

- Q1. Sketch your solutions to Task 4, 5 and 6 below. Compare your tanks with a classmate. What is the same? Is anything different?
- Q2. Shift the task back to Task 1. Create a narrow cylinder, select the play button, and record the time it takes to fill the cylinder. Create a wide cylinder and record the time it takes to fill the cylinder. Which cylinder fills the fastest?
- Q3. Change the sides of the tank so it is no longer a cylinder. What changes do you observe in the graph as you change the sides of the tank?
- Q4. Make a cone-shaped tank that goes from wide at the bottom to narrow at the top. Describe the graph of height vs. time. Select play to fill the tank. Is the rate of change of height increasing, decreasing, or remaining the same?



More questions related to page 2.3

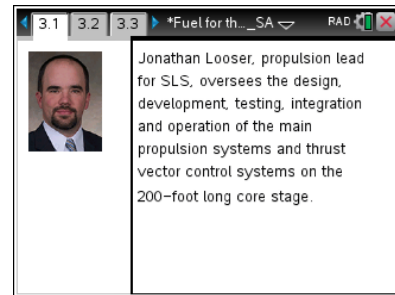
- Q5. Make a cone-shaped tank that goes from narrow at the bottom to wide at the top. Describe the graph of height vs. time. Is the rate of change of height increasing, decreasing, or remaining the same?
- Q6. Make a conjecture about how the cross-sectional area of the tank affects how fast height of fuel in the tank changes over time.
- Q7. Now that you've seen how the shape of the tank is related to the shape of the graph of height vs. time when the fuel rate is constant, which of the following graphs shows how the height of the liquid hydrogen tank of the core stage of the SLS will change over the first 45 minutes if the rate of fill is assumed to be constant? (Hint: try using the file to model the shape of the bottom half of the tank as closely as possible to justify your answer.)





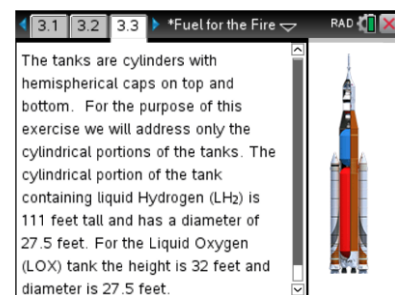
### Move to pages 3.1 – 3.2.

- Pages 3.1 and 3.2 introduce Jonathan Looser, propulsion lead for the SLS. Looser oversees the design, development, testing, integration and operation of the main propulsion systems and thrust vector control systems on the 200-foot long core stage.



### Move to pages 3.3 – 3.5.

- Read the information about the tanks for the core stage of the SLS.

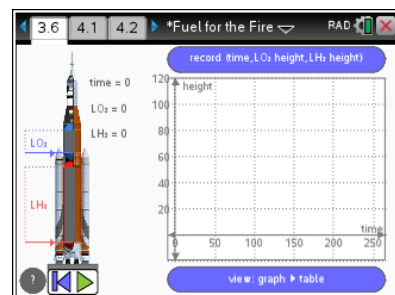


### Questions related to pages 3.3 – 3.5.

- The area of the base of each tank is the same. What is the area of the base of both tanks to the nearest square foot?
- Excluding the hemispherical caps, what is the approximate volume of each tank to the nearest cubic foot?

### Move to page 3.6

- Play the simulation for filling the tanks and observe the graph of height vs. time. Keep in mind this is just addressing the cylindrical portion of each tank. Units are in feet and minutes.





Questions related to 3.6

- Q10. The total time to fill each tank is the same. Based on the graph, how long does it take to fill the tanks?
- Q11. What do you notice about rate of change of height over time?
- Q12. Given that the tanks have straight sides, what else do you think could be causing the graph to have this shape?
- Q13. Replay the simulation and take two data points for height and time filling for each portion of the graph. Be sure your points are within each of the boundaries in each table. Compute the rate of change in height of the fuel in each tank with respect to time,  $\frac{\Delta h}{\Delta t}$ , for the distinct portions of the graph. Complete the table below for each time period.

For LOX:

Time period	$t_1$	$h_1$	$t_2$	$h_2$	Rate of Change $\frac{\Delta h}{\Delta t} = \frac{h_2 - h_1}{t_2 - t_1}$
0 to 45					$\frac{\Delta h}{\Delta t} = \frac{\underline{\quad} - \underline{\quad}}{\underline{\quad} - \underline{\quad}} = \underline{\quad} \frac{ft}{min}$
45 to 75					
75 to 135					
135 to 255					



For LH<sub>2</sub>:

Time period	$t_1$	$h_1$	$t_2$	$h_2$	Rate of Change $\frac{\Delta h}{\Delta t} = \frac{h_2 - h_1}{t_2 - t_1}$
0 to 45					$\frac{\Delta h}{\Delta t} = \frac{\text{---} - \text{---}}{\text{---} - \text{---}} = \text{---} \frac{ft}{min}$
45 to 75					
75 to 135					
135 to 255					

Q14. Between what two times in the filling process is the height of the fuel in the LOX tank changing the fastest? The slowest?

Q.15 In each part of the graph, is the rate of change of height over time for the LH<sub>2</sub> tank increasing, decreasing, or remaining constant?



- Q16. The fill rate,  $\frac{\Delta V}{\Delta t}$ , for each portion of the fueling process is given in the table below. Determine the value of the ratio of fill rate to the rate of change of height for each portion to the nearest  $ft^2$ .

**LOX:**

Time period	$\left(\frac{\Delta V}{\Delta t}\right) \frac{ft^3}{min}$	Value of the ratio $\left(\frac{\Delta V}{\Delta t}\right) : \left(\frac{\Delta h}{\Delta t}\right)$
0 to 45 minutes	35.400	$\frac{\frac{\Delta V}{\Delta t}}{\frac{\Delta h}{\Delta t}} = \frac{35.400 \frac{ft^3}{min}}{\frac{ft}{min}} = \text{_____ } ft^2$
45 to 75 minutes	42.468	
75 to 135 minutes	141.600	
135 to 255 minutes	63.732	

**LH<sub>2</sub>:**

Time period	$\left(\frac{\Delta V}{\Delta t}\right) \frac{ft^3}{min}$	Value of the ratio $\left(\frac{\Delta V}{\Delta t}\right) : \left(\frac{\Delta h}{\Delta t}\right)$
0 to 45 minutes	7.662	$\frac{\frac{\Delta V}{\Delta t}}{\frac{\Delta h}{\Delta t}} = \frac{7.662 \frac{ft^3}{min}}{\frac{ft}{min}} = \text{_____ } ft^2$
45 to 75 minutes	63.672	
75 to 135 minutes	849.003	
135 to 255 minutes	106.140	



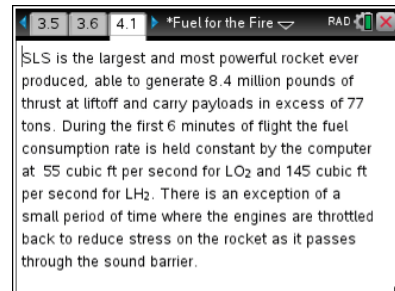
Q17. Does the value of the ratio of fill rate to rate of change of height for both tanks change or remain approximately the same over the entire filling process?

Q18. How is your response to question 17 related to your response in question 8? Is there a feature of the tanks that is related to the value of the ratio? Explain why this makes sense based on the formula for the volume of a cylinder.

Q19. Using your answer to question 18, write an equation in the form  $y = ax$  relating the rate of change of volume of fuel in the tank,  $\frac{\Delta V}{\Delta t}$ , in terms of the rate of change in height in the tank,  $\frac{\Delta h}{\Delta t}$ .

### Read pages 4.1 – 4.3

8. The first official mission for the SLS, EM-1, is an unmanned mission to orbit the moon and return safely to earth. Suppose you are on a flight controller preparing for this mission.



### Move to page 4.4.

9. Use the information you know from filling the tanks to determine a function for computing fuel consumption as a function of rate of change of height in the tank. Complete the function in the form on page 4.4 and start the animation.

