



# Why t?

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# Moderator Bio



## Chris True

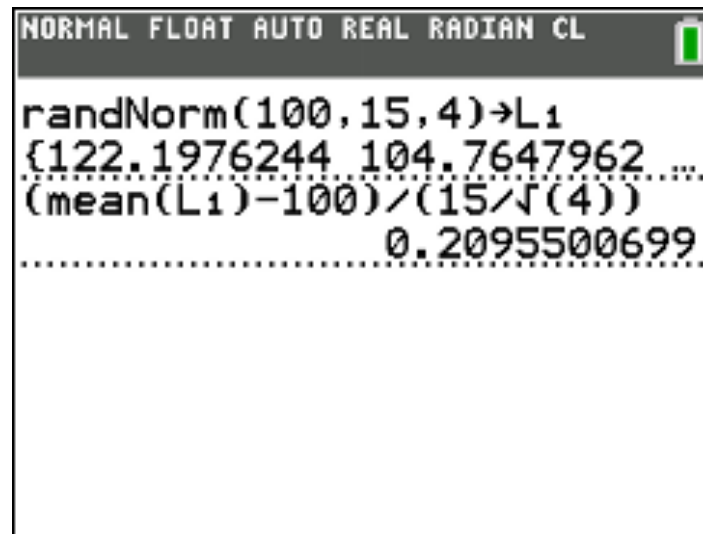
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Chris has taught AP Statistics and AP Calculus for 25 years. He is a Regional Instructor for Texas Instruments, has served as a table leader and member of several rubric teams at the AP Statistics reading, is a consultant for the College Board. He currently teaches multi-variable Calculus at the University of Nebraska-Lincoln. He continually seeks ways to integrate handheld technology into his classrooms.

## Why $t$ ?

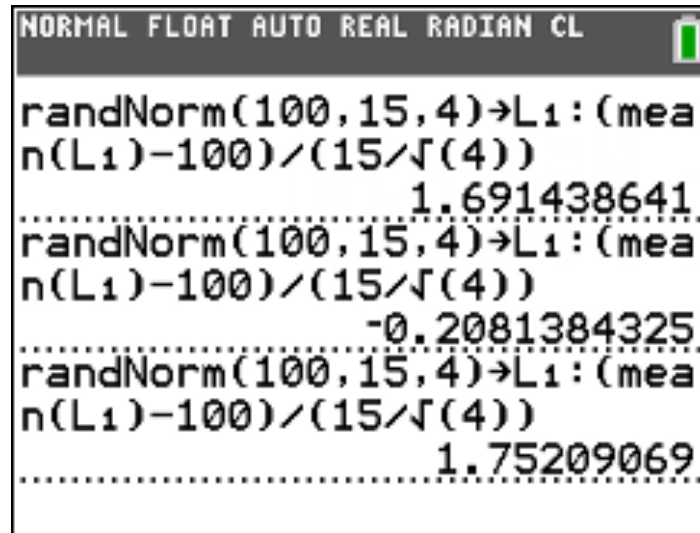
Suppose we sample IQ scores from a population that is known to be approximately normal with  $\mu = 100$  and  $\sigma = 15$ . According to the Central Limit Theorem, the sampling distribution of  $\bar{x}$  is approximately normal with  $\mu_{\bar{x}} = 100$  and  $\sigma_{\bar{x}} = \frac{15}{\sqrt{n}}$ .

We begin by taking samples of size 4, calculating  $\bar{x}$ , and investigating the behavior of the distribution of the standardized z-score =  $\frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ .

A TI-84 Plus calculator screen showing a normal distribution simulation. The top status bar reads "NORMAL FLOAT AUTO REAL RADIAN CL" with a battery icon on the right. The main display shows the command "randNorm(100,15,4)→L1" followed by a list of four values: {122.1976244, 104.7647962, ..., ...}. Below this, the formula "(mean(L1)-100)/(15/√(4))" is entered, and the result "0.2095500699" is displayed at the bottom.

```
NORMAL FLOAT AUTO REAL RADIAN CL
randNorm(100,15,4)→L1
{122.1976244, 104.7647962, ..., ...}
(mean(L1)-100)/(15/√(4))
0.2095500699
```

To investigate the behavior of the test statistic, we want to repeat these steps many times.

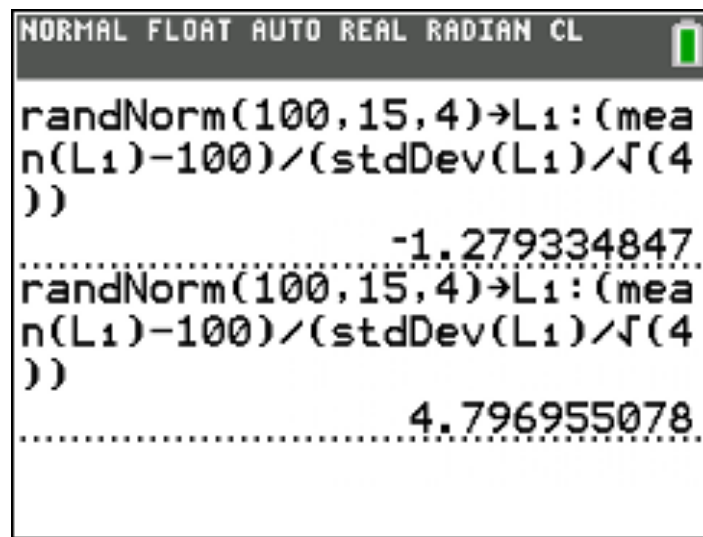


A TI-84 Plus calculator screen showing three iterations of a normal distribution simulation. The screen displays the command `randNorm(100,15,4)→L1:(mean(L1)-100)/(15/√(4))` followed by the resulting z-score for each iteration. The results are 1.691438641, -0.2081384325, and 1.75209069.

Iteration	Command	Result
1	<code>randNorm(100,15,4)→L1:(mean(L1)-100)/(15/√(4))</code>	1.691438641
2	<code>randNorm(100,15,4)→L1:(mean(L1)-100)/(15/√(4))</code>	-0.2081384325
3	<code>randNorm(100,15,4)→L1:(mean(L1)-100)/(15/√(4))</code>	1.75209069

We should not be surprised to see that these values tend to land in the range from -3.00 to 3.00 since they are standard z-scores.

Now, suppose that  $\sigma$  is unknown (which is generally the case). It would be reasonable for us to replace  $\sigma$  with our best estimate (namely, the standard deviation of the sample  $s$ ). Let's consider the consequences of such a substitution. We can go back to our simulation on the TI-84 and replace  $\sigma = 15$  with the standard deviation of the sample and repeat several times to explore the distribution of this test statistic...

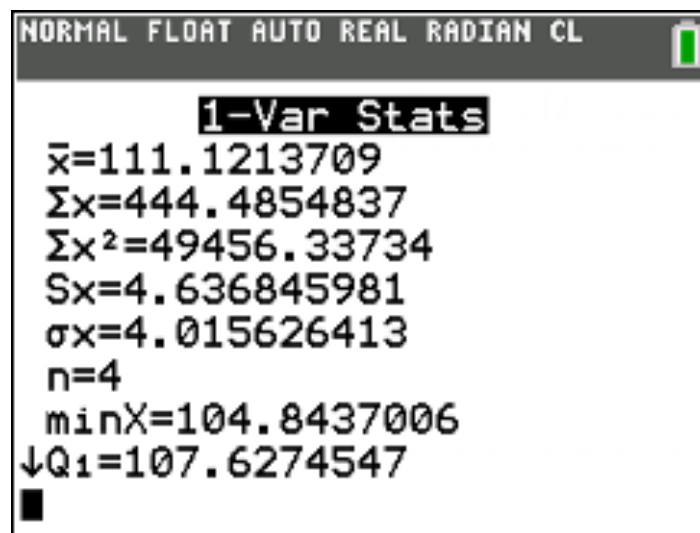


```
NORMAL FLOAT AUTO REAL Radian CL
randNorm(100,15,4)→L1:(mea
n(L1)-100)/(stdDev(L1)/√(4
))
-1.279334847
randNorm(100,15,4)→L1:(mea
n(L1)-100)/(stdDev(L1)/√(4
))
4.796955078
```

Whoa! How did we get a test statistic that was so large?? These kids must be geniuses! Let's see just how high their IQ's must be to get a test statistic of 4.797...



Recall that we are substituting  $s$  for  $\sigma$ . For this sample, the standard deviation appears to be much smaller than 15. In fact,



The mean for this sample is 111.12, which is larger than  $\mu = 100$ . If we had known that  $\sigma = 15$ , we would have obtained a test statistic equal to  $\frac{111.12 - 100}{15/\sqrt{4}} = 1.48$  (not an unusual  $z$  score).

When we substitute  $s = 4.637$  for  $\sigma$ , we get a very different test statistic.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

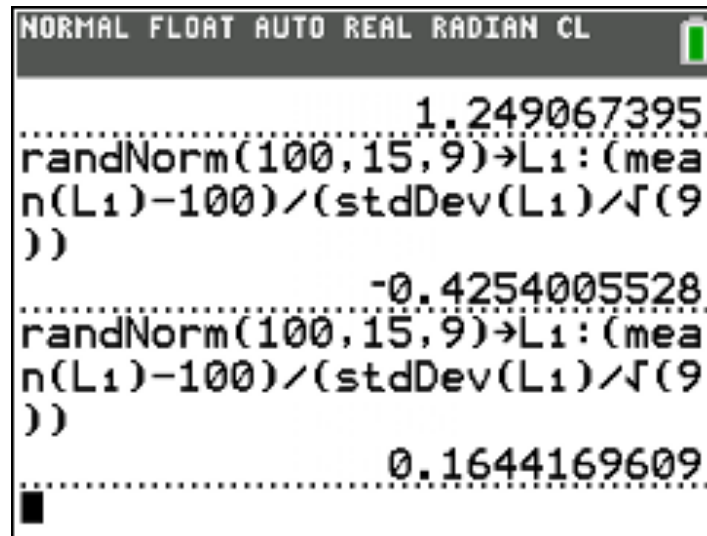
William Gosset discovered the  $t$ -distribution while serving as a chemist and statistician for the Guinness Brewing Company in 1908.





Further investigation...

Suppose the sample size  $n$  is increased from 4 to 9? What happens to the  $t$ -distribution? Let's investigate... Simply change the 4's to 9's...



```
NORMAL FLOAT AUTO REAL RADIAN CL
.....1.249067395
randNorm(100,15,9)→L1:(mea
n(L1)-100)/(stdDev(L1)/√(9
))
.....-0.4254005528
randNorm(100,15,9)→L1:(mea
n(L1)-100)/(stdDev(L1)/√(9
))
.....0.1644169609
█
```

Hitting [ENTER] several times reveals that the distribution seems to “settle” more in the range from -3.00 to 3.00. It seems to follow more of a normal distribution. Why would this be?

Looking at our last sample taken...

NORMAL FLOAT AUTO REAL RADIAN CL					
L1	L2	L3	L4	L5	0
101.01	-----	-----	-----	-----	
95.051					
125.74					
111.19					
115.54					
89.115					
69.851					
102.46					
98.006					
-----					
L3(1)=					

NORMAL FLOAT AUTO REAL RADIAN CL					
<b>1-Var Stats</b>					
$\bar{x}=100.8847897$					
$\Sigma x=907.9631076$					
$\Sigma x^2=93684.73131$					
$Sx=16.14413249$					
$\sigma x=15.22083408$					
$n=9$					
$\min X=69.85093287$					
$\downarrow Q_1=92.08321365$					

Note that as  $n$  increases,  $s$  will become less variable and the  $t$ -distribution will begin to resemble the standard  $z$ -distribution. The  $t$ -distribution is actually a family of curves each with a different degree of freedom, where  $df = n - 1$ .

Thank you!

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