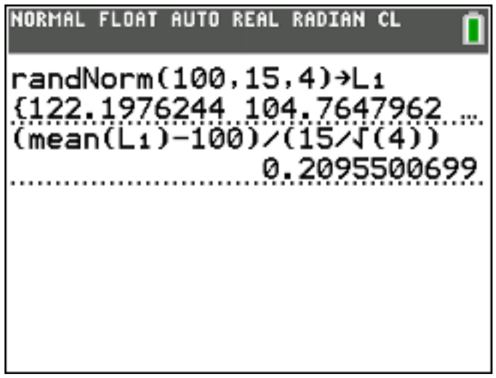
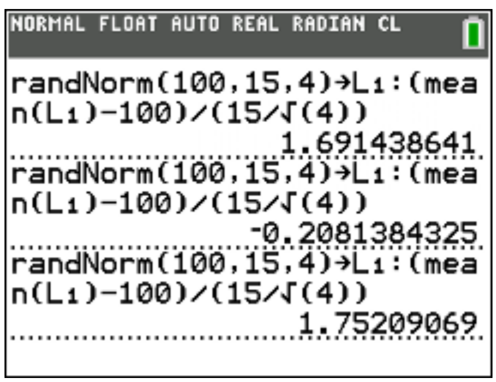
Why *t*?

Suppose we sample IQ scores from a population that is known to be approximately normal with $\mu = 100$ and $\sigma = 15$. According to the Central Limit Theorem, the sampling distribution of $\bar{x}$ is approximately normal with $\mu_{\bar{x}} = 100$ and $\sigma_{\bar{x}} = \frac{15}{\sqrt{n}}$.

We begin by taking samples of size 4, calculating $\bar{x}$, and investigating the behavior of the distribution of the standardized z-score = $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$.

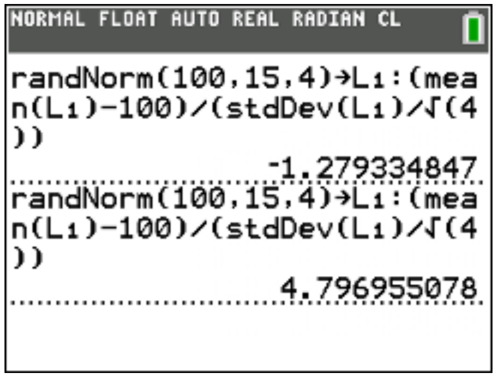


To investigate the behavior of the test statistic, we want to repeat these steps many times.

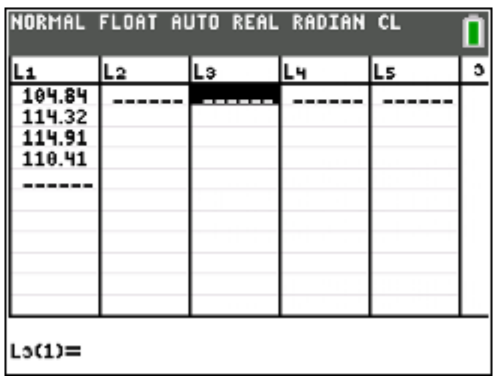


We should not be surprised to see that these values tend to land in the range from -3.00 to 3.00 since they are standard *z*-scores.

Now, suppose that σ is unknown (which is generally the case). It would be reasonable for us to replace σ with our best estimate (namely, the standard deviation of the sample s). Let's consider the consequences of such a substitution. We can go back to our simulation on the TI-84 and replace $\sigma = 15$ with the standard deviation of the sample and repeat several times to explore the distribution of this test statistic…

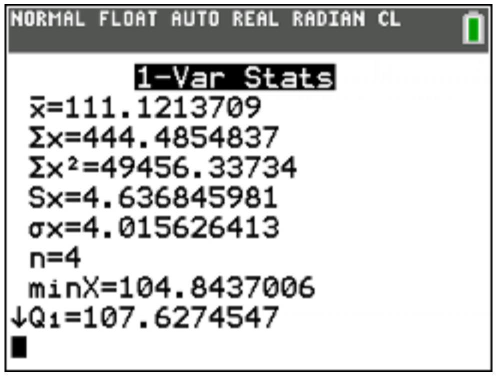


Whoa! How did we get a test statistic that was so large?? These kids must be geniuses! Let's see just how high their IQ's must be to get a test statistic of 4.797…



Wait… Recall that $\mu = 100$ and $\sigma = 15$ for this population. Not one of these kids had an IQ that was more than one standard deviation above the mean. What gives?

Recall that we are substituting s for σ. For this sample, the standard deviation appears to be much smaller than 15. In fact,



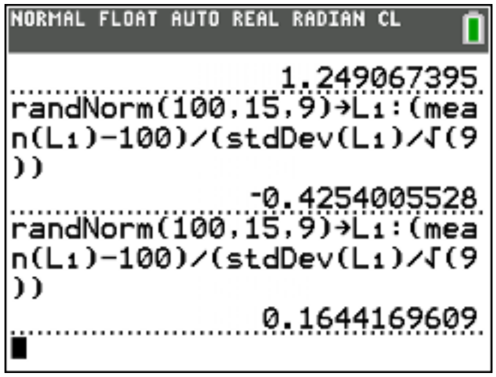
The mean for this sample is 111.12, which is larger than $\mu = 100$. If we had known that $\sigma = 15$, we would have obtained a test statistic equal to $\frac{111.12-100}{15/\sqrt{4}} = 1.48$ (not an unusual z score). When we substitute s = 4.637 for σ, we get a very different test statistic.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

William Gosset discovered the t-distribution while serving as a chemist and statistician for Guinness Brewing Company in 1908.

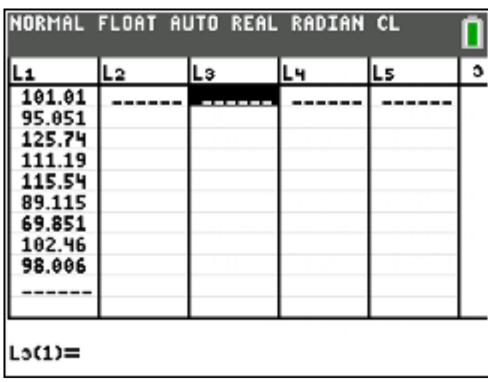
Further investigation…

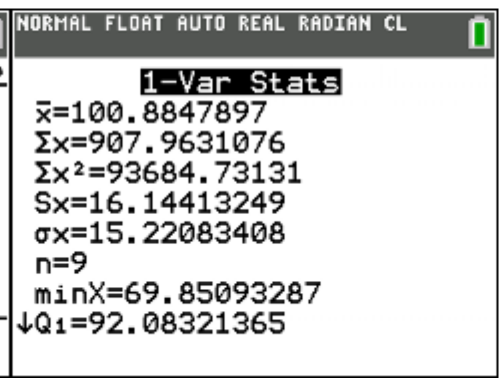
Suppose the sample size *n* is increased from 4 to 9? What happens to the *t*-distribution? Let's investigate… Simply change the 4's to 9's…



Hitting [ENTER] several times reveals that the distribution seems to "settle" more in the range from -3.00 to 3.00. It seems to follow more of a normal distribution. Why would this be?

Looking at our last sample taken…





Note that as *n* increases, *s* will become less variable and the *t*-distribution will begin to resemble the standard *z*-distribution. The *t*-distribution is actually a family of curves each with a different degree of freedom, where $df = n - 1$.