Zeros and Factoring

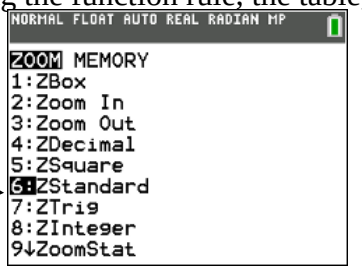
Finding Zeros Graphically, Numerically, and Symbolically

1. The values of x for which $y = 0$ are called the *zeros* of the function.

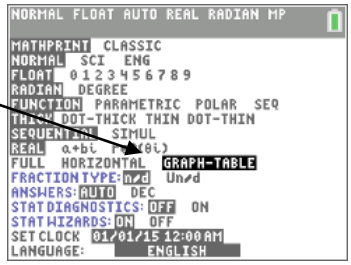
   Identify and report the zeros of the functions below by examining the function rule, the table, and the graph.

   For example, to explore the zeros of $y = (x-3)(x+1)$:

   a. Use the standard viewing window.
      Press ZOOM 6:Standard.



   b. Press MODE and change to Graph-Table Mode.



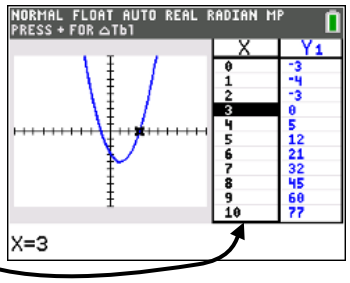
   c. Press 2nd TBLSET.
      Make sure your table starts at 0 with steps of $\Delta\text{Tbl} = 1$

   d. Type the equation in the Y= Editor.

   e. Press 2nd TABLE. Use the up and down arrow keys to move your cursor to any zeros.

      You know your cursor is in the correct place if the pane bordering your table border is dark.

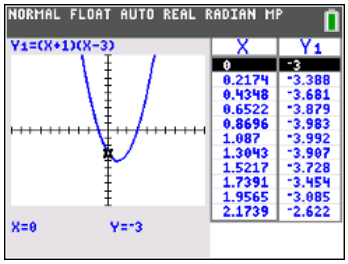
      Yes!



      Troubleshooting:
      If you do not see the cursor on the graph, keep pressing the down arrow.

      If you get noninteger numbers, you might be on the graph screen. You can tell this is the case if the equation is showing at the top of the graph. Upon pressing TRACE, the table follows the noninteger graphing coordinates instead of obeying TblStart=0 and $\Delta\text{Tbl} = 1$.
      To fix it, press 2nd TABLE and press up and down arrows.

      No!



   Find and report any zeros by examining the equation. Check with tables and graphs.

   $y = (x-3)(x+1)$ _____ $y = \frac{1}{10}(2x-2)(x+3)(x-1)$ _____

   $y = (x+2)(x+2)$ _____ $y = (x+1)(x-2)(x^2+1)$ _____

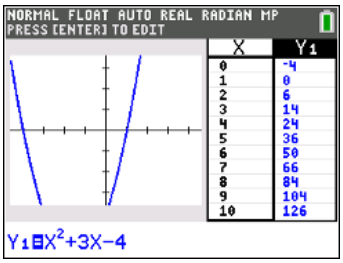
   $y = x(x+4)$ _____ $y = (x^2-4)(x-1)$ _____

   Comment on the statement: *"The number of zeros is equal to the number of factors."*

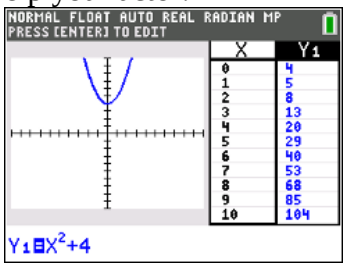
2. Multiply:
$(x-3)(x+1) =$ _____ $(x+2)(x+2) =$ _____ $x(x+4) =$ _____

3. Complete:
x^2-2x-3 has factors of _____ and zeros of _____
x^2+4x+4 has factors of _____ and zeros of _____
x^2+4x has factors of _____ and zeros of _____

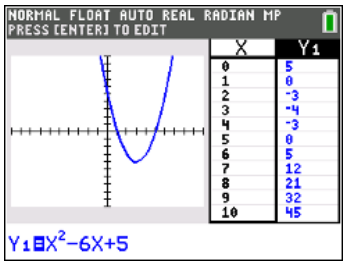
4. Factor, if possible. If it does not factor, state so. Use the screens below to help you factor.
x^2+3x-4 _____



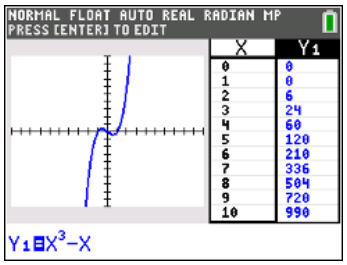
x^2+4 _____



x^2-6x+5 _____



x^3-x _____



5. From your work above, answer the following:
   a. If the zeros of a function are 4 and 8, what are the possible factors of the function?

   b. If the zeros of a function are −2 and −7, what are the possible factors of the function?

   c. If the zeros of a function are $-p$ and q, what are the possible factors of the function?

6. Find all the zeros of each function by examining its equation.
Write the zeros as reduced fractions (and not as decimals).
$y=(5x-2)(x-2)$ _____ $y=(3x-4)(x^2+x)$ _____
$y=(2x+3)(4x-5)$ _____ $y=(ax+b)(cx-d)$ _____

7. Multiply:
$(5x-2)(x-2)$ _____
$(2x+3)(4x-5)$ _____
$(3x-4)(x^2+x)$ _____

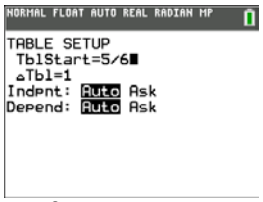
8. Complete:
$5x^2-12x+4$ has factors of _____ and zeros of _____
$8x^2+2x-15$ has factors of _____ and zeros of _____
$3x^3-x^2-4x$ has factors of _____ and zeros of _____

9. From your work above, answer the following:
   a. If the zeros of a function are 3/4 and −1/2, what are the possible factors of the function?

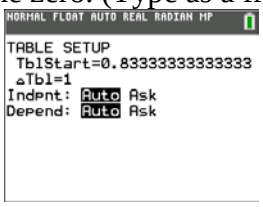
   b. If the zeros of a function are $-a/b$ and $-c/d$, what are the possible factors of the function?

10. There are several ways to *check* noninteger zeros with tables and graphs after you have found them analytically. For example, to check $y = 6x^2 + x - 5$ has zeros 5/6 and 1:

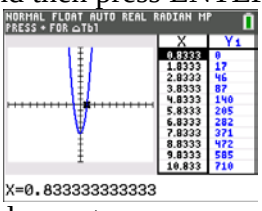
Method 1: In the Y= menu enter `Y1=6X²+X−5`. In Graph-Table mode, press 2nd TBLSET.
Set TblStart to be the value of the zero. (Type as a fraction and then press ENTER).



Press ENTER

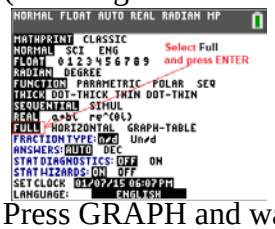


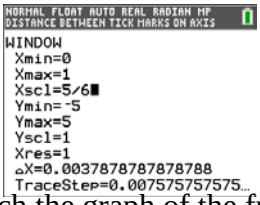
Press 2nd TABLE



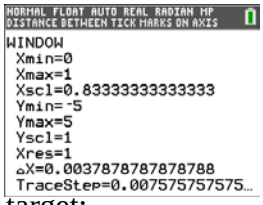
For functions with more than one zero, return to TBLSET and repeat.

Method 2: If the zero is between two integers, change the window settings so that Xmin and Xmax are the two integers on each side of the zero. Change Xscl to be the value of the zero.
(This might be easier to see in full screen mode)

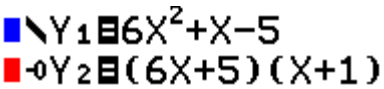


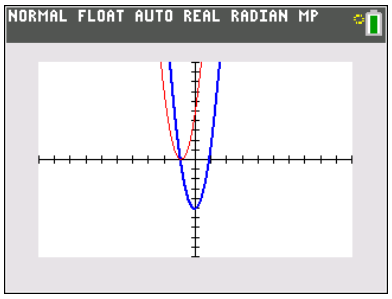


Press ENTER

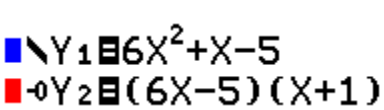


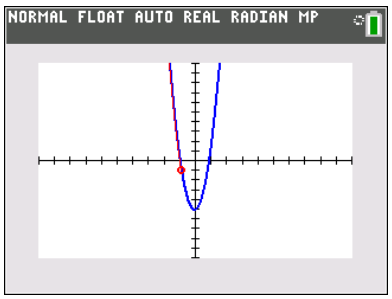
Press GRAPH and watch the graph of the function hit its target:
Type both the original form of the function and the factored form into Y=.
In a standard window Use the TRACER ball for the factored form to check your graphs match.





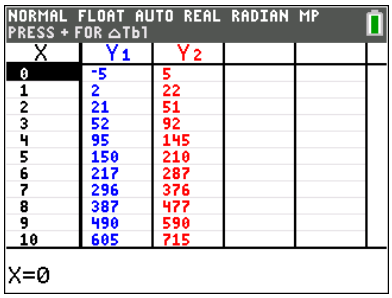
Students can see when they are wrong…





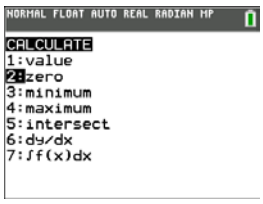
… and when they are correct.

Method 3: Compare tables.
(Again this would be easier to see in full screen mode.)
$y = 6x^2 + x - 5$ has the same table as $y = (6x-5)(x+1)$:

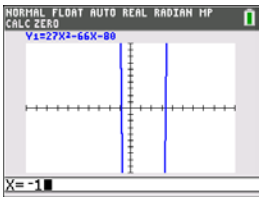


11. To factor $y = 27x^2 - 66x - 80$ with technology, use full screen mode and graph in a standard viewing window. The negative zero is between –1 and 0.

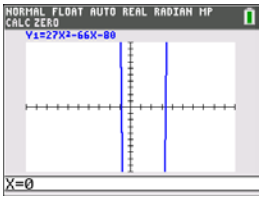
Press 2nd CALC. Press 2:zero.



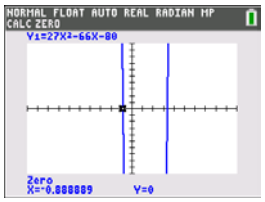
For the left bound, type –1.



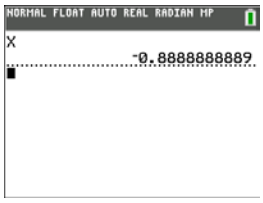
Press ENTER. For the right bound, type 0.



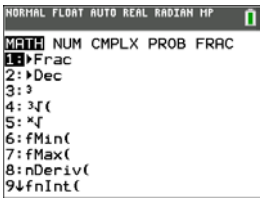
Press ENTER. For the guess, type any number between –1 and 0. Press ENTER.

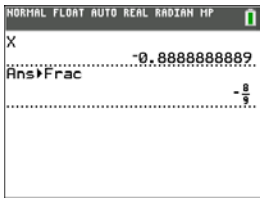


Press 2nd QUIT. The full value of the zero is stored in the graphing variable x.
On the home screen, press X ENTER.



You can convert this to a reduced fraction.
Press MATH 1:Frac ENTER





Knowing one zero is –8/9, we can find that one factor of $27x^2 - 66x - 80$ is (9*x*+8).
From that, we can find the other factor using analytic reasoning.
To complete the other factor for $27x^2 - 66x - 80 = (9x+8)(\quad ? \quad)$,
observe the coefficients 27 and –80. Using analytical reasoning:

a. Explain why we know that $27x^2 - 66x - 80 = (9x+8)(3x \qquad)$

b. Explain why we know that $27x^2 - 66x - 80 = (9x+8)(3x - \quad)$

c. Explain why we know that $27x^2 - 66x - 80 = (9x+8)(3x - 10)$

12. Factor, if possible.

a. $6x^2 - 3x - 3$ _____

b. $10x^2 - 73x - 15$ _____

c. $6x^2 - x - 12$ _____

d. $13x^2 - 45x - 28$ _____

e. $8x^2 + 9x + 4$ _____

f. $x^4 - 3x^3 - 4x^2 + 12x$ _____

g. $x^3 - 3x^2 + 2x - 6$ _____

h. $x^3 + 2x^2 - x - 2$ _____