Results from the 2012 AP Calculus AB and BC Exams

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March, 2013

AP Calculus

Outline

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2. The Reading
   (a) Leadership Structure
   (b) The Flow and Question Teams
   (c) Logistics and Numbers
3. Three Free Response Questions
   (a) The Standard
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   (c) The good, bad, and some suggestions

Exams

AP Calculus Exams

- US Main: United States, Canada, Puerto Rico, US Virgin Islands
- Form A: US Alternate Exam: late test
- Form I: International Main Exam
- Form J: International Alternate Exam

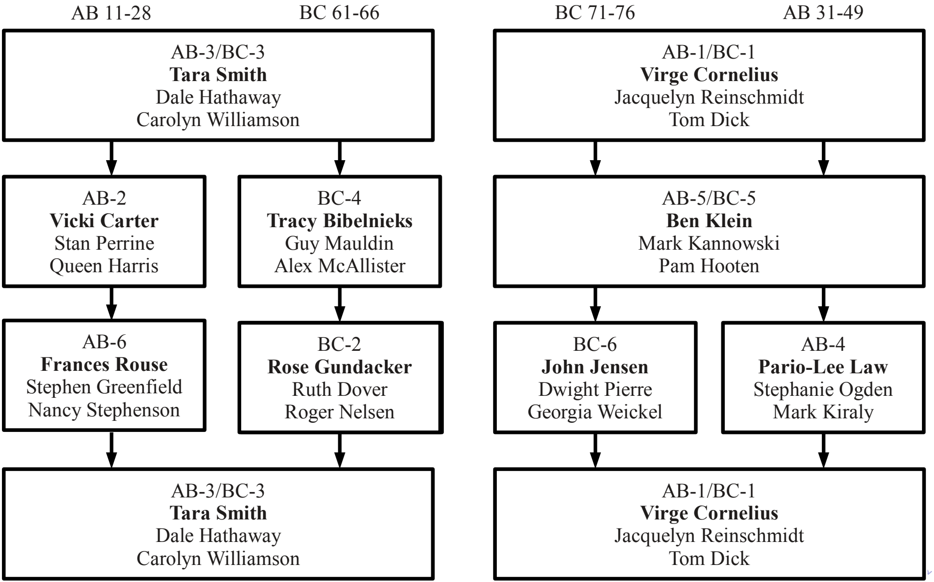
Parts

- Section I: Multiple Choice. Section II: Free Response.
- Calculator and Non-Calculator Sections
- AB and BC Exams.

The Reading Leadership Structure

- Chief Reader (CR)
- Chief Reader Associate (CRA)
- Assistant Chief Reader (ACR)
- Chief Aides (CA)
- Exam Leaders (EL) $(2 \to 5)$
- Question Leaders (QL) $(9 \to 20)$
- Question Team Members (QTM)
- Table Leaders (TL)
- Readers

The Reading: 2012 Grading Flow



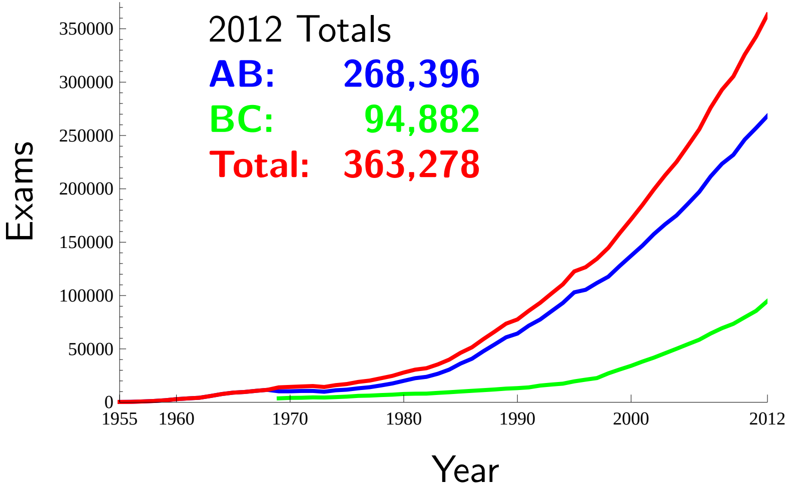
The Reading

Logistics and Participants

- Kansas City:
  Convention Center (versus college campus)
  Westin Hotel (versus college dorms)
- Total Participants: 853
- High School: 55%   College: 45%
- 50 states, DC, and other countries

The Reading

Number of (all) AP Calculus Exams



The Reading

2012 Scores (US Main Exam)

US Main			
Score	AB	BC	AB subscore
5	24.9%	50.6%	60.3%
4	17.0%	16.2%	16.7%
3	17.4%	16.2%	9.0%
2	10.3%	5.4%	5.8%
1	30.5%	11.8%	8.2%

The Reading

General Comments

- We awarded points for good calculus work, if the student conveyed an understanding of the appropriate calculus concept.
- Students must show their work (bald answers).
- Students must communicate effectively, explain their reasoning, and present results in clear, concise, proper mathematical notation.
- Practice in justifying conclusions using calculus arguments.
- Decimal presentation errors, use of intermediate values.

2012 Free Response

General Information

- Six questions on each exam (AB, BC).
- Three common questions: AB-1/BC-1, AB-3/BC-3, AB-5/BC-5.
- Scoring: 9 points for each question.
- Complete and correct answers earn all 9 points.
- The scoring standard is used to assign partial credit.

2012 Free Response Statistics

Question	Mean	St Dev	% 9s	% 0s
AB-1	3.96	2.88	5.6	16.6
BC-1	5.88	2.55	14.7	4.1
AB-2	3.09	3.10	6.9	41.0
AB-3	2.67	2.58	1.3	30.9
BC-3	4.29	2.61	3.8	11.3
AB-4	4.09	2.61	2.8	14.3
AB-5	2.87	2.24	1.4	16.3
BC-5	4.75	2.55	7.1	5.5

2012 Free Response Statistics

Question	Mean	St Dev	% 9s	% 0s
AB-6	3.59	2.82	5.5	19.1
BC-2	5.07	2.66	10.7	6.0
BC-4	5.43	2.84	18.0	7.6
BC-6	4.23	2.70	5.3	11.5

2012 Free Response: AB-1/BC-1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

2012 Free Response: AB-1/BC-1

(a) $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \dfrac{67.9 - 61.8}{6}$

$= 1.017$ (or 1.016)

The water temperature is increasing at a rate of approximately 1.017 °F per minute at time $t = 12$ minutes.

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by 16 °F over the interval from $t = 0$ to $t = 20$ minutes.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{interpretation with units} \end{cases}$

(c) $\dfrac{1}{20}\int_0^{20} W(t)\,dt \approx \dfrac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$

$= \dfrac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$

$= \dfrac{1}{20} \cdot 1215.8 = 60.79$

This approximation is an underestimate since a left Riemann sum is used and the function W is strictly increasing.

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{underestimate with reason} \end{cases}$

(d) $W(25) = 71.0 + \int_{20}^{25} W'(t)\,dt$

$= 71.0 + 2.043155 = 73.043$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2012 Free Response: AB-1/BC-1

Results

- In general, students performed well.
  Multiple entry points. No surprises.

- Part (a): Most students set up a difference quotient.
  Interpretation tougher. Needed correct units.
  Average rate of change (your answer).

- Part (b): Good job recognizing the FTC.
  Interpretation: change, units, and interval.

2012 Free Response: AB-1/BC-1

Results

- Part (c): Left Riemann sum and computation good. Explanation: inadequate or incorrect reasons.
- Part (d): Students did fairly well.

2012 Free Response: AB-1/BC-1

Common Errors

- Interpretation of the answer in the context of the problem.
- Correct units.
- Part (b): The meaning of the definite integral.
- Part (c): Incorrectly assumed the width of each subinterval was the same.
  Explanation associated with an underestimate.
- Part (d): Use of 0 as a lower bound on the definite integral.

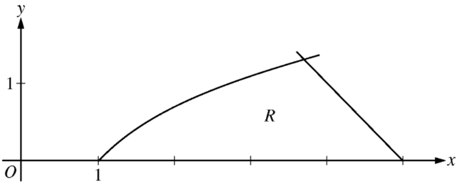
2012 Free Response: AB-1/BC-1

To Help Students Improve Performance

- In general, most students were able to apply appropriate concepts and compute correct numerical answers.
- Interpretation and communication of results.
- Clearly indicate the mathematical steps to a final solution.

2012 Free Response: AB-2

Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

2012 Free Response: AB-2

$\ln x = 5 - x \quad \Rightarrow \quad x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a) $\text{Area} = \int_0^B \left(5 - y - e^y\right)\,dy$

$= 2.986 \;(\text{or } 2.985)$

OR

$\text{Area} = \int_1^A \ln x\,dx + \int_A^5 (5 - x)\,dx$

$= 2.986 \;(\text{or } 2.985)$

3 :
- 1 : integrand
- 1 : limits
- 1 : answer

(b) $\text{Volume} = \int_1^A (\ln x)^2\,dx + \int_A^5 (5 - x)^2\,dx$

3 :
- 2 : integrands
- 1 : expression for total volume

(c) $\int_0^k \left(5 - y - e^y\right)\,dy = \frac{1}{2} \cdot 2.986 \left(\text{or } \frac{1}{2} \cdot 2.985\right)$

3 :
- 1 : integrand
- 1 : limits
- 1 : equation

2012 Free Response: AB-2

Results

- Area / volume problem with two regions: difficult for students.
- Working in x: OK in parts (a) and (b).
- Part (c) was very challenging for students working with respect to x.
- Part (a): Common solution in terms of x, 2 regions. For those in y: OK if correctly found $x = e^y$.

2012 Free Response: AB-2

Results

- Part (b): Students did well.
  Two distinct, separate integrals to find total volume.
- Part (c): Those working in terms of y more successful.
  Some complicated yet correct solutions in terms of x.

2012 Free Response: AB-2

Common Errors

- No calculator use to find (A, B).
  Point of intersection reported as $(4, 1)$.
- Solving for x in terms of y (inverse functions).
- Part (a): $\displaystyle\int_1^5 (5 - x - \ln x)\, dx$
- Incorrect limits:
  0 as a lower bound, 4 as an upper bound.
- Part (b): $\displaystyle\int_1^5 (5 - x - \ln x)^2\, dx$ (constant π)
- Part(c): Equation in terms of x.

2012 Free Response: AB-2

To Help Students Improve Performance

- Practice in solving for x in terms of y (inverse functions).
- Computing areas in which there are distinct right and left boundaries.
- Using the best, most efficient method for finding the area of a region.
- Computing the volume of a solid with known cross sectional area.
- Computing the area of certain known geometric regions at an arbitrary point x.

2012 Free Response: BC-2

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

(a) $\left.\frac{dx}{dt}\right|_{t=2} = \frac{2}{e^2}$

Since $\left.\frac{dx}{dt}\right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$$\left.\frac{dy}{dx}\right|_{t=2} = \frac{\left.dy/dt\right|_{t=2}}{\left.dx/dt\right|_{t=2}} = 3.055 \text{ (or } 3.054)$$

$$3: \begin{cases} 1: \text{moving to the right with reason} \\ 1: \text{considers } \frac{dy/dt}{dx/dt} \\ 1: \text{slope at } t = 2 \end{cases}$$

2012 Free Response: BC-2

(b) $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt = 1.253 \text{ (or } 1.252)$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575 \text{ (or } 0.574)$

Acceleration $= \langle x''(4), y''(4) \rangle$
$= \langle -0.041, 0.989 \rangle$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

(d) Distance $= \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$
$= 0.651 \text{ (or } 0.650)$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2012 Free Response: BC-2

Results

- Student performance generally very good. Understanding of parametrically defined curve.
- Arithmetic, algebra, decimal presentation errors.
- Correct presentations followed by incorrect answers (from calculator).
- Inappropriate use of initial condition.

2012 Free Response: BC-2

Common Errors

- Part (a): Horizontal movement using $\dfrac{dy}{dx}$ or $x(2)$.

  Arithmetic, algebra errors in computing the slope.

- Part (b): Initial condition not used.

  Symbolic antiderivative of $\dfrac{dx}{dt}$

  (abandoned, restart with calculator)

2012 Free Response: BC-2

Common Errors

- Part (c): Incorrect formula for speed.

  Derivatives of $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ analytically.

  OK, but quotient rule or algebra errors.

- Part (d): Use of $\left|\dfrac{dy}{dx}\right|$ as the integrand in computing distance.

  Use of the formula for arc length assuming $y = f(x)$.

2012 Free Response: BC-2

To Help Students Improve Performance

- Decimal presentation instructions and intermediate values.
- More practice with the use of the Fundamental Theorem of Calculus.

  $$x(b) = x(a) + \int_a^b x'(t)\,dt$$

- Concepts of speed and total distance traveled.

AP Calculus Reading

Other Information

- **AP Teacher Community**:
  A new online collaboration space and professional learning network for AP Educators.
  Discussion boards, resource library,
  member directory, email digests, notifications, etc.
  Each community is moderated.
- **2013 Reading**:
  June 11-17, Kansas City, Session II