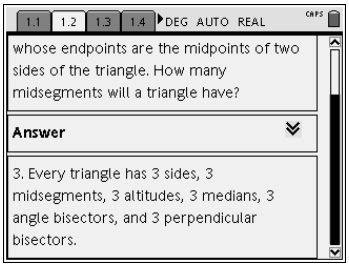
Exploring Midsegments of a Triangle TEACHING NOTES

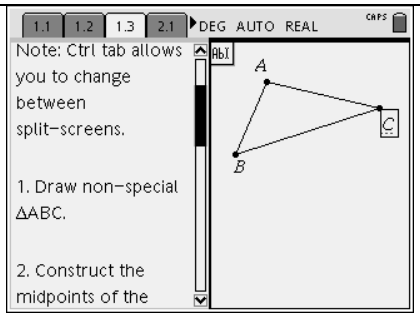
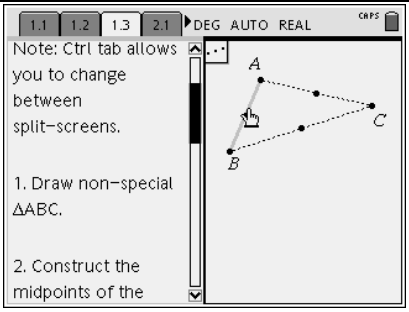
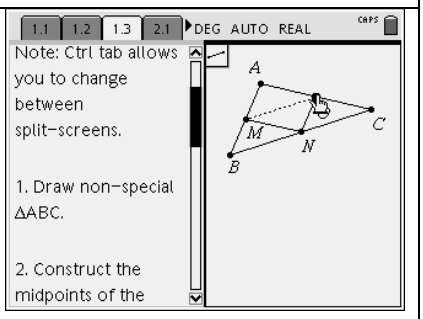
Activity Overview: Students will discover the relationships between a midsegment of a triangle and its third side.
NCTM Content Standard: Geometry, Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
NCTM Process Standards: Reasoning & Proof, Make and investigate mathematical conjectures; Communication, Organize and consolidate their mathematical thinking through communication; Representation, Create and use representations to organize, record, and communicate mathematical ideas.

Supplies & Materials: Copies of student handout, TI-Nspire handhelds with the file Midsegments Student.tns loaded
Activity Time: 1-1.5 hours. Explorations 1-4 will not take very long, but 6, in particular, will take more time to explore in depth. It allows the opportunity for making several connections between different chapters in geometry.

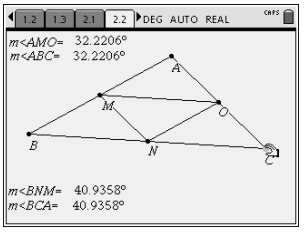
Prerequisite Knowledge: Students should be familiar with the angle relationships formed when parallel lines are cut by a transversal. Students should also be familiar with opening and turning pages in a TI-Nspire document (/¢). Students who are not familiar with constructing and manipulating figures in a Graphs & Geometry page of TI-Nspire will need more teacher-led instruction than those who are.

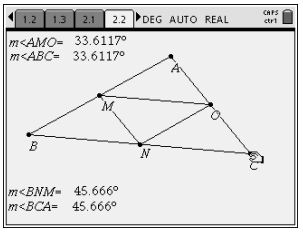



Exploration 1: Students are given the definition of a midsegment and asked to determine how many midsegments a triangle will have. A sample answer is shown. On the left screen of page 1.3, students are given directions for constructing the midsegments of a triangle. Students will construct the triangle on the right screen of page 1.3. /e allows them to change between the split-screens.

| | | |
|---|---|---|
| 1. Draw non-special ΔABC. | b 8: Shapes; 2: Triangle.<br>Press enter at each vertex of the triangle.<br><br>To label the vertices, b 1: Tools; 5: Text.<br>Press enter on top of the vertex and then type the label. |  |
| 2. Construct the midpoints of the three sides of the triangle so that M is the midpoint of AB, N is the midpoint of BC, and O is the midpoint of AC. | b 9: Construction; 5: Midpoint.<br>Press enter on each side of the triangle to construct the midpoints.<br>Label the vertices. |  |
| 3. Draw $\overline{MO}$, $\overline{MN}$, and $\overline{NO}$. | b 6: Points & Lines; 5: Segment.<br>Press enter on the endpoints of each segment. |  |
| 4. We call $\overline{BC}$ the "third side" for midsegment $\overline{MO}$. What is the "third side" for $\overline{MN}$? | | |
| 5. Make some conjectures about the relationship between a midsegment and its third side. | | |

Exploration 2: *In order to explore one of the properties of a midsegment, the following measurements have been calculated for ∆ABC on page 2.2: m<AMO, m<ABC, m<BNM, m<BCA. Adjust the size of the triangle by moving one of its vertices, and watch what happens to the measures of the angles. If you choose, you can also calculate the measures of <CON and <CAB.*

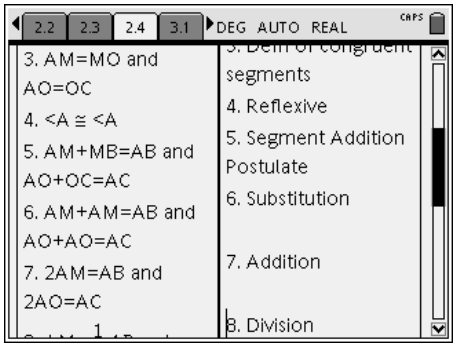


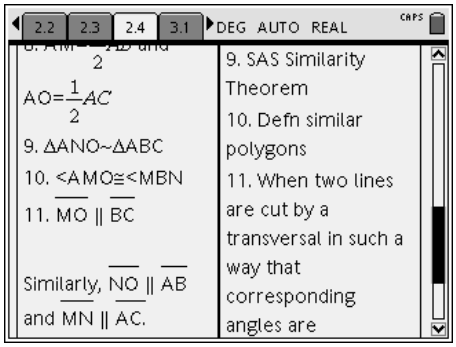



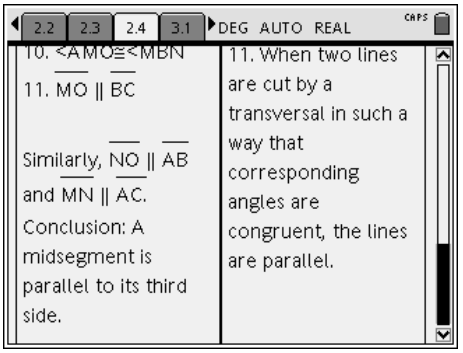

To calculate the measures of additional angles, press b 7: Measurement; 4: Angle. Then select three points on the angle, selecting the vertex second. Move the measurement to a clear part of the screen and press enter to drop it.

So that students can spend more time exploring, the measurements for the remaining explorations are set up. On page 2.2, have students grab a vertex of ∆ABC (/x) to resize the triangle. They should notice that the measures of the corresponding angles remain congruent and conclude that a midsegment will be parallel to its third side.

On page 2.4, students can move between columns in the proof by pressing e. Some of the reasons that students might not have gotten to yet in their geometry course are completed for the students.

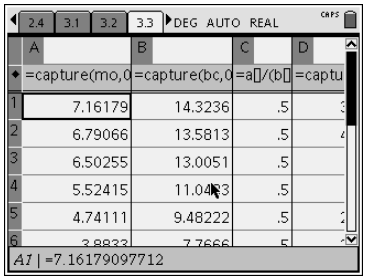



Exploration 3: *In order to explore another of the properties of a midsegment, the lengths of the sides of ∆ABC and its midsegments have been calculated on page 3.2. Adjust the size of the triangle by moving one of its vertices, and watch what happens to the lengths of the sides and midsegments of the triangles.*

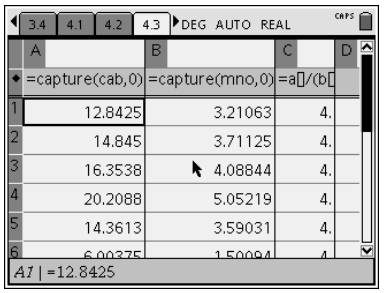
*If the relationship is not apparent to you, then capture a few measurements onto the spreadsheet on page 3.3 by pressing "ctrl .", resizing the triangle, and pressing "ctrl ." again. Capture data for several resized triangles, and then examine the ratio of the lengths of the sides in the spreadsheet.*




Some students will probably notice that the measure of the midsegment is half the length of the third side. However, those who do not can use the data capture feature of the TI-Nspire to explore the relationship in a spreadsheet. When a students presses /^ on page 3.2, the lengths will be captured into the spreadsheet on page 3.3. Have students resize the triangle several times, pressing /^ after each change. When they look at the spreadsheet on page 3.3, they can see in column C that $\frac{MO}{BC} = \frac{1}{2}$, no matter the size of the triangle. Similarly in column F, $\frac{MN}{AC} = \frac{1}{2}$.

Exploration 4: *In order to explore the area of ∆ABC and the area of the triangle formed by its midsegments, the areas of the triangles in the diagram have been calculated on page 4.2. Adjust the size of the triangle by moving one of its vertices, and watch what happens to the areas of the triangles. What can you conclude about the area of the midsegment triangle compared to the area of the triangle?*

*If the relationship is not apparent to you, then capture a few measurements onto the spreadsheet on page 4.3 by pressing "ctrl .", resizing the triangle, and pressing "ctrl ." again. Capture data for several resized triangles.*




Some students will probably notice that the area of the midsegment triangle is one-fourth the measure of the whole triangle. However, those who do not can use the data capture feature of the TI-Nspire to explore the relationship in a spreadsheet. When a students presses /^ on page 4.2, the lengths will be captured into the spreadsheet on page 4.3. Have students resize the triangle several times, pressing /^ after each change. When they look at the spreadsheet on page 4.3, they can see

in column C that $\frac{CAB}{MNO} = 4$, no matter the size of the triangle.

*Because the sides of ∆MNO are proportional to the corresponding sides of ∆CAB, we say that ∆MNO is similar to ∆CAB (∆MNO~∆CAB). You will learn more about similar triangles later in your geometry course. For now, know that if the ratio of two similar figures is a:b, then the ratio of their areas will be a²:b².*

*The ratio of the sides of ∆MNO:∆CAB is 1:2. What is the ratio of their areas?* 1²:2², or 1:4.

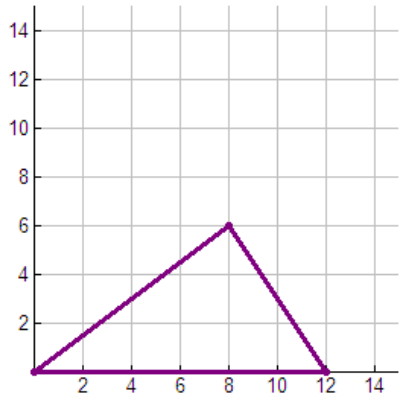
*Does this agree with the relationship you discovered between the areas of the triangles on page 4.2?* Yes, the area of ∆MNO is 1/4 the area of ∆CAB.

Extending the Exploration: You could have students calculate the areas of ∆AMO, ∆ONC, and ∆MBN.

Exploration 5: *Midsegments in the Coordinate Plane*

*Answer the questions on your handout. If necessary, use the graph on page 5.2 to help you answer the questions.*

This exploration is intended to review slope, midpoint and distance formulas. However, students can calculate the measurements on page 5.2 if they want to verify that their use of the formulas is correct.




1. Construct a triangle on the grid with vertices at F(0,0), G(12,0), and H(8,6).
2. What is the length of $\overline{FG}$ ? 12
3. Find the location of the midpoint of side $\overline{FH}$ and call it point M. What are the coordinates of M? $(4,3)$
4. Find the location of the midpoint of side $\overline{GH}$ and call it point N. What are the coordinates of N? $(10,3)$
5. Connect M and N with a line segment. What is the length of $\overline{MN}$ ? 6
6. $\overline{MN}$ is a **midsegment** of △FGH. The corresponding side of $\overline{MN}$ is $\overline{FG}$ . Does the length of the midsegment compared to its corresponding side as you conjectured in the exploration that it would in Exploration 3? Yes; 6 is one-half of 12.
7. Calculate the slope of $\overline{MN}$ . 0 Calculate the slope of $\overline{FG}$ . 0

What can you conclude about $\overline{MN}$ and $\overline{FG}$ ? They have the same slope, and so they are parallel.
Does this conclusion agree with Exploration 2? Yes. The midsegment is parallel to its third side.

8. Calculate the area of △FGH. $\frac{1}{2}(12)(6) = 36 \text{ u}^2$

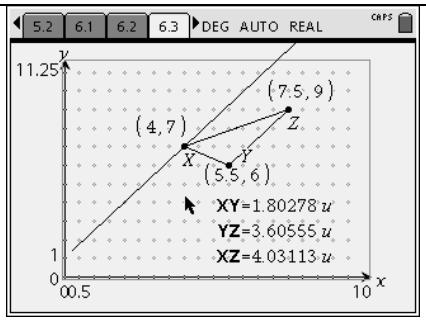
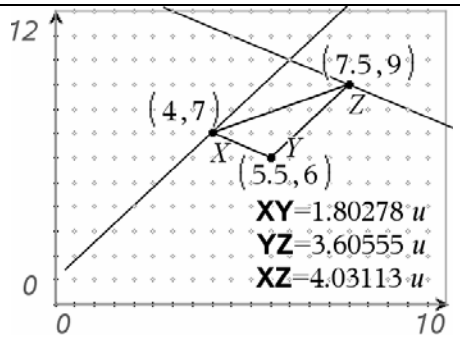
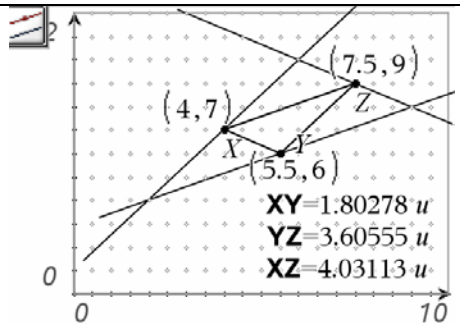
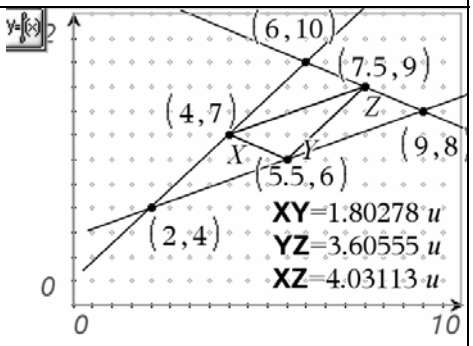
Let O be the midpoint of side $\overline{FG}$ . Draw △MNO. Calculate its area. $\frac{1}{2}(6)(3) = 9 \text{ u}^2$

What is the relationship between the area of △MNO formed by the midsegments of the triangle and △FGH? 9 is one-fourth of 36.
Do the areas agree with your conclusion in Exploration 4? Yes. The midsegment triangle has area that is one-fourth that of the whole triangle.

Exploration 6: *The vertices of the midpoints of a triangle are (4,7), (7.5,9), and (5.5,6). What are the vertices of the triangle?*

*If necessary, use the graph on page 6.2 to help you find the vertices.*

This problem is intended to challenge students. You might talk with them about what they do know and what they need to know. There are several pages in the Instructor file that shows how students can use TI-Nspire to solve the problem.

| | | |
|---|---|---|
| 1. Calculate XY, YZ, and XZ.<br><br>2. Construct a line through X parallel to $\overline{YZ}$. | 1. b 7: Measurement; 1: Length. Select the endpoints of each segment, and move the measurement to a clear part of the screen. To link the values to a variable, press x on top of the value to select it, then press h 1: Store Var and type the name of the length.<br><br>2. b 9: Construction; 2: Parallel. Select X and $\overline{YZ}$. Press ·. |  |
| 3. Construct a line through Z parallel to $\overline{XY}$. | 3. b 9: Construction; 2: Parallel. Select Z and $\overline{XY}$. Pr ess ·. |  |
| 4. To find the third vertex of the triangle, construct a line through Y parallel to $\overline{XZ}$. | 4. b 9: Construction; 2: Parallel. Select Y and $\overline{XZ}$. Pr ess ·. |  |
| 5. Draw the points of intersection for the three lines.<br><br>6. Find the coordinates of the point of intersection. | 5. b 6: Points & Lines; 3: Intersection Point(s). Select each pair of lines to get all points of intersection. Press ·.<br><br>6. b 1: Tools; 6: Coordinates and Equations. Press enter on the point of intersection. |  |

The vertices of the triangle are (2, 4), (6, 10), and (9, 8).
Extending the Exploration. Have students move one of the vertices of the triangle formed by the midsegments. Have students determine how to find the vertices of the triangle without using the TI-Nspire!

Practice: In ΔABC, M is the midpoint of $\overline{AB}$, N is the midpoint of $\overline{BC}$, and O is the midpoint of $\overline{AC}$.

1. If $BC = 37\frac{1}{2}$, then what is $MO$? 18.75

2. If $MN = \frac{\pi}{4}$, then what is $AC$? $\frac{\pi}{2}$

3. If $AB = 8x - 10$ and $ON = 2x + 10$, then what is the length of $\overline{AB}$?

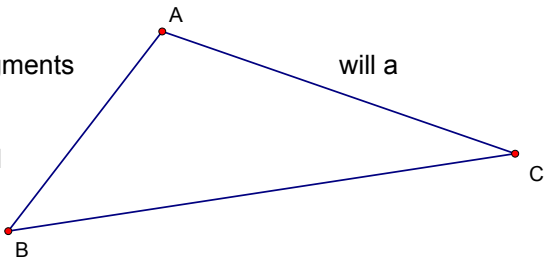
$$2x + 10 = \frac{1}{2}(8x - 10)$$

$$x = \frac{15}{2}$$

A rubric is provided so that students can monitor their progress as they work.

Exploring Midsegments of a Triangle Name_____

Exploration 1: A **midsegment** of a triangle is a segment whose endpoints are the midpoints of two sides of the triangle. How many midsegments will a triangle have?
Draw the midsegments of ΔABC.
Make some conjectures about the relationship between a midsegment and its third side.



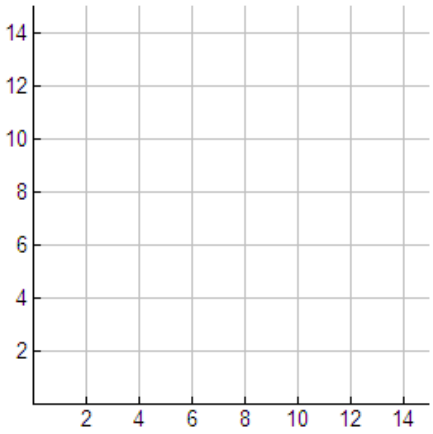

Exploration 2: What can you conclude about a midsegment and its third side?

Exploration 3: What can you conclude about the length of a midsegment Compared to its third side?

Exploration 4: What can you conclude about the area of the midsegment triangle compare to the area of the whole triangle?

Exploration 5: Midsegments of a Triangle in a Coordinate Plane

1. Construct a triangle on the grid with vertices at F(0,0), G(12,0), and H(8,6).
2. What is the length of $\overline{FG}$ ?
3. Find the location of the midpoint of side $\overline{FH}$ and call it point M. What are the coordinates of M?
4. Find the location of the midpoint of side $\overline{GH}$ and call it point N. What are the coordinates of N?
5. Connect M and N with a line segment. What is the length of $\overline{MN}$ ?
6. $\overline{MN}$ is a **midsegment** of △FGH. The corresponding side of $\overline{MN}$ is $\overline{FG}$ . Does the length of the midsegment compared to its corresponding side as you conjectured in the exploration that it would in Exploration 3?
7. Calculate the slope of $\overline{MN}$ . Calculate the slope of $\overline{FG}$ .




What can you conclude about $\overline{MN}$ and $\overline{FG}$ ?
Does this conclusion agree with Exploration 2?

8. Calculate the area of △FGH. Let O be the midpoint of side $\overline{FG}$ . Draw △MNO. Calculate its area. What is the relationship between the area of △MNO formed by the midsegments of the triangle and △FGH?
Do the areas agree with your conclusion in Exploration 4?

Exploration 6: The midpoints of a triangle are X(4,7), Y(5.5,6), and Z(7.5,9). What are its vertices?
How can you find the vertices without your calculator?

Practice: In ΔABC, M is the midpoint of $\overline{AB}$ , N is the midpoint of $\overline{BC}$ , and O is the midpoint of $\overline{AC}$ .

1. If $BC = 37\frac{1}{2}$ , then what is $MO$ ?
2. If $MN = \frac{\pi}{4}$ , then what is $AC$ ?
3. If $AB = 8x - 10$ and $ON = 2x + 10$ , then what is the length of $\overline{AB}$ ?

Read each statement and rate your work 4 if you agree with the statement, 3 if you somewhat agree, 2 if you somewhat disagree, or 1 if you disagree. Use NA, not applicable, if the statement does not apply in this situation. Circle one response for each description of your work.

| While Problem Solving, I ... | agree | somewhat agree | somewhat disagree | disagree | not applicable |
|---|---|---|---|---|---|
| Stayed on task. | 4 | 3 | 2 | 1 | NA |
| Tried different strategies, without the help of my instructor. | 4 | 3 | 2 | 1 | NA |
| Communicated with others about mathematics. | 4 | 3 | 2 | 1 | NA |
| Tried to solve the problem. | 4 | 3 | 2 | 1 | NA |
| Gave clear, coherent soluations. | 4 | 3 | 2 | 1 | NA |
| Actively participated in the lesson. | 4 | 3 | 2 | 1 | NA |
| Understand what a midsegment of a triangle is, and also its properties. | 4 | 3 | 2 | 1 | NA |