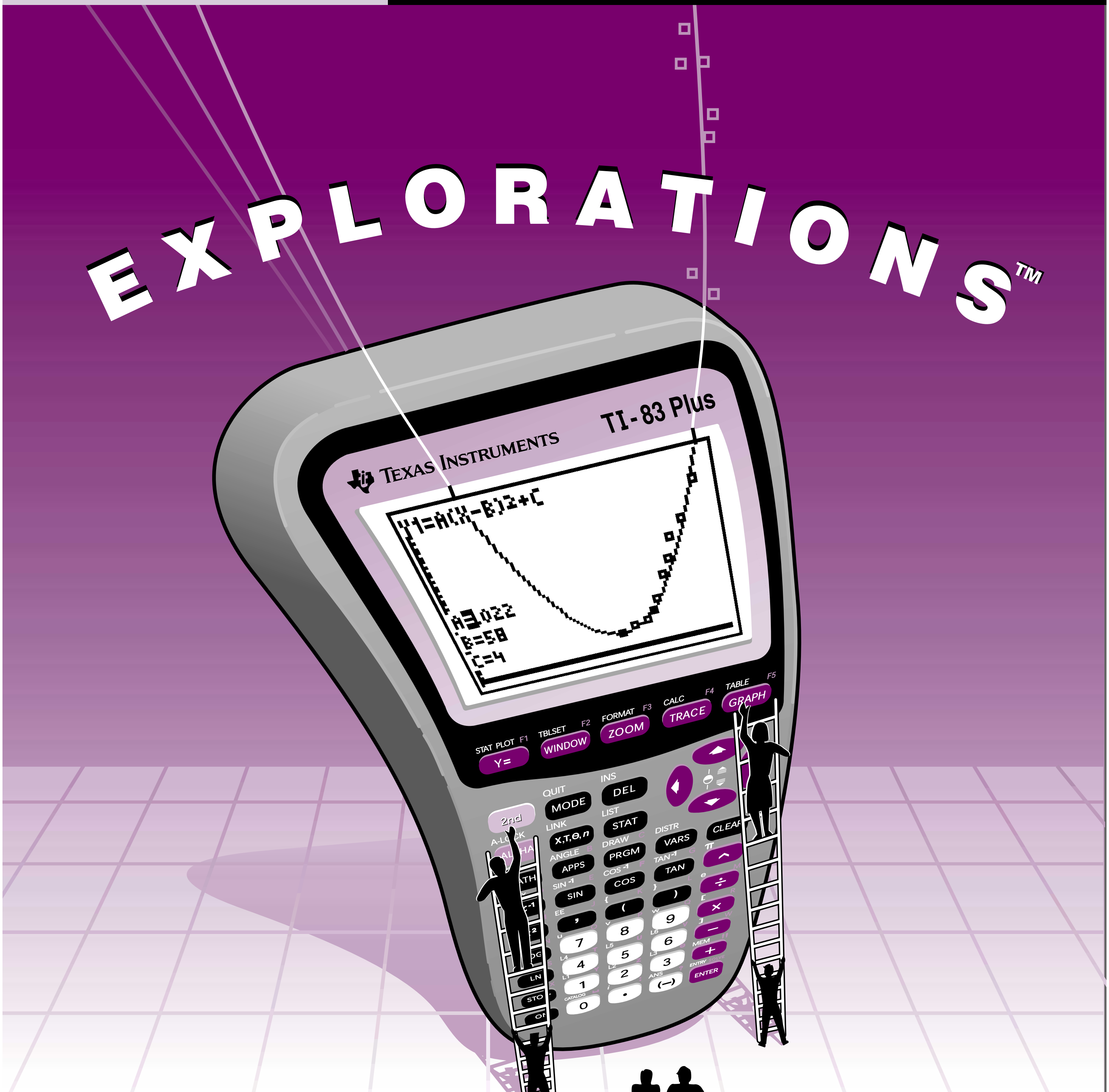
TI-83 Plus

TRANSFORMATION GRAPHING
APPLICATION SOFTWARE

A Hands-On Look at Algebra Functions: Activities for Transformation Graphing

Allan Bellman



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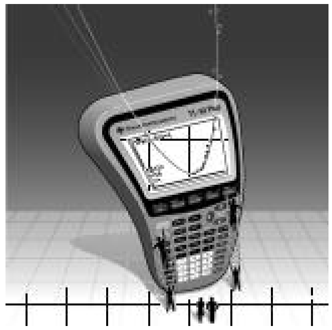

A Hands-On Look at Algebra Functions: Activities for Transformation Graphing

Allan Bellman
James Hubert Blake High School
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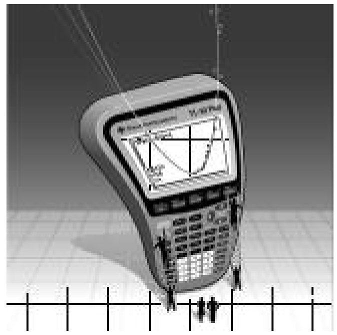
Visit the TI World Wide Web home page. The web address is **education.ti.com**.



Contents

Preface

Graphing calculators have given today's students a power to visualize mathematics that was unheard of in previous generations. Transformation Graphing is one of many applications that will be written for the various FLASH calculators that will greatly enhance this power. The ability to easily interact with a graph, which this application provides, will move the visualization and interactive capabilities of the graphing calculator to a new level. This will have a very positive effect on the way students will study functions in the near future.

Students will instantly feel comfortable with this application. It will remind many teachers of interactive computer programs that they have used in their Algebra classes in the past.

The six activities in this book provide an interactive look at the three functions usually studied in beginning Algebra: linear, quadratic, and exponential. In these activities, students will first visually study the functions and the effects of the various parameters on them, and then they will use Transformation Graphing as a modeling tool. While this book was written for those classes that study these functions, the techniques that are introduced can be used with any function.

I hope these activities will illustrate ways Transformation Graphing can be used, and that you will greatly expand on what is presented here in your own classes. Any function can be studied in this manner; I hope to only get your interest started. I have found that my students have enjoyed this view of functions and use this application to refine their models, even when they start the modeling process with other methods.

The explorations are organized in three sets of two activities: Activities 1 and 2 deal with linear functions, Activities 3 and 4 the quadratic function, and Activities 5 and 6 the exponential function. The first activity in each set enables the students to explore the effect of the function's parameters. For example, what effect does changing the value of A or B have on the graph of the line Y=AX+B. Each of the six activities has a modeling problem or problems included.

Each activity begins by presenting an example that walks the students through the keystrokes that are needed to use Transformation Graphing. Following this "handholding," they are given problems and homework with no keystrokes provided. In this manner, I feel an activity can be given to a student as an exploration that they complete on their own, or the explorations can be used as part of a group assignment or a class activity.

The keystrokes that are needed to teach with real data with the TI-83 Plus are listed both in the activities, when they are needed, and in an Appendix. You might want to duplicate the keystroke pages in the Appendix to give to your students for all their calculator work, not just while they are using Transformation Graphing.

Part of activities 2, 4, and 6 involve work with data that can be collected with CBL or CBR. Data for these activities can be found at **education.ti.com** if you choose not to collect the data yourself. If you decide to use the data at this site, go to the actual page in the book at this site to get a link to the sample data. If you choose to collect the data, I encourage you to have your students store the data either as a program or in a group so they can return to it at a later date. The keystrokes for both of these methods can be found in the Appendix.

One word of warning: While using Transformation Graphing, only one function can be graphed at a time. Also, be sure to inactivate the application when you are finished using it. Inactivation is called uninstall by the application, but don't worry—this does not remove it from the calculator.

I would like to thank those who have helped in the production of this book and the application, especially the staff at Texas Instruments, and Pam Harris and Judy Wheeler, who gave valuable advice on these activities when they were first being developed.

I hope your students find these activities and using Transformation Graphing as rewarding and fun as mine have.

— *Allan Bellman*

About the Author

Allan Bellman has taught for thirty-one years in Montgomery County, Maryland, as a Mathematics and Computer Science teacher. He presently teaches at James Hubert Blake High School, where his recent teaching interests have focused on Statistics and Algebra. Throughout his career, the use of technology of all types has been a special interest.

He is a co-author of several secondary mathematics textbooks published by Prentice Hall and South-Western Educational Publishing and is a frequent speaker at regional and national NCTM and T^3 (Teachers Teaching with Technology) conferences.

He has been an outreach instructor on Mathematical Modeling for the Woodrow Wilson National Fellowship Foundation since 1988 and an instructor in the T^3 program since 1996.

Downloading and Installing Transformation Graphing

Downloading the Application

1. Connect to the TI Online Store at **http://epsstore.ti.com**
2. Shop for TI-83 Plus software, and follow the link for **For Purchase** software.
3. Scroll down and locate **Transformation Graphing**. Click the download button.
4. Follow the on-screen instructions, and then choose the operating system of your computer (Windows or Macintosh).
5. Record the location where you download the file for future reference.

Installing the Application for Windows®

1. Connect the TI-GRAPH LINK™ cable between your computer and TI-83 Plus, and make sure the TI-83 Plus is on the Home Screen.
2. Launch the TI-GRAPH LINK for the TI-83 Plus software.

   Note: If you do not have TI-GRAPH LINK software for the TI-83 Plus, you can download it from the TI Online Store.
3. Click on **Link**, **Send Flash Software**, **Applications and Certificates**.
4. Navigate to the location where you downloaded the software.
5. Click on the downloaded file to select it, click **Add**, and then click **OK**. Your computer will send the application to your TI-83 Plus. You will see a progress bar while the application is loading.

Installing the Application for Macintosh®

1. Connect the TI-GRAPH LINK cable between your computer and TI-83 Plus, and make sure the TI-83 Plus is on the Home Screen.
2. Launch the TI-GRAPH LINK for the TI-83 Plus software.

   Note: If you do not have TI-GRAPH LINK software for the TI-83 Plus, you can download it from the TI Online Store.
3. Navigate to the location where you downloaded the software.
4. Drag the application to the calculator window in TI-GRAPH LINK. Follow any on-screen instructions that are given.

EXPLORATIONS

Activity 1

How Many Drivers? Investigating the Slope-Intercept Form of a Line

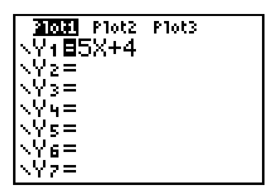
Any line can be expressed in the form $y = mx + b$. This form is named the *slope-intercept* form.

In this activity, you will study the *slope-intercept* form of the line. When you are finished, you should understand the effect of *m* and *b* on the graph of a line. You also will be able to fit a line to data by selecting values for these two parameters by using the guess-and-check method.

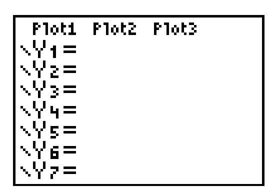
Investigating the Effect of the Values of m and b on the Graph of a Line

1. Begin this investigation by examining one instance of $y = mx + b$, when $m = 3$ and $b = 5$. On your TI-83 Plus, enter the equation **Y1 = 3X + 5**.

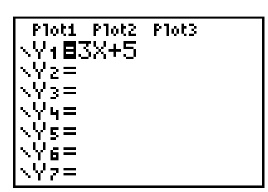
   a. Begin in Function (**Func**) Mode and turn off any plots or equations previously turned on. (This screen shows both an unwanted equation and an unwanted plot.)



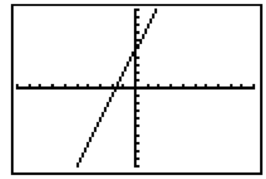
   To turn off a plot, press [Y=] and use the cursor control keys ([▲] and [▼]) to move to the highlighted plot and press [ENTER]. To clear an equation, press [▲] or [▼] to move to the first part of the equation and press [CLEAR].



   b. Move the cursor to **Y1=** and enter **3** [X,T,Θ,*n*] [+] **5**.



2. Display the graph by pressing [ZOOM] **6:ZStandard**. Both the x-axis and y-axis will be displayed with a range of -10 to 10.



3. Enter and graph two more examples of $y = mx + b$.

   a. Y2 = 3X + 2

   b. Y3 = 5X + 2

Can you tell the effect of the *m* or the *b* on the graph of the line? Can you guess why they call this form the slope-intercept form of a line?

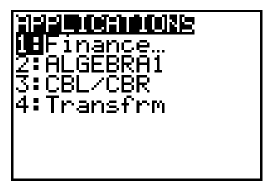
You probably are not ready to answer these questions because it is difficult with only three examples. You will be ready to answer these after a short investigation. On a piece of paper, write a guess about the effect of *m* and *b* on the graph of a line. You will come back to this sheet later to check your hypothesis.

Using Transformation Graphing to Investigate m and b

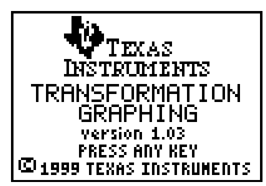
To investigate the effects of *m* and *b* you will use a TI-83 Plus with the Transformation Graphing application to graph many examples. You must have Transformation Graphing installed and running on your calculator to complete this activity.

To start Transformation Graphing:

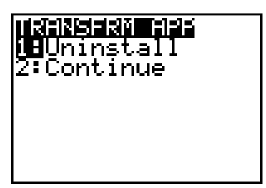
1. Press [APPS]. Press the number in front of the application **Transfrm**. (This number may vary based on the applications you have loaded.)



2. Press any key (except [2nd] or [ALPHA]) to install the application.



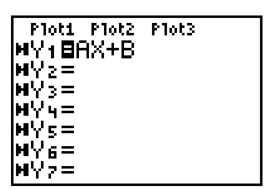
   If the application is already running, press **2:Continue**.



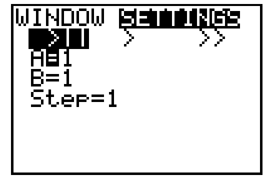
To graph the example:

1. Press [Y=], clear the **Y=** editor, and turn off all plots.
2. For **Y1**, enter **AX + B**. Press [ALPHA] **A** [X,T,Θ,*n*] [+] [ALPHA] **B**.

   Note: You entered Y=AX + B in place of Y=MX + B, which is the form commonly found in textbooks, because Transformation Graphing only uses the coefficients A, B, C, and D.

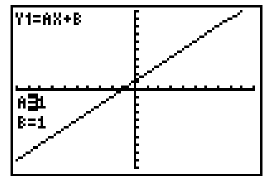


3. Set Transformation Graphing for Play-Pause Mode. Press [WINDOW] [▲] to display the SETTINGS screen. If necessary, move the cursor until you highlight >|| and press [ENTER].



   As a starting place, set the other SETTINGS as pictured. To make these selections, press [▼] 1 [▼] 1 [▼] 1. This defines the starting values for the coefficients and the increment by which you want to observe the change in the coefficients.

4. Press [ZOOM] **6:ZStandard** to display the graph. If **A=** is not highlighted, press [▼] until the **A=** is highlighted.



5. Press [▶] to increase the value of **A** or [◀] to decrease the value of **A**. With each new **A** value, notice the change on the graph.

6. Enter values for **A** by hand and notice the effect on the graph. Let **A** take both positive and negative values between -10 and 10, including decimal values. To enter values, press the key for the value you want and then press [ENTER].

Questions for Discussion

1. What effect did **A** have on the graph?

   a. As the value gets larger, what happens to the graph?

   b. If the value is decreased what effect will that have on the graph?

   c. If the value is changed to a negative, what will the result be on the graph?

2. Use the same method to investigate the effect of **B** on the graph. Set **A = 1**. To change **B**, press [▼] and then press [▶] to increase the value of **B** or [◀] to decrease the value of **B**. You may also enter values for **B** by hand as you did for **A**.

   What effect does **B** have on the graph? As the value gets larger what happens? As it gets smaller? Negative?

3. Go back to the sheet of paper where you wrote your hypothesis about the effect of M and B. How close were you to the actual effect?

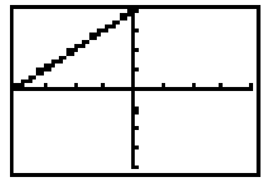
   Why do you think they call this the slope-intercept form of the line?

Problems

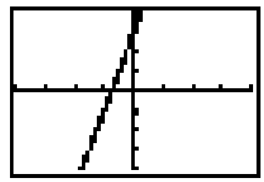
For the following problems, use your hypothesis to guess the correct answers and then use your calculator to confirm your guess.

1. Match the graph from the second column to its equation in the first. Notice that the window range for both X and Y is from -4 to 4 with a scale of 1.

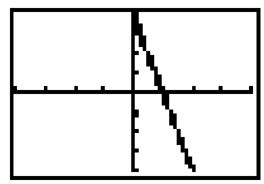
a. $y = 4x + 3$

1 

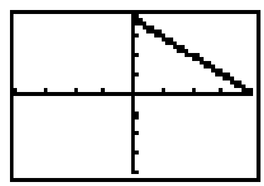
b. $y = 4x + 1$

2 

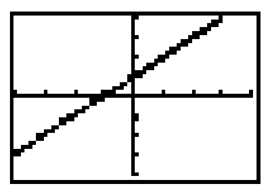
c. $y = 1x + 4$

3 

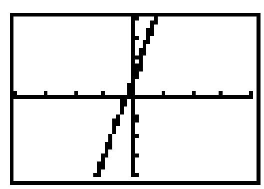
d. $y = 1x + 1$

4 

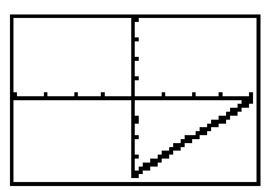
e. $y = 1x - 4$

5 

f. $y = -1x + 4$

6 

g. $y = -4x + 4$

7 

2. Where does $y = 4x - 1$ cross the y-axis?

3. Where does $y = -3x + 2$ cross the y-axis?

4. Which line is steeper, $y = 3x - 1$ or $y = x + 5$?

5. Which two lines slant the same direction?
   $y = 2x + 5$ $y = 5x - 1$ $y = -x + 3$

Use Your New Skill

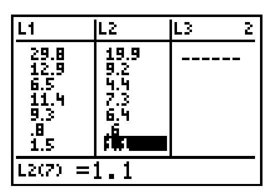
Most students turn 16 and start thinking about getting a driver's license. The table shows the population from the 1990 census and number of licensed drivers in a few selected states.

State	Population (in millions)	Licensed Drivers (in millions)
California	29.8	19.9
Florida	12.9	9.2
Georgia	6.5	4.4
Illinois	11.4	7.3
Michigan	9.3	6.4
Montana	0.8	0.6
New Mexico	1.5	1.1
New York	18.0	10.3
Pennsylvania	11.9	7.8
Texas	17.0	11.1

Source: Used with permission from *World Alamanac and Book of Facts, 1992.*

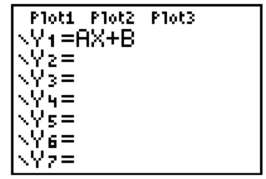
Create a scatter plot for this data and determine if you think you see a relationship or pattern. Which variable do you think should be the independent variable?

1. Enter the data in two Stat Lists of your TI-83 Plus. Press [STAT]. Select **1:Edit**. Enter the values from the table into an empty list in the stat editor. Clear two lists if no lists are empty. To clear a list, press [▲] to highlight the list name, press [CLEAR], and then press [ENTER].

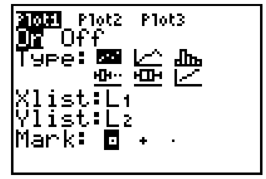


2. Press [MODE] and verify that you are in **Func** mode. (**Func** is highlighted if you are in Function mode.)

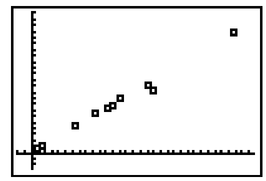
3. Press [Y=] and deselect any functions that have been turned on. Verify that all plots are off (not highlighted).



4. To display the plot, press [2nd] [Stat Plot]. Select **1:Plot1** and press [ENTER]. Turn the plot on and set the setup menu as shown at the right.



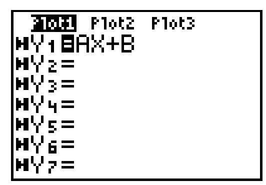
5. Press [ZOOM] **9:ZoomStat** to display the plot. Does there appear to be a relationship between a state's population and the number of licensed drivers in that state?



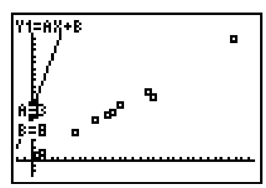
6. There appears to be a linear relationship between the two variables. Estimate values for the slope and the y-intercept of a linear model to represent the data.

7. Press [Y=] to display the **Y=** editor. Enter **Y1** = **AX** + **B** for the equation. (Press [ALPHA] **A** [X,T,Θ,n] [+] [ALPHA] **B**.) Leave Plot1 on.

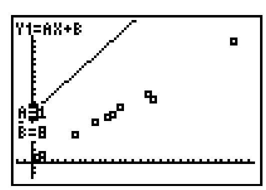
   Press [◄] to move to the left of **Y1**= and continue to press [ENTER] until Transformation Graphing is in Play-Pause mode (>||).



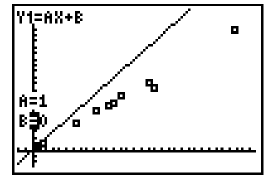
8. Since the window was just set to display the plot, you do not need to change the WINDOW settings. Display the plot with Transformation Graphing enabled by pressing [GRAPH]. The line **Y** = **AX** + **B** will be displayed with the most recent values of A and B. This line will have no relationship to your plot.



9. In step 6, you estimated values for the slope and y-intercept. Enter these values for A and B. For the purpose of this sample, assume your estimates are A=1 and B=0. To enter these values use the up and down cursor control keys ([▲] and [▼]) to move the highlight to **A**=. When **A**= is highlighted, press **1** [ENTER].



10. Press [▼] to highlight **B** and press **0** [ENTER]. This is your starting point as you look for a reasonable model.



11. Keep selecting and entering new values for **A** and **B** until you feel you have a reasonable model with values for **A** and **B** estimated to the hundredths place.

Homework Page

Name ________________________

Date ________________________

How Many Drivers?

Most students turn 16 and start thinking about getting a driver's license. The table shows the population from the 1990 census and number of licensed drivers in a few selected states.

State	Population (in millions)	Licensed Drivers (in millions)
California	29.8	19.9
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New Mexico	1.5	1.1
New York	18.0	10.3
Pennsylvania	11.9	7.8
Texas	17.0	11.1

Source: Used with permission from *World Alamanac and Book of Facts, 1992.*

1. About how many licensed drivers does your model predict the state of Maryland, with a population of 4.9 million, should have?

2. Go to the Internet and find the population of your state. Use your model to predict the number of licensed drivers in your state.

3. Population is constantly changing. Do you think the slope of the line you used as a model would change very much if you used newer data, such as the 2000 census? Explain your answer.

4. What do you think is the meaning of the slope of your model? What else could the slope represent?

5. What is the unit of measure of the slope in this example?

Notes for Teachers

This activity is an introduction to the slope-intercept form of the line. Its purpose is to enable the student to recognize the effect of changes in the slope and/or the y-intercept on the graph of a line. At the end the student will use Transformation Graphing to eyeball fit a linear model to actual data.

At this point, no attempt is made to have the students graph a line by hand, just to recognize the effect of the parameters and start modeling.

This activity will help the development of what makes a reasonable model. You should spend time talking about different models that are selected and what makes some better then others—but be careful that you do not look for a best model. As part of the activity, various models should be entered in the overhead calculator, with Transformation Graphing deactivated, to discuss the reasonableness of each model.

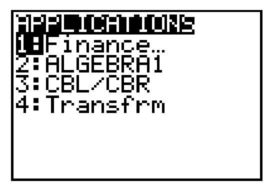
The activity will take approximately 50 minutes.

There is one modeling exercise at the end of the activity. Only one was given as the next activity picks up from here and continues with more practice. If you would like to give more practice, any linear data set from your textbook could easily be used.

Be sure students realize that only one equation can be graphed at a time while Transformation Graphing is installed and activated.

To de-activate Transformation Graphing:

1. Press [APPS] and select the number preceding **Transfrm**.



2. Select **1:Uninstall**.

Answers

Questions for Discussion

1. **A** affects the slope (slant) of the line. The larger **A** is, the steeper the line. When **A** is negative, the line slopes downward from left to right. Both **A** and **-A** have the same steepness; but one slants up from left to right and the other down.

2. **B** affects where the line crosses the y-axis. The larger the value of **B**, the higher the line appears on the graph.

3. **B** gives the y-intercept and **A** is the slope.

Problems

1. a. 2
   b. 6
   c. 1
   d. 5
   e. 7
   f. 4
   g. 3
2. -1
3. 2
4. $y = 3x - 1$
5. $y = 2x + 5$ and $y = 5x - 1$

Use Your New Skill

11. Reasonable values for A and B will be approximately **A** = .65 and **B** = .10.

Homework Page

1. Approximately 3.3 million.
2. Answers will vary.
3. No, the slope of the line probably relates to the percentage of people in the state over 16 and this percent probably wouldn't change over short time spans.
4. The change, in millions, in the number of drivers per every 1 million change in the population. This is the percentage of licensed drivers in the total population. See the answer to Question 3.
5. The unit of measure of the slope would be drivers/population.

EXPLORATIONS

Activity 2

Lines, Models, CBR — Let's Tie Them Together

In this activity, you will practice finding models using the "eyeball" method on data you collect. The activities will allow you to apply what you have learned about the slope-intercept form of a line.

One, of many, methods that can be used to find a mathematical model is the "eyeball" method.

This method is quite simple to do by hand as long as you are only looking for the line, not its equation. When working with linear data, you can use spaghetti to help "eyeball" a best fit line. To do this, move a piece of spaghetti over the data until you have what you feel is a reasonable model.

In this activity, you will use your TI-83 Plus with the Transformation Graphing application to "eyeball" fit a linear model. This method will also give you an equation for your model.

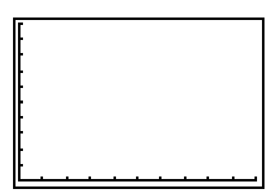
First you need some data to "eyeball." Use the CBR to collect "linear" motion data.

Collecting the CBR data

A CBR will record the distance from the CBR to a walker walking away from it (or towards it). In this activity the walker should try to walk at a constant rate.

The CBR/TI-83 Plus combination plots time as the independent variable and the walker's distance from the CBR as the dependent variable.

Questions for Discussion

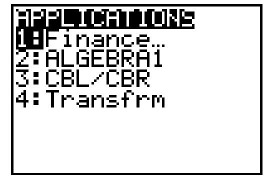


1. How should the plot appear if the walker successfully walks at a constant rate away from the CBR? Sketch your guess on the coordinate system.
2. Why do you think the grid contains only the first quadrant?

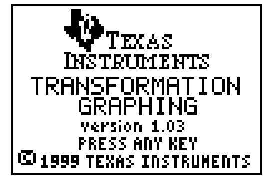
Procedure

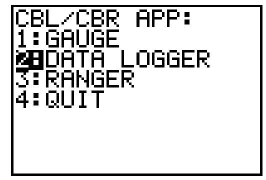
1. Use the unit-to-unit link cable to connect the CBR to the TI-83 Plus.

2. Press APPS and then choose **CBL/CBR**.

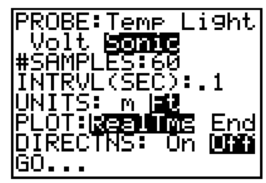


3. Press any key to advance past the introduction screen and then select **2:DATA LOGGER**.

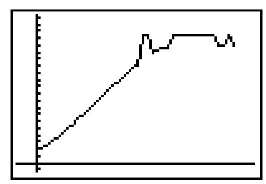




4. Make the selections in the setup menu as illustrated. These settings will allow the CBR to collect and record one reading every 0.1 second for 6.0 seconds.

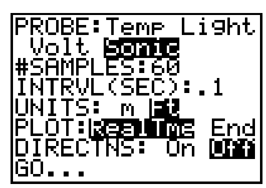


5. Place the CBR on a table facing into an open area of the room. A student (the walker) should stand approximately 1.5 feet from the CBR, facing away from the CBR. Remind the walker that they are trying to walk slowly at a constant pace, away from the CBR. Tell them that when you say "go" they should start. Use the cursor control key (▼) to move to **GO**. Press ENTER and tell the walker to go.

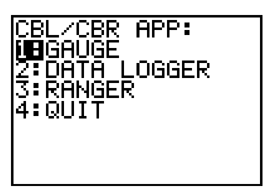


6. The CBR will start recording the distance to the walker. Once the 6 seconds has passed, the plot will re-scale the window automatically using **ZoomStat**.

7. You are still running the CBL/CBR Application. You should exit this application.



   Press ON 2nd [QUIT] **4:QUIT**.



8. The points from the plot are connected by the CBL/CBR application. It is easier to "eyeball" a model with the points in unconnected form. Press 2nd [STAT PLOT] **1:Plot 1** and then select the unconnected scatter plot option.

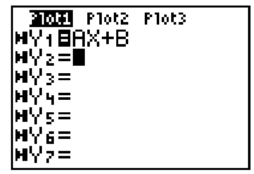
Repeat the previous steps until you have a reasonable scatter plot.

9. Once you have reasonable data, link the calculators so everyone in the class has the same data.

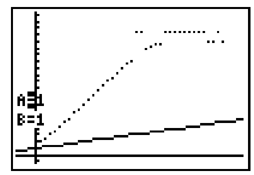
Finding The Model

Use the data you just collected to find your model.

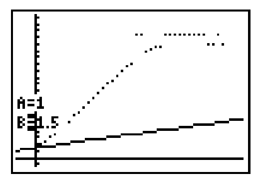
1. Activate the Transformation Graphing application.
2. Enter the slope-intercept form of a line in **Y1**. Press [Y=]. Clear all equations from the **Y=** editor. Press [▲] to move to **Y1** and enter **Y1** = [ALPHA] **A** [X,T,Θ,n] [+] [ALPHA] **B**. Press [◄] to move to the left of **Y1** and continue to press [ENTER] until you have selected the Play-Pause mode (>||).



3. Display the graph by pressing [ZOOM] **9:ZoomStat**. The plot will be displayed and a line will be graphed. The line will contain the present **A** and **B** values from the calculator. These values have absolutely nothing to do with the activity and thus the graph should have little relationship to the points.



   What is the physical meaning of the y-intercept (**B**) in this problem? What would be a good starting value for this coefficient?



   The y-intercept tells how far from the CBR the walker started *if the walker and the CBR started at exactly the same time*. The walker was asked to start 1.5 feet from the CBR, so 1.5 would be a good starting value for **B**.

4. Use the up/down cursor keys ([▲] [▼]) to move to the **B**= and enter 1.5. Adjust this value to match the y-intercept more accurately.

   Note: A later section of this activity shows an example where the walker and the CBR do not start together.

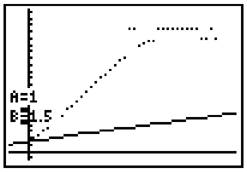
   a. What is the physical meaning of the slope (**A**) of the line in this example? Can you make a good estimate for its value?

   b. If the CBR is measuring in feet and taking readings in seconds, **A** is in what unit of measure? It is very important that measurements have a unit of measure.

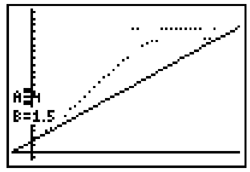
      The slope (rate of change of the function) shows the speed of the walker. With Transformation Graphing, you can easily make many different estimates for the slope.

   c. The line in the sample does not appear to be steep enough if **A=1** is used. What does that mean about your model? Does your model have the walker walking too fast or too slow?

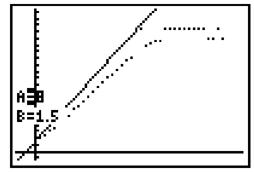
5. The scatter plot in the example needs a model with a steeper slope, meaning the walker was moving faster than 1 foot/sec. To refine the model, press [▲] to go to **A=** and enter different values for **A** until you like the fit. Remember, to enter values, type a number and press [ENTER]. The graph will move to reflect the new value.



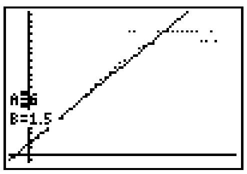
A = 2



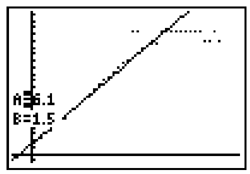
A = 4



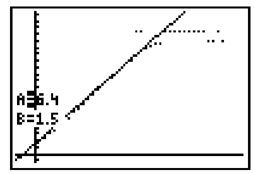
A = 8



A = 6



A = 6.1



A = 6.4

6. Once you have a reasonable value for **A**, you might decide you need to revise your value for **B**. Press [▼] to move to **B** and refine that value.

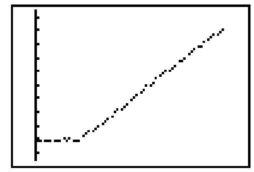
Homework Page

Name ________________________

Date ________________________

Walker 2

Using the same CBR setup, have a second walker try to duplicate the pattern pictured at the right. Once you have good data, link the data to the rest of your class and use Transformation Graphing to help find a model for this "walk."



1. What does the walker need to do to match the picture?

2. Could you tell from watching which of the two walkers in your classroom moved the fastest? If it was noticeable, which one walked the fastest?

3. How fast was each walking?

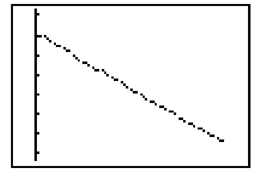
4. Does the y-intercept of your linear model have any meaning in this example? Explain your answer.

5. Tyler's Algebra II teacher said he should use a piecewise defined function for his model. What did she mean? With respect to the physical situation, why would that make sense?

6. You have used the slope of your model as the speed of the walkers. Is there another way to find the walker's speed?

Walker 3

Using the same CBR setup, have another walker try to match the pattern pictured at the right.



1. What will they have to do?

Once you collect good data, link the data to the rest of the class and use Transformation Graphing to help find a model.

2. What does the negative slope mean?

3. Which of the three walkers walked fastest? Slowest? Did you disregard the sign of the slope for this question? Why or why not?

4. How can you tell if the walker walked at a constant speed?

5. Sketch a graph that would represent a walker walking at a non-constant rate.

Notes for Teachers

Note: You can use either a CBL or CBR to collect the data. The directions you are using assume the TI-83 Plus APP named CBL/CBR is used. There are many other programs available that can be used, including the HIKER program and activity, available as a free download from the Texas Instruments Web site at ***education.ti.com****. There are other workbooks in the Exploration series that use material that could be helpful in the development of this activity.*

This activity is intended as a follow-up to Activity 1. The purpose is to give the student practice with the "eyeball" method of modeling. This method is a great aid in the development of a sense of the effects of the parameters of a function and will be continued throughout this book. At some point later in your study, you might want to compare their "eyeball" models to those found using linear regression.

After each student walk, the data should be linked to each calculator in the classroom so all students can work with the same data.

If enough CBL/CBRs are available, this activity should be continued with the students in groups.

This activity is the second of two on the slope-intercept form. In this activity, the use of student walkers to create the data set allows the students to visualize the relationship between a physical action and the slope and y-intercept of the equation use as a model. The students can (and will want to) do many variations on their walks.

You might want to have the walkers walk along a number line on the floor. In this way you can measure where they start and how far they walk as a method of relating the equation to the "real world."

Be sure to emphasize the importance of units of measure and labeling units on the axis. No units labels were given in the activity. You can set Data Logger to collect data in either meters or feet.

Walker 2 presents an opportunity to discuss the issue of domain and range of a model. Although the equation that is developed to match the walking part of the graph has a domain of all reals, it only has a direct meaning for values while the walker is moving.

There are two distinct parts of the graph for walker 2: the walker standing still, and then the walker moving. There should be two distinct parts for the model over $0 < x < 6$ if you try to model the complete graph. This could be used to introduce piecewise defined functions *if appropriate for your students.*

Answers

Collecting the CBR Data: Questions for Discussion

1. Linear data.
2. The CBR cannot record distances behind it; thus there are no negative y-values. The CBR cannot go back in time; thus there are no negative x-values.

Finding the Model

3. The y-intercept shows how far the walker started from the CBR.
4. a. The slope is an estimate of the walker's speed.
   b. **A** is in feet/sec. A unit of speed or velocity.
   c. You need to make **A** larger. The data in the example shows a walker who is walking faster then the model says.

Homework Page

Walker 2

1. The walker must first stand motionless and then move at a constant rate away from the motion detector.
2. Answers will vary.
3. Answers will vary.
4. In this problem the y-intercept has no meaning. The walker did not start moving until after t=0. The model has meaning only while the walker is moving, and since t=0 is not where the model has meaning, the y-intercept has no meaning.
5. One function would be used to model while the student was standing still and another function used to model the walking portion. Thus the plot could be considered to have two pieces.
6. $$\text{speed} = \frac{\text{distance traveled}}{\text{time traveled}}$$

   You can use **TRACE** to find the point the walker started to move and the last point recorded. Insert these values into this formula:

   $$\text{speed} = \frac{\text{final position - starting position}}{\text{final time - starting time}}$$

   This would only be accurate if the walker moved at a constant rate.

Walker 3

1. Move toward the CBR at a constant rate.
2. The walker was walking toward the CBR.
3. Answers will vary. You should disregard the sign of the slope. The sign of the slope tells the direction, the magnitude gives the speed or rate.
4. The more constant the speed, the more linear the data.
5. Answers will vary. Any non-linear graph.

EXPLORATIONS

Activity 3

Exploring the Vertex Form of the Quadratic Function

Any quadratic function (parabola) can be expressed in $y = a(x - h)^2 + k$ form. This form of the quadratic function (parabola) is known as the *vertex form*. In this activity, you will discover how this form shows both the location of the vertex and the width and direction of the curve. As you make discoveries, record your findings on a piece of paper for later practice sessions.

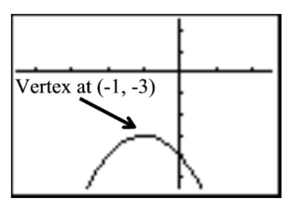
At the end of this activity, you should be able to locate the vertex of any parabola that is expressed in vertex form and determine the direction it opens.

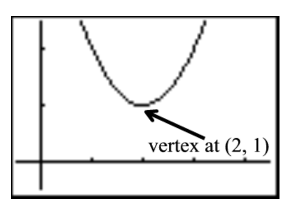
Since you will be able to find the vertex and tell the direction a parabola opens, you will be able to find the maximum or minimum value of quadratic functions expressed in vertex form.

You will end this activity with a beginning knowledge of the general concept of translation and be able to apply it to new situations.

The Graph of a Quadratic Function in Vertex Form

Quadratic functions come in two basic shapes: those that open up and those that open down. One of the important points in a quadratic function is its vertex. The *vertex* is the lowest point (minimum value) if the function opens up and it is the highest point (maximum value) when it opens down.





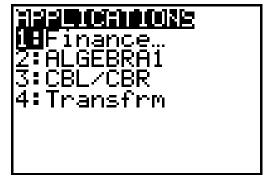
Using your TI-83 Plus, graph $y = (x - 2)^2 + 1$.

Questions for Discussion

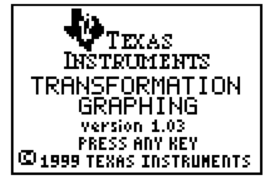
1. Where is its vertex? What direction does it open?

Studying the Effect of A, B, and C

1. Press [APPS] and select Transformation Graphing by pressing the number at the left of **Transfrm.** Press any key (except [2nd] or [ALPHA]) to start Transformation Graphing.



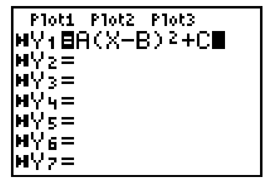
*Note: If you do not see the screen illustrated at the right, select **2:Continue**.*



2. In **Func** mode, press [Y=] to display the **Y=** editor. Clear any functions that are listed and turn off any plots. Enter the general vertex form of the quadratic function, $Y = A(X - B)^2 + C$. Press [ALPHA] **A** [(] [X,T,Θ,n] [−] [ALPHA] **B** [)] [x^2] [+] [ALPHA] **C.**

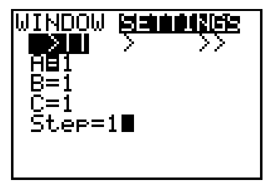
   If the Play-Pause Mode is not selected at the left of **Y1** (>||), press [◄] until the cursor is over the symbol and then press [ENTER] until the correct symbol is selected.

   Note: You entered Y=A(X - B) 2 + C in place of Y=A(X - H) 2 + K, which is the form commonly found in textbooks, because Transformation Graphing only uses the coefficients A, B, C, and D.

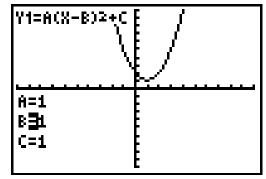


3. Press [WINDOW] [▲] to display the settings screen for Transformation Graphing.

   As a starting place, set the SETTINGS as pictured. To make these selections, press [▼] **1** [▼] **1** [▼] **1** [▼] **1.** This defines the starting values for the coefficients and the increment by which you want to observe the change in the coefficients.



Studying the Effect of B



4. Press [ZOOM] **6:ZStandard** to display the graph. The graph will show the pre-selected values of **A**, **B**, and **C**. Both the X and Y-axis range from -10 to 10 with a scale of 1.

   Press [▼] to move down one space and highlight **B=**. You will start your study with the effect of **B**.

5. Press [▶] to increase the value of **B** by the pre-selected Step value (1 in this example). The graph is automatically redrawn showing the effect of this change on **B**. Continue to press [▶] until you have an idea of how changing **B** effects the graph.

6. Press [◀] to decrease the value of **B** by the pre-selected Step value. Did the graph move the direction you would have expected?

Questions for Discussion

1. Changing the value of **B** has the curve move in what direction?

   This moving of the curve is called a translation in the X-direction or horizontal translation.

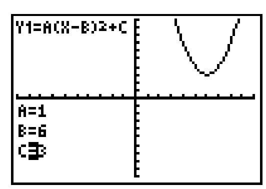
2. Use the cursor keys ([◀] [▶]) to change the value of **B** again. As you change **B** notice the x-coordinate of the vertex.

   If **B=3**, where is the vertex? How about **B=5**? And when **B=-2**?

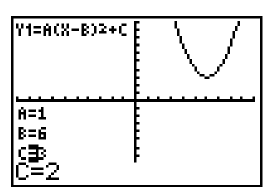
Make a hypothesis about the relationship between **B** and the vertex of the parabola. Test your hypothesis by entering **B=1**, **B=3**, **B=5**, **B=-1**, and **B=-2**. Were you correct?

In vertex form $Y = A(X - B)^2 + C$, the value of **B** gives the x-coordinate of the vertex. Be careful, though. Notice the form has X - B. If you had the equation $Y=(X - 3)^2$ then B=3 and the vertex is at X = 3. For the equation $Y=(X + 1)^2$ the **B** would be **-1** with the vertex at X = -1.

Studying the Effect of C



1. Press [▼] to highlight the **C=**. Press [▶] several times and notice the change in the graph. Press [◀] several times and notice this change.



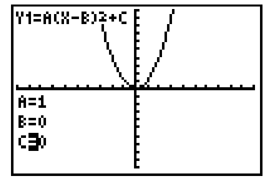
2. Predict where the vertex of the function will be if you let **C=2**. Enter **2** for **C** and check your prediction.

3. Make some conclusions about the effect of changes in **C** on the vertex. Check your conclusions by entering test values for **C**.

Changes in **C** create a vertical translation of the curve. When **C** increases the curve moves up. When **C** decreases the curve moves down. The value of **C** is the y-coordinate of the vertex.

Check Your Understanding So Far

The graph of $y = x^2$ is shown at the right. The following equations are vertical and horizontal translation of $y = x^2$. Use what you have discovered about translation of the vertex of a quadratic function to predict the vertex of the graph of each equation. Check your prediction using Transformation Graphing.



$y = (x - 2)^2$

$y = (x - 2)^2 + 3$

$y = (x + 1)^2 +3$

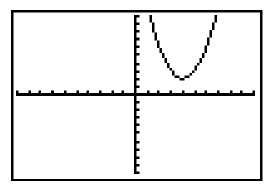
$y = (x + 1)^2 -2$

$y = (x + 5)^2$

$y = x^2 - 2$

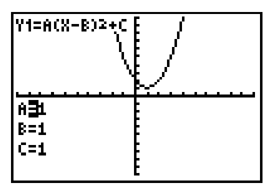
What is the equation of the parabola (quadratic function) graphed at the right?

Note: The scale is 1.



Studying the Effect of A

1. Return to the Transformation Graphing Screen and press [▲] until the **A=** is highlighted.



2. Use the same discovery method you used with **B** and **C** to investigate the effect of **A** on the graph of the parabola. Be sure to let **A** be both negative and positive. Once you have a hypothesis and have checked it for the effect of **A**, continue with the next question.

Questions for Discussion

1. What effect does changing the value of **A** have on the graph? Be sure to discuss both magnitude and sign changes.

The value of **A** determines the direction of the parabola and its width. The larger the magnitude of **A**, the narrower the curve. The smaller the magnitude of **A**, the wider the curve. A positive sign means the parabola is opening up. A negative sign means the parabola is opening down.

Deactivate Transformation Graphing before continuing.

1. Press [APPS] and select the number preceding **Transfrm**.

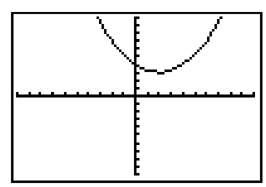
2. Select **1:Uninstall**.

Check Your Understanding

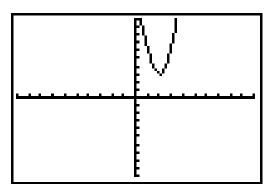
Match the equation from column 1 with its graph in column 2. Be careful that you look at all the equations and compare them before you answer any questions. Do these first without using your calculator, and then verify your answers using your calculator.

Note: These examples only investigate A.

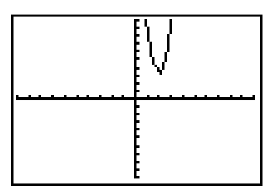
1. $Y = 3(X - 2)^2 + 3$

a. 

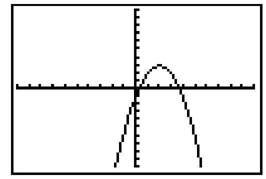
2. $Y = -(X - 2)^2 + 3$

b. 

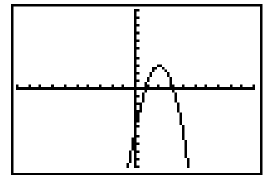
3. $Y = .25(X - 2)^2 + 3$

c. 

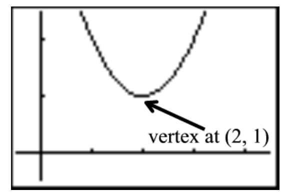
4. $Y = -2(X - 2)^2 + 3$

d. 

5. $Y = 6(X - 2)^2 + 3$

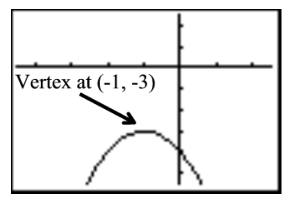
e. 

Maximum and Minimum Values Come Into Focus



When a parabola opens upward, the vertex will be the lowest point on the curve. Any other point will have a larger value for *y*. In the graph at the left, the y-value of the vertex is 1. This is the lowest value *y* can obtain and it is thus called the *minimum* value of the function.

The graph shows a parabola, quadratic function, with a minimum value of 1 when **x=2**.



Likewise, when a parabola opens down there will be a largest or maximum value for *y*. This graph shows a function with a maximum value of -3 when **x= -1**.

Check This Out

Complete the table for each parabola.

Equation	Opens up/down	Function has a maximum/ minimum	Maximum/ minimum value
$Y = 2(x - 3)^2 + 2$	Up	minimum	2
$Y = -3(x + 1)^2 + 10$			
$Y = 10(x + 4)^2 - 36$			
$Y = -16(x - 2)^2 - 100$			

A Quick Application

The equation $y = -16(x - 4)^2 + 259$ models the flight of a model rocket where *y* is the height of the rocket and *x* is the time since it was launched. Graph the function with a reasonable domain. What is the maximum height of the rocket? How long after it was launched did it reach its maximum? What does this have to do with this activity?

Homework Page

Name ______________________

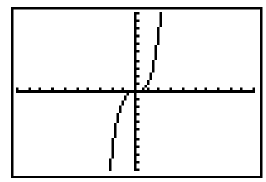
Date ______________________

Look at some equations of linear functions and see how translation applies.

1. Use your graphing calculator to graph **Y1 = X** and **Y2 = X + 3** on the same axis. In what two ways is the second equation a translation of the first?

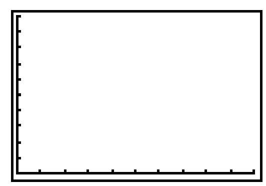
Now let's look at some functions you might not have already studied and see if you can apply your knowledge in a new situation.

2. The graph of the function $y = x^3$ goes through the origin (0, 0). Look at the graph of $y = x^3$ at the right and using the point at the origin as the point you translate (like you did the vertex) sketch the graph of $y = x^3 + 2$. Check your answer by graphing $y = x^3 + 2$ on your TI-83 Plus.

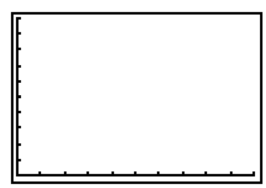


Note: You can either use [^] **3** *for the power of three or* [MATH] *3:* ^{3}.

3. Sketch $y = (x - 2)^3$ and check your answer with your graphing calculator.



4. Sketch $y = (x + 1)^3 - 5$ and check your answer.



Notes for Teachers

The purpose of this activity is to allow the students to "play" with the vertex form of the parabola and discover how the vertex, direction, and width can be determined by looking at the parameters.

Once the activity is complete, the students should be very capable of making predictions about the location of the vertex of a parabola expressed in vertex form. They will need much more work with the effect of **A.** After this activity, they will only be capable of comparing two graphs; they will not be able to graph a parabola in vertex form by hand. Students will need classroom instruction to graph the actual width.

The concept of translation is basic to the general study of functions, and the activity ends with a quick look at translation in general. These closing questions on translation are not really part of the study of the vertex form; you can easily leave them out. They serve as a quick view of translation for those who use translation as part of their study of function.

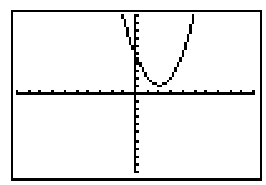
The work with translation, at the end, encourages the students to make a hypothesis and use their calculator to check it.

The skills and knowledge developed in this activity will be used in the next activity.

Answers

The Graph of a Quadratic Function in Vertex Form: Questions for Discussion

1. Vertex is at (2, 1). The parabola opens up.



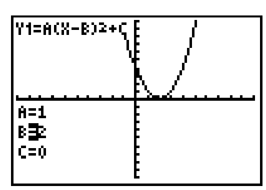
Studying the Effect of B: Questions for Discussion

1. The curve moves in the X-direction.
2. When **B=3**, the vertex is at -3. **B=5** gives a vertex of -5. **B=-1** gives a vertex at 1. The vertex is 1 if **B=-1**.

Check Your Understanding So Far

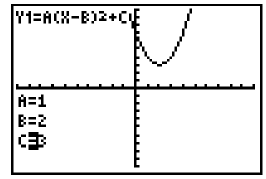
$y = (x - 2)^2$

Vertex is at (2, 0)



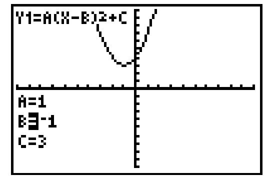
$y = (x - 2)^2 + 3$

Vertex is at (2, 3)



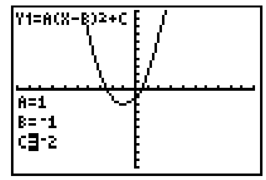
$y = (x + 1)^2 +3$

Vertex is at (-1, 3)



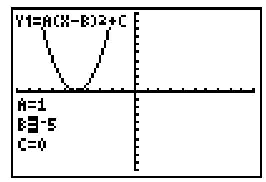
$y = (x + 1)^2 -2$

Vertex is at (-1, -2)



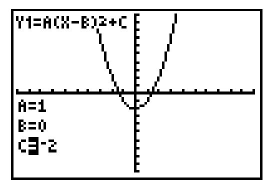
$y = (x + 5)^2$

Vertex is at (-5, 0)



$y = x^2 - 2$

Vertex is at (0, -2)



What is the equation of the parabola (quadratic equation function)?

$Y = (x - 4)^2 + 2$

Studying the Effect of A: Questions for Discussion

1. The value of **A** determines the direction of the parabola and its width. The larger the magnitude of **A**, the narrower the curve. The smaller the magnitude of **A**, the wider the curve. A positive sign means the parabola is opening up. A negative sign means the parabola is opening down.

Check Your Understanding

1. b
2. d
3. a
4. e
5. c

Check This Out

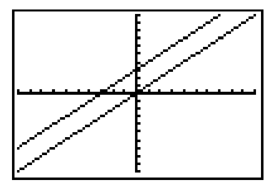
Equation	Opens up/down	Function has a maximum/minimum	Maximum/minimum value
$Y = 2(x - 3)^2 + 2$	Up	Minimum	2
$Y = -3(x + 1)^2 + 10$	Down	Maximum	10
$Y = 10(x + 4)^2 - 36$	Up	Minimum	-36
$Y = -16(x - 2)^2 - 100$	Down	Maximum	-100

A Quick Application

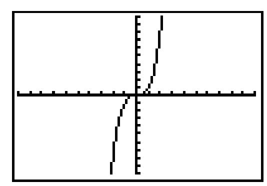
The maximum height is 259 feet. It takes 4 seconds to reach the maximum. The vertex is the maximum or minimum value of the function. Since **A** is negative, the parabola opens down and has a maximum.

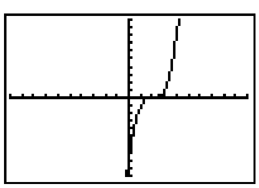
Homework Page

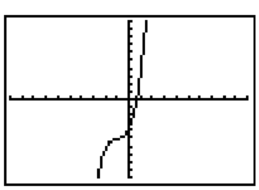
1. The second function *($y = x + 3$)* can be viewed either as a translation 3 units up or a translation 3 units to the left. If it is viewed as a translation 3 units up, you would be viewing the equation as $y = x + 3$. If you view it as a translation 3 units to the left (x-direction), you would be seeing the equation as $y = (x + 3)$.



2. The graph of $y = x^3 + 2$ is a vertical translation of $y = x^3$ two units up. The point at the origin should be moved to (0, 2).



3. 

4. 

EXPLORATIONS

Activity 4

Exploring Quadratic Data with Transformation Graphing

In Activity 3, you explored the effect of the parameters A, B, and C on the graph of the general quadratic function in vertex form, $y = a(x - b)^2 + c$. In this activity, through translation and dilation of the general parabola, you will try to "eyeball" fit data that can be modeled with a quadratic function.

This activity will serve as practice with both the vertex form of a parabola and "eyeball" modeling. Its purpose is to continue your study of the quadratic function (parabola) and its properties by developing quadratic models.

This is also a good opportunity to think about what makes a good or reasonable model.

When you move to Data Model 2, think about the function properties of domain and range and how important they are in modeling.

If you have not completed Activity 3, you should do so before starting this activity.

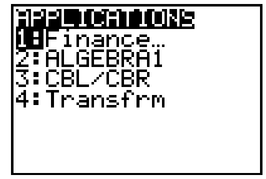
Data Model 1 - Ball Bounce

Equipment Needed

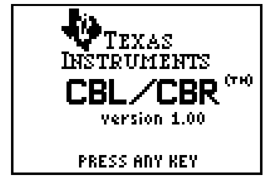
- TI-83 Plus with CBL/CBR and Transformation Graphing applications installed
- CBR
- Ball - a racquet ball or a basketball will work fine and are the ones that are usually used for this activity. Do not use a "fuzzy" ball such as a tennis ball.

Experiment Set-up

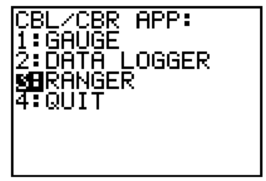
1. Attach the CBR to the TI-83 Plus with the unit-to-unit link cable.



2. Press [APPS] and choose **CBL/CBR**.
3. Press any key to advance past the introduction screen.

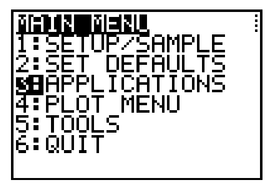


4. Select **3:Ranger**.

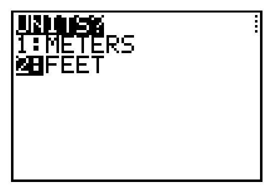


5. Press [ENTER] to advance past the introduction screen.

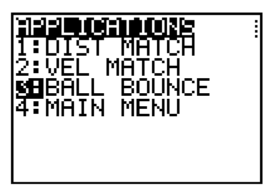
6. Select **3:APPLICATIONS**.



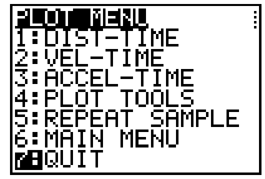
7. Select either **1:METERS** or **2:FEET**.

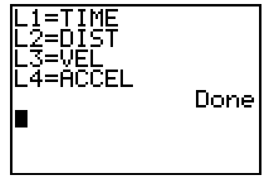


8. Select **3:BALL BOUNCE**. Follow the instructions with the **BALL BOUNCE** program.



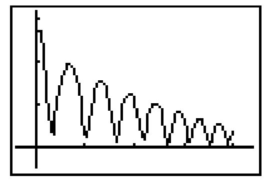
9. Once you have good data appearing on the calculator screen, exit the program by pressing [ENTER] **7:QUIT**. The data you will work with is stored in **L1** and **L2**.

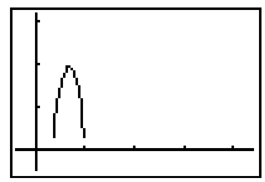




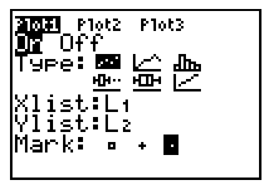
10. Use the built-in **Select()** feature of the TI-83 to select the first "good" bounce that is collected.

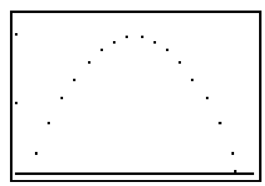
Note: Keystrokes are given below and on the next page.





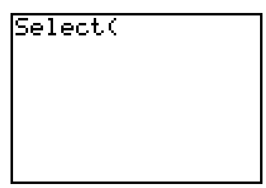
11. The **CBL/CBR** program leaves the data connected, press [2nd] **[STAT PLOT] 1:Plot1** and select the unconnected scatter plot option.

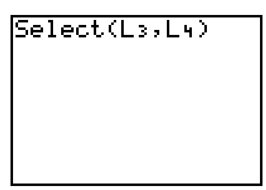




Keystrokes for Using the Built-In Select Feature

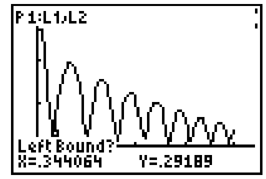
1. With the plot displayed, press [2nd] [LIST] [▸] **8:Select(** and enter where you want to store the selected data. To use **L3** and **L4**, press [2nd] [L3] [,] [2nd] [L4] [)] [ENTER].

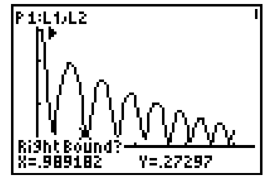


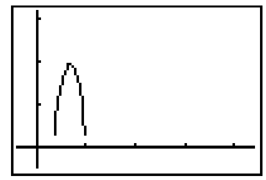


2. To actually select a part of the graph you will use, press [▶] to move to the left end of the data you want to keep. Press [ENTER]. This sets the left bound. Press [▶] to move to the right end of the data you want. Press [ENTER]. The selected data will be placed in **L3** and **L4**, and then this data will be displayed.

   *Note: You can also find instructions for using **Select()** and setting up Stat Plots in the TI-83 Plus Guidebook.*







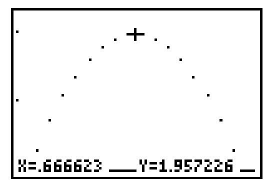
Developing the First Model

1. Using the free-moving cursor, try to position the cursor's + at the approximate location of the vertex. Record these coordinates.

   To use the free-moving cursor, press the cursor keys ([◀] [▲] [▶] [▼]) to move the cross hatch around the screen.

   a. Do you think the vertex will always be one of the points that is collected by the CBR? Why or why not?

   b. How does the fact that the parabola has an axis of symmetry help you estimate the location of the vertex?

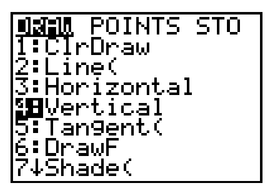


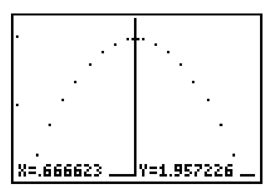
2. Using the **DRAW** feature, sketch the axis of symmetry through the point you have selected as the vertex. Press [2nd] [DRAW] **4:Vertical**. The line will be drawn by the TI-83 Plus at the same location as the cursor. Does the line appear to be the axis of symmetry? If not, move the line left and right with the cursor control keys until it appears where you envision the axis of symmetry.

   If you need to move your axis of symmetry, be sure to update the starting location of the vertex accordingly.

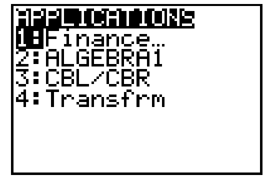
   Record the approximate coordinates of the vertex.

   X = ___ Y = ___

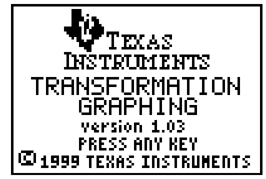




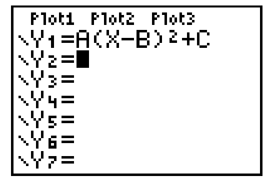
3. Press [APPS] and select Transformation Graphing by pressing the number at the left of **Transfrm.** Press any key (except [2nd] or [ALPHA]) to start Transformation Graphing.



*Note: If you do not see the screen illustrated at the right, select **2:Continue**).*

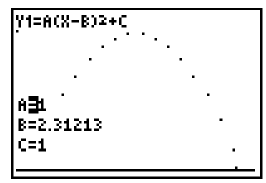


4. In **Func** mode, press [Y=] to display the **Y=** editor. Clear any functions that are listed. Enter the general vertex form of the quadratic function, **Y = A(X - B)**^{2} + **C**. Press [ALPHA] **A** [(] [X,T,Θ,n] [−] [ALPHA] **B** [)] [x^2] [+] [ALPHA] **C.**



   If the Play-Pause Mode is not selected at the left of **Y1** (>||), press [◄] until the cursor is over the symbol and then press [ENTER] until the correct symbol is selected.

5. Press [GRAPH] to display the plot and the graph. The graph will start with the most recent values of **A**, **B**, and **C**. These values do not necessarily relate to the problem you are working on and thus the graph might not appear on the screen.

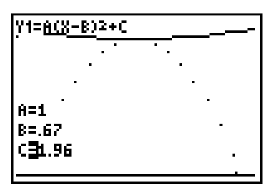


   a. Why do you think you recorded the approximate coordinate of the vertex? How will you use them in this activity?

   b. The "eyeball" method to find a model for quadratic data works quickest when you are working with the vertex form of the parabola. You have already recorded a starting point for the vertex and thus have starting values for **B** and **C**. Which value is **B** and which is **C**? Why?

6. Press [▼] to highlight **B=.** Enter your estimate. In this example, you would press [.] **6 7** [ENTER].

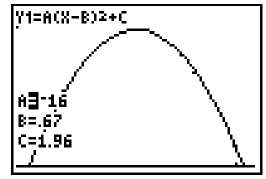
7. Press [▼] to highlight **C=.** Enter your estimate. For this example, press **1** [.] **9 6** [ENTER].



   a. You now have a parabola with its vertex matching the vertex of your data (in the example at .67, 1.96). You need a value for **A**. What would be a good starting value for **A**?

   Because the parabola is opening down, you need a negative value. If you have taken Physical Science, you know that the coefficient of the quadratic term for a free falling body is half the acceleration due to gravity. The acceleration due to gravity is approximately 32 ft/sec^2 or 9.8 m/sec^2. This may give you an idea for a starting value for **A**.

8. Press [▲] [▲] to highlight **A=**. Enter your estimate. In the example, press [-] **1 6** [ENTER]. How good is your model?



If your model is not as good as the one displayed, you should refine your model by selecting new values for **A**, **B**, or **C**.

Internet Research: Once you finish this activity, look up free fall or projectile motion on the Internet and discuss how this activity relates to what you find.

Homework Page

Name ____________________

Date ____________________

Stamp Prices

In early 1999, the U.S. Post Office raised the price of a first class stamp to $0.33. As it was in 1995 when the price was raised to $0.32, many people complained that there was no reason to increase the stamp prices. The table shows each year that the cost of a first class stamp has changed since 1958. Enter the data into the Stat Lists of your TI-83 Plus and then use Transformation Graphing to develop a quadratic model for the relationship between year and price.

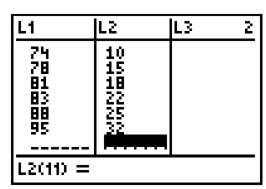
Use your model to predict the price of a first class stamp in 1999 and decide if the price increase seems justified.

Year	Cost of a First Class stamp
1958	4
1963	5
1968	6
1971	8
1974	10
1978	15
1981	18
1983	22
1988	25
1995	32

Source: Reprinted with permission from *World Almanac and Book of Facts, 1998*.
©1998, 2000 World Almanac Education Group, Inc.

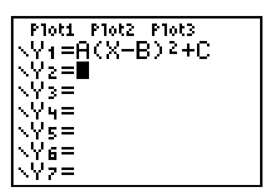
1. Which variable is the independent variable?

   a. Enter the data in the Stat List of your TI-83 Plus. Press [STAT] **Select 1:Edit.** Enter the values from the table into an empty list in the stat editor. Clear two lists if no lists are empty.

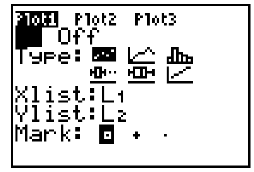
   

   Note: the example shows the year data without the first two digits. (1974 is entered and 74.)

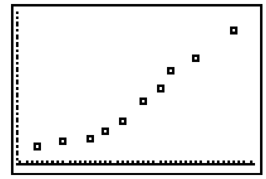
   b. Press [Y=] and deselect any functions that have been turned on.

c. To display the plot, press [2nd] [Stat Plot]. Select **1:Plot1**. Turn the plot on and set the plot menu as shown at the right.



d. Press [ZOOM] **9:ZoomStat** to display the plot.



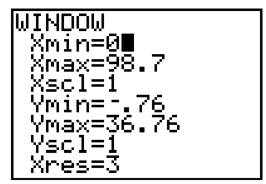
2. Does there appear to be a relationship between the year and the cost of a first class stamp? Which point(s) seems the most "out of line?"

3. Do you feel a quadratic is a reasonable model? Explain.

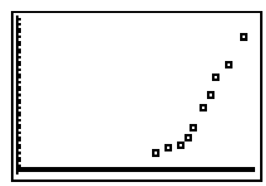
4. Would you visualize a different model if the data stopped with 1983?

Use Transformation Graphing and the general vertex form of a parabola to develop a model for the data.

Important: Before you start, press [WINDOW] and set **Xmin** = **0**. This will allow a better view of the data points and the coefficients with Transformation Graphing installed.



As a starting point, pick one of the given points as the vertex and then start refining your model.



Once you have found your model, answer the following questions.

5. What is your model? Why do you think it is reasonable?

6. What does your model predict for the price of a first class stamp in 1999? To have your calculator help answer this question, with the graph of your model showing, press [TRACE] [▲] **99** [ENTER].

*Note: your **Xmax** must be larger than 99 for this to work.*

7. Using your model, do you think the cost increase was reasonable? Explain mathematically.

8. Some people would say that, in theory, an exponential model would be better. Why do you think they would make this claim?

9. The table listed the years as 58, 63, … instead of 1958, 1963, …. Why do you think this was done? What type of operation was performed on the data?

10. What would you put in your table for the year 2000 or 2010?

11. Would you use your model to predict the cost of a first class stamp when the U.S. Post Office issued the first stamp? Hint: The first stamp was issued well before 1950. You might want to do an Internet search to learn the history of the Post Office Service.

Notes for Teachers

This activity is a perfect follow-up for the previous activity. In Activity 3, the student explores the effect of the various parameters on the graph of a parabola. In this activity, they will use what they have just learned to find a quadratic model using Transformation Graphing.

The main intent of this activity is to give the students practice with the parameters of the vertex form of the parabola and their effect on the shape of the curve. "Eyeball" fitting models helps reinforce these relationships and helps the student discover some of the properties of the particular curve being studied.

The methods used in this activity should be used before any more formal model development takes place. Doing "eyeball" fits also helps to develop a sense of when a model is reasonable.

Be careful not to look for one correct answer. Any reasonable model will have some strong points and can be justified even though no sort of formal optimization takes place.

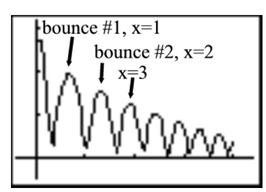
There are two data sets. Set 1 is collected by the students in the classroom. The instructions for this data set assume the use of a CBR but a CBL and motion detector can be used. If you are using the CBL, you will need the ball bounce program named **BOUNCE**. This program can be found on the TI website.

More detailed instructions on the ball bounce experiment can be found in the *CBL™ System Experiment Workbook* that comes with the CBL, or in the books *Real-World Math with the CBL™ System* or *Math and Science in Motion: Activities for Middle School,* available from Texas Instruments. The *CBL™ System Experiment Workbook* can also be downloaded for free from Guides section of the Texas Instruments web site at **education.ti.com.**

Hint: The instructions tell the student to use the free-moving cursor to approximate the vertex. It would probably be easier to go to the **DRAW** menu and use the vertical line to try to visualize the axis of symmetry. This was not done at the start of they activity as the attempt was to not bring in too many new or less familiar keystrokes early on, but to let them stay with familiar operations and thus concentrate more on the vertex form.

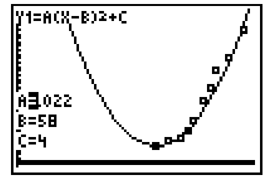
The models for the bouncing ball should all be very good and easy to find.

For the Future: You might want to save the ball bounce data and re-visit it when you study the exponential function. The data set formed by the number of the bounce and the maximum height of the ball for each bounce can be modeled with an exponential function.

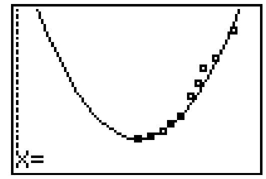


There are any numbers of reasonable models. Do not expect all students to get the same model, or in fact models whose predicted cost in 1999 are very close. Any reasonable model should be accepted.

Using the first data point (58, 4) as the vertex will be a very popular method. A reasonable model can be found using this approach. Both models shown here will predict a stamp cost of $.41 for 1999.



You might want to show your students how to use the "value" option under the CALC menu if you feel they do not already have this skill.

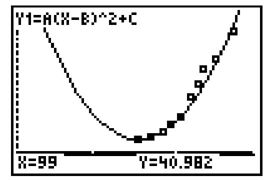


To use the value option:

a. With the graph showing, press [2nd] [CALC]**1:value**.

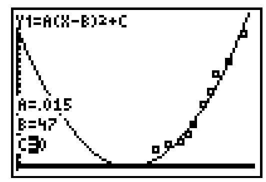
b. Enter the number you want to use to evaluate the function (in this example, **x = 99**).

c. Press [ENTER].

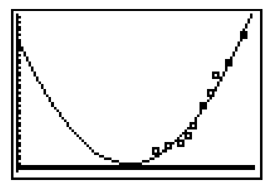


In the activity the same result was found using the **Trace** option.

Remember, there are any number of equally reasonable models. This model gives a predicted 1999 cost of $0.41.



Using the built-in regression gives a model in a different form *(y = ax*^{2}+ *bx* + *c*) that does not appear to be that much better then the eyeball models. The predicted cost for this model is $0.39.



An exponential regression on the data gives a model that predicts a cost of $0.50 in 1999, while a logistic regression predicts a cost of $0.34.

In any case, the cost increase does not seem to be out of line.

The quadratic model, no matter what they have, gives a perfect opportunity to discuss domain and range issues. If no restriction is placed on the domain any quadratic model will show that many years before those listed on the table the price of a stamp was much larger and that in fact the cost had dropped going into the 1940s or 1950s.

With unrestricted domains, the logistic or exponential models would make more sense for years previous to those in the table.

A couple of Internet searches are suggested during the activity. As a third Internet activity, you might have your students research the cost of stamps earlier than 1958, add that data to their plot, and then determine a new model. If you use this option, you might want to discuss the modeling concept of maintaining, updating, and revising a model.

Answers

Answers to most of the questions follow them in the activity. Those that do not are listed below.

Developing the First Model

1. a. There is no reason to believe that any particular point would be collected by the CBR. You will probably collect a point close to the vertex, which can be used as a starting place for the vertex, but not the vertex itself.

   b. Since the vertex is on the axis of symmetry, if you visualize the axis of symmetry you can more easily find the x-coordinate of the vertex. It is often easier to start looking for your model by visualizing the axis of symmetry because you can use all the collected points.

5. a. You can use this point as a starting place for **B** and **C**.

   b. **B** is the X-coordinate of the vertex and **C** is the **Y**.

7. Any value that is negative because the parabola opens down.

Homework Page

1. Date is independent, cost dependent.
2. Yes, there seems to be a relationship: as X increases, Y increases at an increasing rate. Looking at the total picture, the price increase in 1983 seems the most out of line.
3. A quadratic model seems reasonable, but it would have a restricted domain.
4. The year 1983 seems out of line with the total data set. If the data had stopped in 1983, you would probably have a steeper curve at the end.
5. Answers will vary. As an example **Y** = $.023(X - 58)^2 + 4$.
6. Answers will vary. For the example in #5, $0.43.
7. Answers will vary. For the example in #5, very reasonable. For probably any model you might have, the model will predict a price greater than $0.33.
8. Most people have heard that the cost of living grows exponentially. Thus they expect the cost of any particular item to grow exponentially over time also.
9. This performs a horizontal translation of the data to keep the numbers small. The quadratic relationship would hold before or after this translation.
10. The year 2000 would be 100 and 2010 would be 110. The data in the table has been shifted to the left 1900 years.
11. Answers will vary, but the model is working with a restricted domain. Many students will use the first data point (or somewhere close) as the vertex, which means any year before the vertex will have a higher cost. It is safe to assume the cost of a stamp did not drop in 1958.

EXPLORATIONS

Activity 5

Exploring the Exponential Function

When the value of a variable increases or decreases by a constant percent, the variable is said to change *exponentially*. When you have completed these activities, you will be able to identify exponential growth or decay from equations. In these activities, you will practice developing "eyeball" models for situations involving exponential growth.

If you invest money in a bank savings account, it grows by a constant percentage. For example, you might invest money at a simple 6% annual rate. If you invested this way, each year the bank would give you interest that amounted to 6% of the money in your account. Your money would grow exponentially.

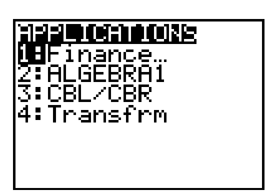
If you drop a ball and let it bounce, like you did in Activity 4, with each bounce it rebounds less and less and the ball gets lower and lower. However, each bounce rebounds at a fixed percentage of the previous height. This is an example of *exponential decay*.

If a cup of hot chocolate was removed from the stove and allowed to cool, the temperature of the chocolate over time would illustrate exponential decay.

Exponential growth and decay are all around us. In this activity, you will explore exponential functions of the form $y = ab^x$. In this form, the constant b is called the growth factor.

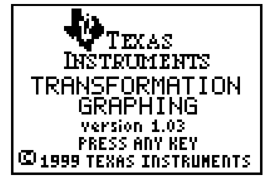
Investigating the Effect of a and b on the Graph of $y = ab^x$

1. Press [APPS] and select Transformation Graphing by pressing the number at the left of **Transfrm**.



2. Press any key (except [2nd] or [ALPHA]) to start Transformation Graphing.

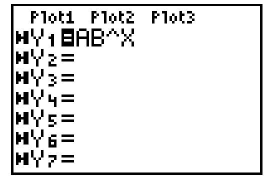
   *Note: If you do not see the screen illustrated at the right, select **2: Continue**.*



3. In **Func** mode, press [Y=] to display the **Y=** editor. Clear any functions that are listed and turn off any plots.

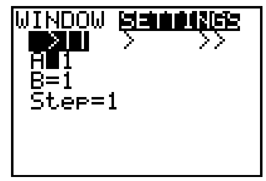
4. Enter **Y = AB**^{X}. Press [ALPHA] **A** [ALPHA] **B** [^] [X,T,Θ,*n*].

   If the Play-Pause Mode is not selected at the left of **Y1** (>||), press [◄] until the cursor is over the symbol; then press [ENTER] until the correct symbol is selected.

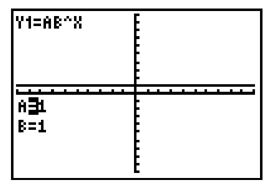


5. Press [WINDOW] [▲] to display the settings screen for Transformation Graphing.

   Set the settings as shown. To make these selections press [▼] **1** [▼] **1** [▼] **1**. This defines the starting values for the coefficients and the increment by which you want to observe the change in the coefficients.



6. Press [ZOOM] **6:ZStandard** to display the graph.



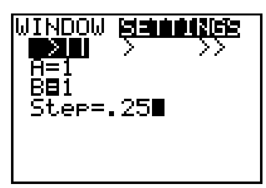
Questions for Discussion

1. The graph appears to be a line. Why? Explain your answer.
2. If **B** remains 1 and **A** changes, what will happen to the graph? Make a hypothesis and then press the cursor control keys (either [◄] or [►]) a number of times to check.
3. Changing the coefficient **A** does not seem to affect the rate of change of the exponential function? What does **A** affect?

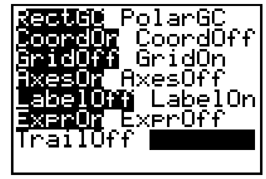
Investigating B > 1

1. Press [WINDOW] to display the settings screen for Transformation Graphing.

   Set the **SETTINGS** as shown. To make these selections, press [▲] [▼] **1** [▼] **1** [▼] **.25**. This defines the starting values for the coefficients and the increment by which you want to observe the change in the coefficients.



2. Press [2nd] [FORMAT] ▼ ▼ ▼ ▼ ▼ ▼ ► to highlight **TrailOn**. Press [ENTER]. This activates the Trail capability. With **TrailOn**, as the value of **B** is changed and the graph redrawn, a dotted line will remain in the location of the last curve.

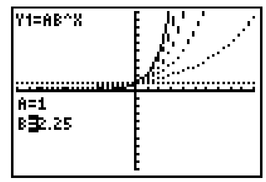


   *Note: **TrailOn** is a feature that exists only with Transformation Graphing.*

   You can use [2nd] [DRAW] **1:ClrDraw** to clear the trails or repeat the steps above and highlight **TrailsOff**.

3. Press [GRAPH] to return to the graph screen.

4. Highlight the **B=**. Press ► a few times to increase the value of **B**. Pause after each increase to notice the change in the graph. You only need to change the value about 5 times.



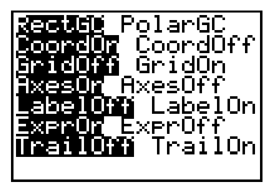
   What happens to the graph as **B** increases?

The value of **B** controls the rate at which the function grows and is called the growth factor. As **B** increases the function bends upward more steeply.

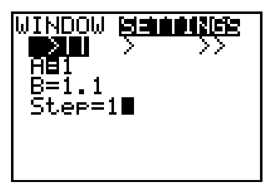
So far we have only investigated values of B>1. We will save our exploration of B<1 for later in these activities.

Investigating the Effect of A

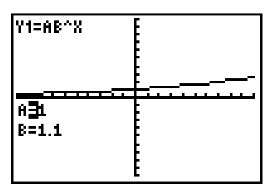
Before starting this investigation turn **TrailOn** to **TrailOff**. Press [2nd] [FORMAT] ▼ ▼ ▼ ▼ ▼ ▼ [ENTER].



1. Press [WINDOW] ▲ to display the settings screen for Transformation Graphing. Set the **SETTINGS** as shown.

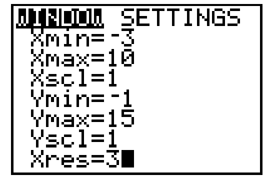


2. Press [GRAPH] to return to the graph screen. Highlight **A=**.



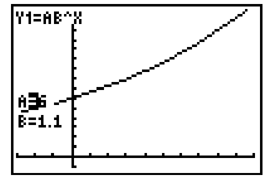
3. Press ► a few times to see the effect on the graph as **A** increases. Pause each time the curve is re-graphed and take note of the change. Pay special attention to the value of the y-intercept with each move.

4. Press [WINDOW] and change the graph window to match the graph window at the right.



5. Press [GRAPH] to re-display the graph with the new window.

6. Enter specific values for **A** by typing the number and then pressing [ENTER]. Pay special attention to the y-intercept. Enter the values 2, 6, 10.



What effect does **A** have on the graph of $\mathbf{Y = AB^x}$?

The value **A** is the y-intercept of the curve. The y-intercept is the value when $\mathbf{X = 0}$, $\mathbf{Y = A*B^0 = A*1 = A}$. The y-intercept of an exponential function is frequently called the starting value of the dependent variable, the variable that is growing exponentially.

As an example, if you invested $100 in a bank, at time t=0 you would have $100. To set up an exponential model for the growth of your money, you would let **A=100**, the starting value. The value of **B** would reflect the interest rate.

Now look at cooling as a second example. If the temperature of a cup of hot chocolate was 60°C when it was taken from a stove and then placed outside the house on a table where the temperature was 0°C, the starting point for our equation would be time=0, temperature = 60. Expressed as y= for the calculator, the value of A would be 150, the starting temperature. In this case, **B** would be affected by the rate at which the chocolate was cooling. The hot chocolate would be losing heat, an example of decay. The next activity will look at decay.

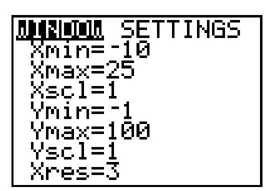
Revisiting B

You have considered exponential decay but have not viewed its graph. Before you proceed any further, look at an example of decay and see if you can sketch its graph.

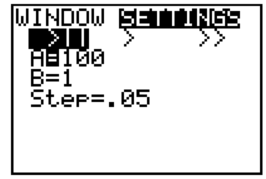
The maximum height of each bounce of a bouncing ball illustrates exponential decay. Assume the ball is dropped from a height of 6 feet and rebounds to half that height, or 3 feet. On the next bounce, the same thing happens, it rebounds halfway again, or 1.5 feet. This pattern continues until the ball no longer bounces. Draw a rough sketch of the maximum height of the ball with each bounce. Let **Y** be the height and **X** be the number of the bounce.

Now that you have an idea of how an exponential decay curve would look, let's continue our investigation.

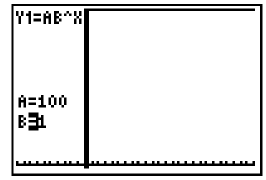
1. Press [WINDOW] and set the window settings as shown at the right.



2. Press the cursor up key (▲) until you see the settings menu for Transformation Graphing. Set the settings as shown at the right.



3. Press GRAPH to return to the graph window.
4. Press ▼ until the **B=** is highlighted.



5. Press ◄ several times to lower the value of **B**. Observe values of **B** between 1 and 0. Pause after each graph change and notice the effect on the graph. You might want to enable **TrailOn**. If you do, be sure to turn that feature off when you finish this particular part of the activity.

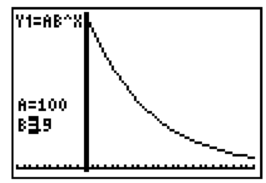
   What effect does lowering the value of **B**, in the range 0 < B < 1, have on the graph of the exponential function $Y = AB^x$?

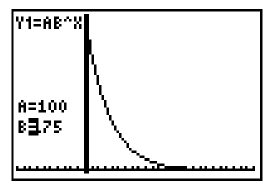
   When 0 < B < 1 the values of the function, **Y**, decrease as **X** grows. The closer to B = 0, the faster the **Y** values decrease and thus the faster the rate of decay of the graph.

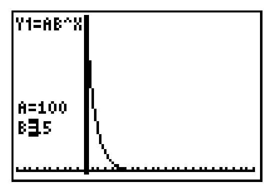
   *Note: **B** can not be negative. Allow **B** to be negative and investigate the function values with the table feature of your calculator.*

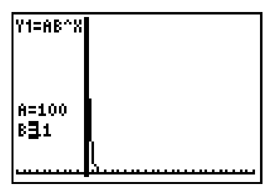
   Assuming A > 0 and 0 < B < 1, can Y ever be negative for $Y = AB^x$? Explain your answer.

   Since 0 < B < 1, the more you multiply **B** by itself, the smaller the term becomes, but it can never be less then 0. As an example take $.9^1$, $.9^2$, $.9^3$,…,$.9^{10}$, when you evaluate them you get .9, .81, .729,…,.349. The value of each successive term is getting smaller but will never reach 0. Thus as long as **A** is not negative there is no way for AB^x to ever be negative, only very small.







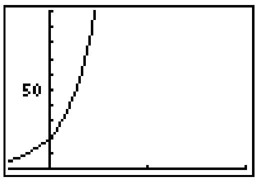


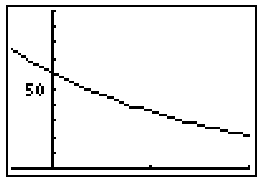
Curves that approach a value but never reach it are said to be *asymptotic.* The exponential equation $Y = AB^x$ is asymptotic to 0 because it approaches 0 as **X** gets large, but never reaches 0.

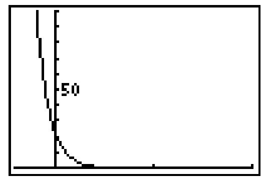
How Are You Doing?

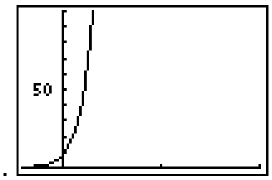
Each graph is an example of an exponential function. Tell:

1. If the graph is an example of growth or decay.
2. The value of A.
3. Is B < 1 or B > 1?

a. 

b. 

c. 

d. 

4. Which graph has the larger value for **B**, b or c? How about a or d? Explain your answers.

Homework Page

Name ______________________

Date ______________________

The table shows the population of the world, in billions, since 1940. Use Transformation Graphing to develop an exponential model for the data.

Internet Search: Look for world population on the Internet. Use the data you find if it is different from that shown in the table below.

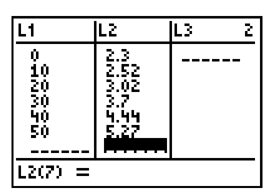
Year	Population (in billions)
1940	2.30
1950	2.52
1960	3.02
1970	3.70
1980	4.44
1990	5.27

Source: *The World at Six Billion*. Population Division, Department of Economics and Social Affairs, United Nations Secretariat. Working Paper ESP/P/W.154, New York, 1999. Reprinted with permission.

Before you enter the data, think about the parameters of the exponential function $Y = AB^x$. The value **A** is considered the starting value of the dependent variable. In this problem, the starting value is 2.3 billions when the year = 1940. But you would like the starting value to be when your independent variable, the year in this case, is 0. Subtract 1940 from each year.

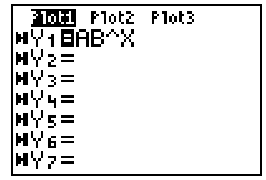
Year	Translated Year (x)	Population (in billions) (y)
1940	0	2.30
1950	10	2.52
1960	20	3.02
1970	30	3.70
1980	40	4.44
1990	50	5.27

1. Enter the data in the Stat List of your TI-83 Plus. Press [STAT] and select **1:Edit.** Enter the values from the table into an empty list in the stat editor. Clear two lists if no lists are empty.

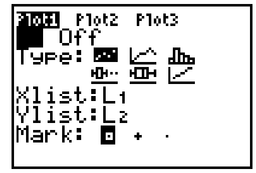


Note: To clear a list, move up to highlight the list name , press [CLEAR] [ENTER].

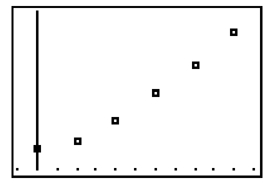
2. Press [Y=] and clear all functions. Enter the general form of the exponential function.



3. To display the plot, press [2nd] [Stat Plot]. Select **1:Plot1**. Turn the plot on and set the plot menu as shown. Press [ZOOM] **9:ZoomStat** to display the plot.

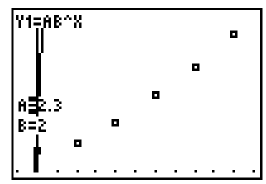


It appears that an exponential model might be appropriate.



4. Be sure Transformation Graphing is installed and in Play-Pause Mode. Press [GRAPH] to re-draw the plot with the graph.

5. Select starting values for **A** and **B**. As an example let **A** = **2.3** and **B** = **2**. Use the up/down cursor control keys to first highlight **A**= and enter **2.3**, and then highlight **B**= and enter **2**. This gives you a starting point in your investigation.



Should the value of **B** be larger or smaller than shown in the graph? Explain.

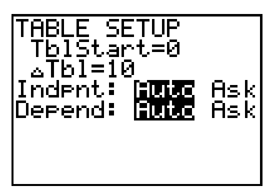
The graph seems to be growing too fast—it is too steep. The value of **B** should be less than 2.

6. Revise your values for **A** and **B** until you have a reasonable model. Revise **B** first and then revise **A**. Express **B** to the thousandth place to get a good model.

What equation did you decide is a good model for the data?

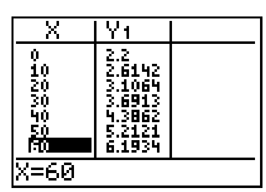
7. Once you have a reasonable model, use the table feature of your calculator to estimate the world population in the year 2000. Hint: If 1940 was entered as 0 in our table, the year 2000 should be 60.

To use the table feature press [2nd] [TBLSET] and set the **TABLE SETUP** as shown.



To read the table, press [2nd] [TABLE].

Note: Your table might have different values depending on your model.



8. What does your model predict the population will be in 2010?

9. What does your model predict the population was in 1930?

Modeling Practice

The population of the mystical city of Transform is shown in the table.

Year	Population
1970	125,000
1975	201,300
1980	324,000
1985	522,000
1990	841,000
1995	1,354,000

10. Find a model for the population of Transform.

11. What will the population be in 2000?

12. At what rate is the population growing?

    Note: Start by representing the year 1970 as 0.

Notes for Teachers

The purpose of this activity is to start a study of the exponential function. Once students have completed the activities they should be able to:

- recognize exponential growth or decay from an equation.
- tell which coefficient controls the rate of growth/decay.
- tell the effect of changes in the value of **A**.
- compare two graphs by looking at their equations.
- eyeball model a situation involving exponential growth.

There is no strong attempt to discuss the magnitude of B, just that it affects the rate at which the function grows. There is no work done with the actual rate; it is not meant to be part of the activity.

Optional Presentation Method

Activities 5 and 6 could be divided into three sections: growth, decay, and non-zero asymptotes. If you decide to use this division the sections would be as follows:

- *Growth:* Activity 5 from start to the section *Revisiting B*. The homework from Activity 5 would be homework for this section.
- *Decay:* Start where the new unit on Growth ends. Continue into Activity 6. End in Activity 6 with the section *Modeling the Experiment: Casting Out Sixes with Special Number Cubes.* The homework on page 61 would be homework for this section.
- *Non-zero Asymptotes:* This section would start where the previous section ended and continue for the rest of the activity. The homework for this section would be the CBL Practice Activity listed on page 63. This activity, *Experiment M5: Coffee To Go* can be found in the *CBL™ System Experiment Workbook*. This activity can also be downloaded for free from the Texas Instruments web site at **education.ti.com**. Any "cooling" experiment could also be used for this homework.

Warning!!!! This activity mentions the cooling of a cup of chocolate but *does not develop* the idea of asymptotes. On page 45, the statement about the hot chocolate has the chocolate starting at 60°C and cooling to 0°C. In this way the asymptote is at $y = 0$. It is felt that investigating decay completely, with non-zero asymptotes, would be too much for one set of activities. Also, most Algebra I courses do not deal with non-zero asymptotes. The discussion of asymptotes occurs in Activity 6. With this activity, you are trying to start the study of the exponential function and only want the students to realize there is decay and growth, and how to tell which is which.

Answers

Investigating the Effect of a and b on the Graph of $y=ab^x$: Questions for Discussion

1. When A=B=1, you have the equation $y = 1(1)^x$. No matter what value x takes, 1^x will remain 1. Thus the equation becomes y = 1(1) or the line y = 1. In this case there is no growth or decay.
2. Since B=1, you will continue to have the line y = A. The general form of the equation becomes $Y = AB^x = A*1 = A$. **B** is the growth factor; as long it remains 1, there is no change.
3. **A** is the Y-intercept.

Investigating B>1

4. As **B** increases the curve gets steeper, the function grows at a faster rate.

Investigating the Effect of A

6. **A** is the y-intercept of the curve.

How Are You Doing?

1-3 Growth, 20, b>1

Decay, 60, b<1

Decay, 20, b<1

Growth, 10, b>1

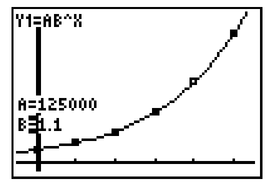
4. The value of **B** is greater in b because the curve doesn't bend as much, it decays slower. The value d is greater because the curve bends faster, it grows faster.

Homework Page

5. The value for **B** should be smaller. The graph is growing too quickly.
6. Answers will vary. $Y = 2.1(1.019)^x$ could be a reasonable model. Any reasonable answer should be accepted. There is no one correct answer. Answers in the neighborhood of $y = 2.2(1.0174)^x$ are exceptionally good. With exponential growth, the number of decimal places is very important.
7. The prediction of the population in 2000 should be in the 6 billion range. Be sure students label the answer as billions.
8. Approximately 7.4 billion. Be sure students label the answer as billions.
9. Approximately 1.85 billion. Be sure students label the answer as billions.

Modeling Practice

10. A reasonable model is $y = 125000(1.1)^x$.
11. The population in 2000 will be approximately 2,181,000.
12. The population is growing at approximately 10% per year.



X	Y1
0	125000
5	201314
10	324218
15	522156
20	840937
25	1.35E6
30	[illegible]

Y1=2181175.28361

EXPLORATIONS

Activity 6

Modeling Exponential Decay with a Look at Asymptotes

In the previous activity, you started your study of the exponential function, modeling exponential growth. In this activity, you will model exponential decay and learn more about asymptotes.

You should work Activity 5 before you begin.

Modeling the Experiment: Casting Out Sixes

Perform the following experiment. Before each roll, record the number of rolls that have taken place and the number of dice left.

Equipment Needed

- TI-83 Plus with Transformation Graphing installed.
- 36 number cubes (dice)

Experiment Steps

1. Count the number of dice.
2. Pick up all the remaining dice and roll them.
3. Remove any die that shows a 6 on its top face.

Number of rolls	Number of dice left
0	36
1	

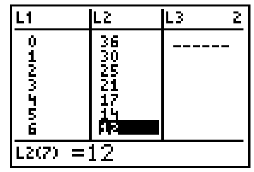
Repeat the previous steps until no dice remain or you have rolled 15 times, whichever occurs first.

Sample Results:

Number of rolls	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of dice left	36	30	25	21	17	14	12	11	9	8	6	5	3	3	2	2

Use the data you collected for your analysis.

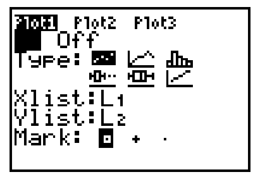
1. Enter the data in the Stat List of your TI-83 Plus. Press [STAT] and select **1:Edit**. Enter the values from the table into an empty list in the stat editor. Clear two lists if no lists are empty.



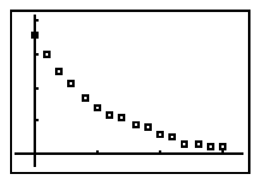
   Note: To clear a list, use the arrow keys to highlight the list name, then press [CLEAR] [ENTER].

2. Press [Y=] and clear all functions.

3. To display the plot, press [2nd] **[Stat Plot].** Select **1:Plot1.** Turn the plot on and set the plot menu as illustrated.



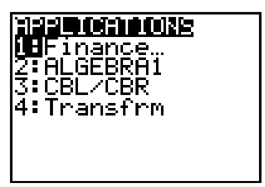
   Press [ZOOM] **9:ZoomStat** to display the plot.



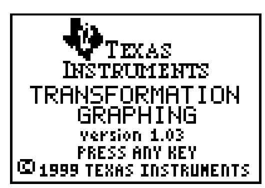
   Looking at the plot, does an exponential model appear to be reasonable? Does the problem situation lend itself to an exponential model?

   Yes, you should be taking approximately 1/6 of the dice away each time, so there seems to be a constant percent decrease. The curve should be pseudo-asymptotic to Y = 0.

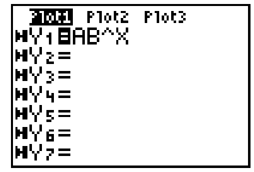
4. Press [APPS] and select Transformation Graphing by pressing the number at the left of **Transfrm.** Press any key (except [2nd] or [ALPHA]) to start Transformation Graphing.



   *Note: If you do not see the screen shown at the right, select **2:Continue**.*



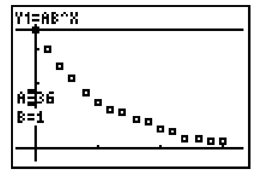
5. In **Func** mode, press [Y=] to display the **Y=** editor. Enter the general form of the exponential function, **Y1 =** $\mathbf{AB^x}$. Press [ALPHA] **A** [ALPHA] **B** [^] [X,T,Θ,*n*].



   If the Play-Pause Mode is not selected at the left of **Y1** (>||), press [◄] until the cursor is over the symbol and then press [ENTER] until the correct symbol is selected.

6. Press [GRAPH] to re-draw the plot and the graph.

7. Select an appropriate value for **A**.

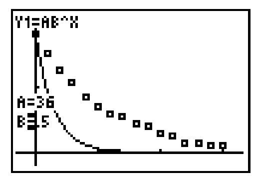


   If you started with 36 dice, then the data point (rolls = 0, dice left = 36) seems reasonable. The value of **A** in an exponential function is the value of the dependent variable, (in this case number of dice) when the independent variable is 0.

   To start with A=36, press the cursor up key [▲] until **A**= is highlighted. Enter **3 6** [ENTER].

   Does the y-intercept seem appropriate? If yes, continue. If no, enter a new value.

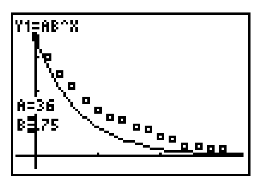
8. You could make a very reasonable estimate for **B** for this particular experiment before you start. For practice, assume that you have no idea of the value of **B**. Here you will learn to systematically try values for **B** and enter them in Transformation Graphing as a check. When the model looks reasonable, stop revising **B** and check the reasonableness of the final value.



   Since the data appears to be an example of decay, **B** must be in the region 0 < B < 1. To start let **B = .5**, the middle value in the interval.

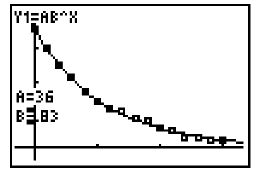
   Press [▼] to highlight **B**=. Press [.] **5** [ENTER].

9. Does your model appear to be reasonable? If yes, revise **A**. If not, test a new value for **B**. In the example, the data does not seem to be decaying as quickly as the model. Therefore, **B** must be greater than .5. **B** must be in the region .5 < B < 1. As a second guess let **B = .75**, the middle of the revised interval.



   Press [.] **7 5** [ENTER]

10. Evaluate the model again. If you think you can get a better model, continue until you feel you have as good a model as you can find. Find **B** to two decimal places.



By repeated, but systematic, guess and check, a reasonable model for the sample data would be $Y = 36(.83)^x$.

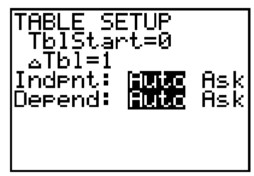
The model has **B=.83**. Does this value make sense with respect to the problem? Explain.

The sample data never reached zero. An exponential model never reaches 0. But the actual experiment would reach zero. How can we use this model to estimate when the dice should all be picked up?

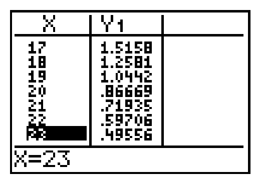
To determine when there should be no dice left, determine when **Y< .5**. The assumption is that as long as **Y** rounds to 1, there will be dice left. When it rounds to less than 1, we will assume no dice are left.

Use the table feature of the calculator to determine when **Y<.5**.

11. To use the table to find **Y<.5**, press [2nd] [TBLSET] to use the **TABLE SETUP** menu. Set the menu as shown at the right.



12. Press [2nd] [TABLE], press [▼] to move down until a value less than .5 appears in the **Y1** column.



The model predicts it will take approximately 23 rolls to remove the dice.

It is true that the exponential model never reaches zero and this situation actually does, but this model is reasonable for the situation. Some data sets are said to be *pseudo-asymptotic.* This is one of those situations.

Note: The experiment is a discrete data set but the exponential function is a continuous model. This fact should be taken into account when the model is used.

What does the note mean and was the model used properly?

Modeling the Experiment: Casting Out Sixes with the Special Number Cubes

Perform the following experiment. Before each roll, record the number of rolls that have taken place so far and the number of dice left.

Equipment Needed

- TI-83 Plus with Transformation Graphing installed
- 36 "special" number cubes available from your teacher

Experiment Steps

1. Count the number of dice.
2. Pick up all the remaining dice and roll them.
3. Remove any dice that show a 6 on their top face.
4. Repeat the steps until no dice remain or you have rolled 15 times, whichever occurs first.

Number of rolls	Number of dice left
0	36
1	

Sample Results:

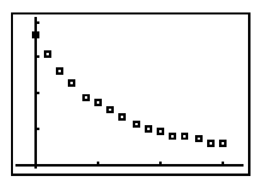
Number of rolls	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of dice left	36	31	26	23	19	17	15	13	11	10	9	8	8	7	6	6

Use the data you collected for your analysis.

Solution

Note: Keystrokes that are the same as were needed for the first part of this activity will not be repeated.

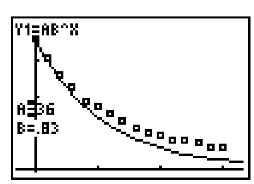
1. Plot the data.



2. Does it appear that an exponential function would be a reasonable model?

   Yes, although the data does not necessarily appear to be asymptotic to **Y = 0**. Compare this plot to the data plotted in the last activity. This set appears to remain higher.

3. Activate Transformation Graphing and enter the general exponential model, **Y = AB**^{x}. The last values entered for **A** and **B** were from the previous dice problem. The two problems appear the same; thus these values should be reasonable.



4. Does the model appear reasonable? Not really. All exponential decay curves are asymptotic, but the data and the proposed model do not appear to have the same asymptote. The asymptote for the data appears to be larger than the **Y= 0** from the previous examples.

   Since the dice were all 6-sided, the probability appears to be correct. The value **B=.83** should be reasonable.

   There are two possible methods to improve the model: try different values for **B**, or try asymptotes different from **Y = 0**.

   Observing the previous examples of exponential decay, the problem appears to be the wrong asymptote.

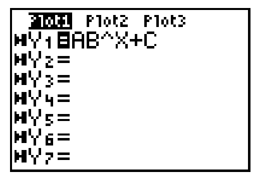
5. The equation for an exponential model with an asymptote other then Y = 0 is $Y = AB^x + C$ where **C** is the value of the asymptote.

   B remains the growth or decay factor.

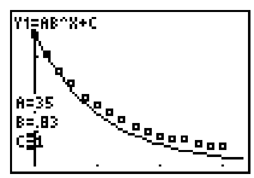
   A is the starting point after adjustments for a non-zero asymptote. But it is no longer the y-intercept. The y-intercept is now A + C. When X = 0 the equation becomes $Y = A + C$, ($B^0 = 1$ for all values of **B**).

   For this example $A + C = 36$, since we started with 36 dice.

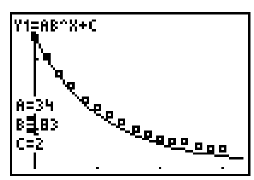
6. Revise your model to $Y = AB^x + C$. Try combinations of A and C where $A + C = 36$.



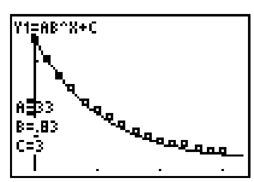
   A = 35 C = 1



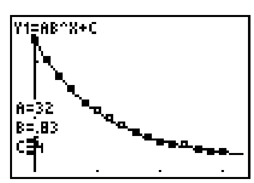
   A = 34 C = 2



   A = 33 C = 3



   A = 32 C = 4

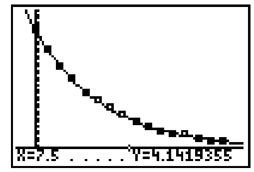


Do any of the revisions appear to depict a reasonable model? Yes, when *A = 32* and *C = 4*.

In this example, the asymptote will be *Y = 4*. This means no matter how many times the dice are rolled there will always be 4 remaining.

7. Use the table feature of the TI-83 Plus to determine when the last die containing a 6 was removed.

A Method for Visualizing the Asymptote



With the graph and stat plot showing, press [2nd] **[DRAW]** **3:Horizontal**. Use the [▼] to move the horizontal line downward with until it appears to be the asymptote. This method will give a rough idea of the location of the asymptote. You could have used this method to get a starting value for **C** when you started to develop your model.

Note: Horizontal asymptotic behavior is an end behavior. With exponential decay it is best observed with large values of x.

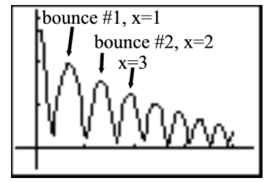
8. How do you explain the existence of a non-zero asymptote?

Homework Page

Name ________________________

Date ________________________

The data for this activity comes from the data that was collected in Activity 4. If you saved the ball bounce data in Activity 4, reload it now. If the data was not saved, redo the Ball Bounce data collection.



The data set formed by the number of the bounce and the maximum height of the ball for each bounce can be modeled with an exponential function. Trace along the data set and record the apparent maximum for each bounce. Record the points as (bounce number, bounce height).

1. When tracing the data, did you find the actual maximum of each bounce?

2. What is the asymptote for the ball bounce experiment?

3. Find a model for this new data set.

Notes for Teachers

The purposes for these activities are:

- Re-enforce the meaning of the parameters a and b in the general exponential equation, $y = ab^x$.
- Introduce the non-zero asymptote form of an exponential function, $y = ab^x + c$.
- Give the students practice using Transformation Graphing systematically as a modeling tool.
- Give the students an example of a situation where a continuous model is used for a discrete experiment.
- Give the students a second look at asymptotes, particularly non-zero asymptotes.

The first activity is worked out almost in its entirety with sample data to give the students their first actual systematic look at modeling with Transformation Graphing. Students should repeat this experiment in small groups and go through the same steps with their own data. This will allow for a rich discussion since each group will have a different set.

The first part of this activity introduces the idea of discrete data sets being used with continuous function models. This concept is very important and should be a point of emphasis.

The sample data was very carefully collected. Students must be sure that with every roll all the dice truly roll. Also for good data, you need good dice.

This activity introduces the student to the opportunity to use modeling to try to understand a situation, not check to see if the data fits a situation.

Equipment for Experiment 2

The students will need dice without a six. The teacher decides how many dice with 6's and how many without each group gets.

- You can get dice like these at some joke, game, or magic stores. The best dice will have 1 - 5 with one number appearing twice.
- A second way to get dice like these is to make them out of cubes that can be bought at party stores or mathematics supply houses. The cubes are called number cubes and you put the numbers on yourself. If you use this method put the numbers 6 - 11 on most of the dice and 7 - 12 on a few.
- A third method would be to take a few of the dice and a drill and drill the two sides to look like the five side. Then do "casting out 2's."
- Another way is to take actual dice and put colored dots on them. Use 6 colors and tell them to pick up a die when a blue dot appears on top. Be sure not to put any blue dots on some dice and one blue dot per die on most of them.

- You can also make them yourself from molding you can buy at a hardware supply store.

Be sure to have your students observe all the graphs of exponential decay so they can visualize the difference between an incorrect asymptote and an inappropriate value for **b**.

In both experiments the students probably stopped before all die were removed. After they predict how many rolls would be needed to remove all the die, have them continue where they left off in the process. Remember the second experiment will not go to zero. They roll until, in our example, four dice remained.

The second activity may be done with or without the students being aware of the nature of the "special dice" set. Doing the experiment "blind" provides an opportunity for a true investigation. You might give different groups different mixes from the "special dice" set. One group might have 4 of the non-six dice, while another has 2. This will allow for different non-zero asymptotes.

CBL Practice Activity

A good CBL activity that can be used as a follow-up to this Activity is:

Experiment M5:Coffee To Go from the *CBL™ System Experiment Workbook.* The workbook was packaged with your CBL. It can also be downloaded for free from the Texas Instruments web site at **education.ti.com.**

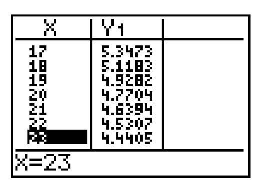
Answers

Modeling the Experiment: Casting Out Sixes

10. **B** is the growth or decay factor. B shows what percent of the dice should remain after each roll. Since there are 6 possible faces and only 1 has a 6, the probability a die will remain is 5/6. Thus 5/6 of the dice should remain with each set of rolls. 5/6 = .83, which is the percentage that would be expected.

Modeling the Experiment: Casting Out Sixes with the Special Number Cubes

7. The last die that had a six on it would be removed, in this sample, approximately on the 23rd roll.



8. Some of the dice had no sixes. In the sample, there were 32 dice with a 6 and four dice with no 6. Therefore, four dice will never be removed.

Homework Page

1. You do not necessary have the maximum, but you have a value that is close to the maximum. Do not assume any special property for any point collected with a CBR.
2. The asymptote is 0. The ball will eventually stop bouncing. It is actually a pseudo asymptote.
3. Answers will vary depending on the data set used.

EXPLORATIONS

Appendix

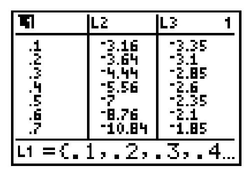
Commonly-Used Keystrokes for Curve Fitting/Data Analysis

This section outlines keystrokes that you will need to be familiar with to work the activities in this book.

Entering Data

Data is entered and displayed from the STAT lists. To enter the data:

1. Press [STAT] **1:Edit**. This will take you to the stat editor.
2. If there is any data presently in the lists that you wish to use, you should first clear the lists. To do this, move the cursor up so it highlights the heading on the list you would like to clear. Press [CLEAR] [ENTER] to clear the list. Repeat these steps on any list you want to clear.

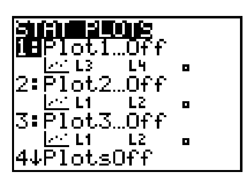


3. Once the lists are clear, move the cursor to the list where you want to place the data and enter the numbers one at a time. Be sure to press [ENTER] after each number is entered. Once one column is entered, move to the next column and start data entry again.

Displaying the Data as a Scatter Plot

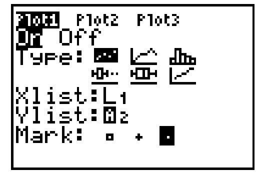
This assumes you already have the data entered.

1. Press [2nd] [STATPLOT] to display the STAT PLOT menu.
2. Turn off any plots that are on by selecting **4:PlotsOff** and pressing [ENTER].



3. If you had to turn off any STAT PLOTs in step 2, press [2nd] [STATPLOT] again.

4. Press **1** to activate the Plot 1 menu.
5. Use the cursor to move to each highlighted object. Press [ENTER] to set your calculator as shown at the right. When your cursor is at **Xlist**, press [2nd] [L1] and then move to **Ylist** and press [2nd] [L2].



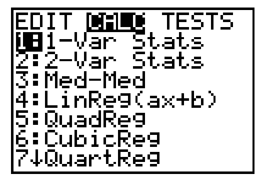
6. Once your calculator settings match the screen at the right, press [ZOOM] **9:ZoomStat** to plot the points.

Fitting an Equation to the Data

Note: In my class the students are never allowed to use the built-in regression methods until:

- they can fit the same type of curve with other methods, and
- they understand (or at least we have discussed) how the calculator does each fit.

1. Press [STAT] [▸] to display the STAT CALC menu. This menu shows the type of curves the calculator can fit to data sets.

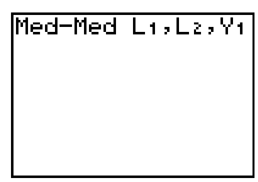


2. Press the number associated with the type of curve you want to fit to the data.

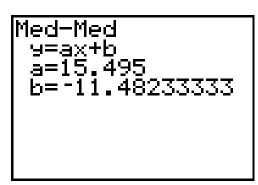
3	Median Fit	**9**	Natural Log
4	Linear Regression	**0**	Exponential
5	Quadratic	**A**	Power
6	Cubic	**B**	Logistic
7	Quartic	**C**	Sine
8	Linear Regression in y = ax + b form		

For example, if you want to fit a median-fit line to the data you would select **3:Med-Med.** This will return you to the HOME screen with the type of curve showing and the cursor blinking, waiting for more input.

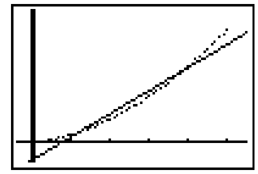
3. You need to tell the calculator which two lists you would like to fit an equation to and where you want to store the equation. If you want to let x = L1, y = L2, and the equation to be placed in Y1 you would press: [2nd] [L1] [,] [2nd] [L2] [,] [VARS] [▸] **1:Function 1:Y1.**



4. When you press [ENTER], the screen will show the equation of that particular family of functions that best fits the data.

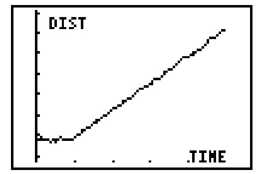


5. Press [ZOOM] **9:ZoomFit** and data will be displayed with the curve superimposed on the plot.



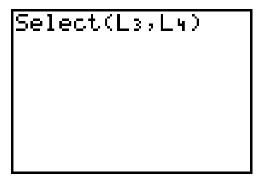
Using the Built-in Select Feature of the TI-83

This data is fairly representative of what might be recorded by a person walking away from a CBR. The data shows two parts. One part is when the person was standing still before they started. The second part is when they actually started to walk. Let's say you want to only work with the second part of the data.

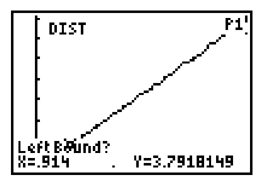


The depicted data is in L1, L2. We will select it out the second part of the data and place it in L3, L4.

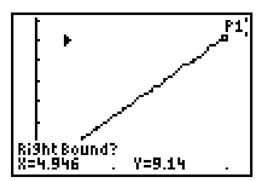
1. Press [2nd] [LIST] [▸] **8:Select(**. Now you enter where you want to store the selected data. To use L3, L4, press [2nd] [L3] [,] [2nd] [L4] [)] [ENTER].

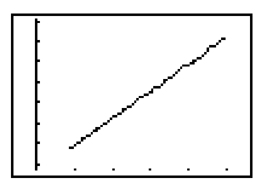


2. To actually select the part of the graph you will use, press [▸] to move to the left end of the data you want to keep and press [ENTER]. This sets the left bound.



   Move to the right end of the data you want and press [ENTER]. This selects the data you want, places it in L3, L4, and displays L3, L4.

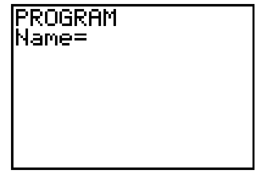




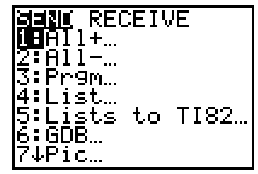
Keystrokes for Linking Two TI-83 Calculators

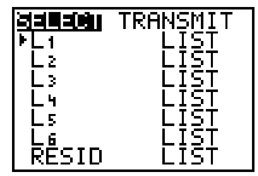
Connect the two calculators with the link cable. *Push both ends of the cable in firmly.*

1. On the RECEIVING calculator press: [2nd] [LINK] [▸] to display the RECEIVE menu. Press **1** to select **1:Receive.** The message **Waiting…** will be displayed.

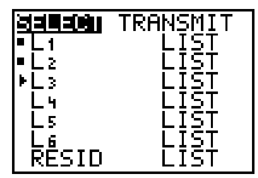


2. On the SENDING calculator press: [2nd] [LINK] to display the SEND menu. Press the number in front of the type of object you want to send. For this example, assume you want to send **L1** and **L2**. To do this, press **4:List**.

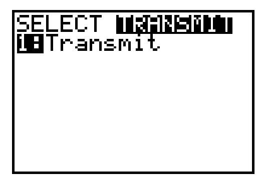




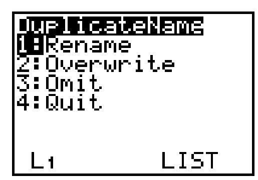
3. Press [▾] to move down the list, pressing [ENTER] when you are in front of each item you want to send. In our example, you would press [ENTER] in front of **L1** and **L2**. The selected items will have a shaded box in front of them when they have been selected.



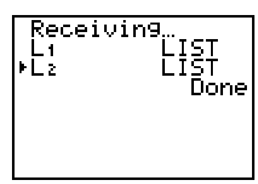
4. Once you have marked the items you want, press [▸] to move to the right and display the TRANSMIT menu. Press [ENTER]. This will start the transmission.



5. Watch the screen of the receiving calculator. If there is already an object with the same name, you will get a message saying you are trying to link a DUPLICATE OBJECT. Press **2:Overwrite** to overwrite the object and thus get the new object. This will erase the old object with that name. If you needed to keep the old object, press **4:Quit**, then rename the old object and start the process again.



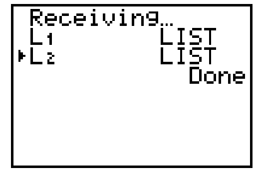
6. Once the data is transferred, you will see a DONE message.



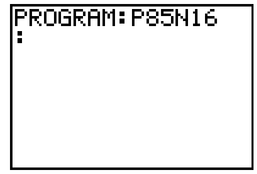
Turning Data into a Program

Note: These instructions assume the data you intend to save is in L1, L2.

1. Press [PRGM] [▸] [▸] [ENTER].



2. Type the name you want and then press [ENTER].

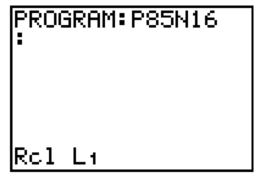


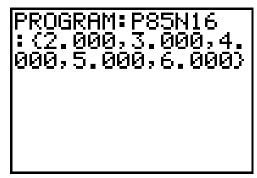
3. Press [2nd] [RCL] [2nd] [L1] [ENTER] [2nd] [RCL] [2nd] [L1] [ENTER].

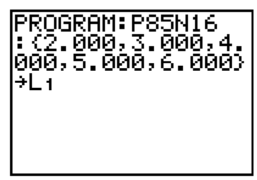
   This recalls the data in L1 and puts it in your program.

   Press [2nd] [RCL] [2nd] [L2] [ENTER] [2nd] [RCL] [2nd] [L2] [ENTER].

   This recalls the data in L2 and puts it in your program.







4. Press [2nd] [QUIT] to exit.

   This ends the simplest version of the program. The data has been placed in the program. When the program is executed the data will be placed back in L1, L2.

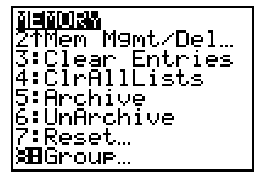
TI-83 Plus Grouping and Archives

Saving Objects as Groups

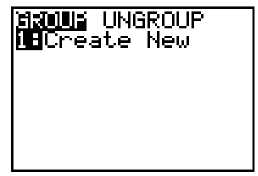
You have just finished a class where you used a CBL program (BALLDROP) to collect data for a freefalling object. The data is in L1, L2. You fit a model to the data (Y1) and the bell is about to ring. You need to clear this information out of the calculator and prepare for your next class.

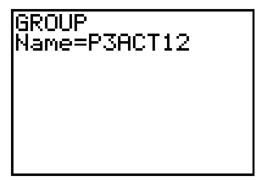
Your options are to run to a computer and save everything, or save it in archive memory as a group on the TI-83 Plus.

1. Press [2nd] [MEM] **8:Group**

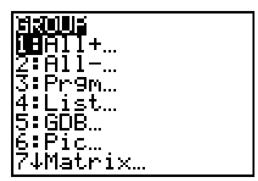


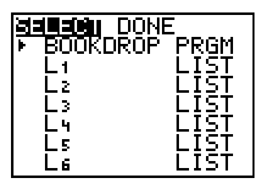
2. Select **1:Create New** and enter the name you want for the total data you are about to save. In this example, I named it **P3ACT12** for period 3 activity 12. Press [ENTER].



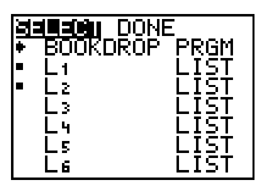


3. Select what you want to save with this name. This works the same as the TI-GRAPH LINK. Since I am storing three different type of object (list, equation, program), I pressed **2:All-.**



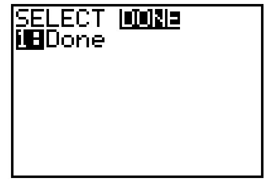


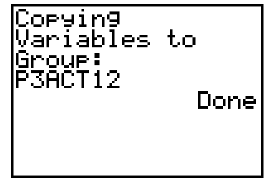
4. Move down the list, pressing [ENTER] beside any object that you want stored. When they are selected, the ▸ will turn into a ■.



5. When you have finished your selection, press [►] and [ENTER]. You are finished.

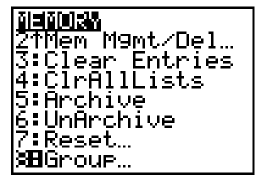
   The group **P3ACT12** has now been saved in archived memory.

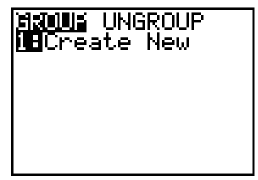




Ungrouping

1. Press [2nd] [MEM] **8:Group.**





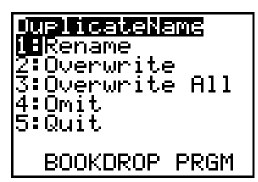
2. Press [►] to select UNGROUP.

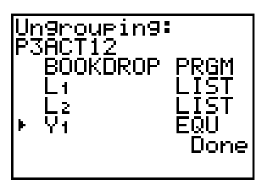
   *Note the * by the names that shows they are in archived memory, not the regular RAM.*

3. Press [▼] to move down and select the group you want. Press [ENTER] when you are beside an object you want to unarchive.

   If the object you are bringing back already exists in RAM, select **2:Overwrite** [ENTER].

   Note: Before you ungroup, be sure to group anything that is in your memory that you want to save.





To Archive or Un-Archive

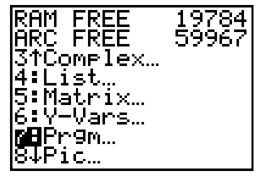
1. Press [2nd] [MEM] **2:Mem Mgmt/Del**.

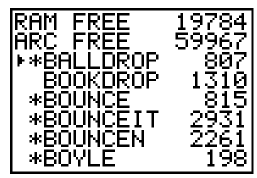
2. Press [▼] to move down to select the type of object you are going to either archive or unarchive.

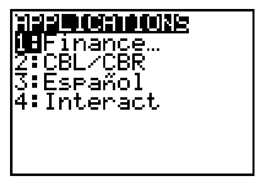
   To look at programs, for example, press **7:Prgm**.

   You now see a list of all the programs. Those marked with a ∗ are already in the archive and thus do not take up the working RAM. Those with no ∗ are in the working RAM. To change from archived to unarchived, cursor down to the object you want to change and press [ENTER].

   You now see a list of all the programs. Those marked with a ∗ are already in the archive and thus do not take up the working RAM. Those with no ∗ are in the working RAM. To change from archived to unarchived, cursor down to the object you want to change and press [ENTER].







Once an object has been unarchived, it can be used. Until it is unarchived, it cannot be used.

Note 1: Ungrouping unarchives the objects in the group. Grouping makes an archived copy of the objects.

Note 2: An application is run without unarchiving it.

