Modeling Garbage and Recycling

Finding good topics for modeling is difficult, but garbage is a problem everywhere. We will mathematically investigate the problem of how garbage increases and how clean-up and/or recycling can help solve it.

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Where are you from?

What do you teach?

<table>
<thead>
<tr>
<th>Math, STEM, Science, Other</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>Higher</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>
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Consider using a conversation “hook”

• Throw a ball of paper at the trash can and ask, “How much paper do you think our school trashes every day?”
• Ask, “Where does our school’s (household’s) garbage go? Is there a lot of garbage? Is it a problem?”

Look up some statistics

  • 4.4 pounds: The amount of trash generated daily, on average, by every American. Packed in cubed feet it would be the height of the Leaning Tower of Pisa.
  • 254 million tons: The amount of trash that Americans generate in a year.
  • 22 billion: Plastic bottles thrown out yearly.
Let’s look at a hypothetical problem...

Ann went to the park one day to find someone had dumped 100 pounds of garbage. Frustrated, Ann picked up 20 pounds, but that’s all she had time to get.

If, each night, someone dumps 100 more pounds of garbage, but each day, Ann brings one more additional friend and each person picks up 20 pounds of garbage, will they ever be able to clean up the park?

Think about what happens each day...

Ann went to the park one day to find someone had dumped 100 pounds of garbage.

Frustrated, Ann picked up 20 pounds, but that’s all she had time to get.

• The next day someone dumped another 100 pounds of garbage on top of the 80 pounds from the day before, but Ann brought a friend to help her pick up garbage. So, by the end of day 2, although there was 180 pounds of garbage, they were able to pick up 40 pounds of it, so they left 140 pounds for the next day.

• At the beginning of day 3, there were 240 pounds of garbage, but Ann brought 2 friends, so they were able to clean up 60 pounds.

• Each night, someone continues to dump 100 more pounds of garbage, but Ann continues to bring one additional friend each day and each person can clean up 20 pounds of garbage.

Take a minute or two to think about how your students might solve this problem... 😊

Based on the grade you teach, what would your students try?

• Would they look at it numerically?

• Would they use a table?

• Would they look at a graph?

• Would they make equations?

• Would they look at sequences?
Suggestions...

• Make a table or chart to organize the numerical data.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trash Dumped</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Daily Trash</td>
<td>100</td>
<td>180</td>
<td>240</td>
<td>280</td>
<td>300</td>
<td>300</td>
<td>280</td>
</tr>
<tr>
<td>Picked up</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>140</td>
</tr>
<tr>
<td>Trash left</td>
<td>80</td>
<td>140</td>
<td>180</td>
<td>200</td>
<td>200</td>
<td>180</td>
<td>140</td>
</tr>
</tbody>
</table>

• Finish the table.
• Look for patterns...
  • Trash dumped—what is happening?
  • Picked up—what is happening?

Let’s create lists... and graph!

Let’s see the additional lists...
Let’s look at the graphs...

Can we write functions?

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>100</td>
<td>180</td>
<td>240</td>
<td>280</td>
<td>300</td>
<td>300</td>
<td>280</td>
</tr>
<tr>
<td>Trash</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>140</td>
</tr>
<tr>
<td>Picked up</td>
<td>80</td>
<td>140</td>
<td>180</td>
<td>200</td>
<td>200</td>
<td>180</td>
<td>140</td>
</tr>
</tbody>
</table>

• Look for patterns first.
  • Initial trash – what is happening?
  • What type of function is this?
  • Picked up – what is happening?
  • What type of function is this?
  • Total trash left – what is happening?
  • What type of function is this?

How can we find these equations?

• Use the regression equation capability.
• Use Quick-Plot and Fit?
• Use the transformation APP.
• Let’s look at sequences.
• Do they know the summation formula?
How well do our functions model the data...

Check to see what we notice from the graphs as compared to our data.
Look at the table – or lists.

Suggestions...

- Consider a different table...

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total trash deposited</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>400</td>
<td>500</td>
<td>600</td>
<td>700</td>
</tr>
<tr>
<td>Total trash picked up</td>
<td>20</td>
<td>20+4</td>
<td>0</td>
<td>60</td>
<td>60+60</td>
<td>120</td>
<td>120+80</td>
</tr>
<tr>
<td>Trash left</td>
<td>80</td>
<td>140</td>
<td>180</td>
<td>200</td>
<td>200</td>
<td>180</td>
<td>140</td>
</tr>
</tbody>
</table>

If total trash grows linearly and trash pick up grows quadratically, will we always be able to clean up the trash?

Let’s look at Sequence Mode!
Let's look at the sequence mode...

How can we find these equations?

• Do they know the summation formula?
Remember this table...

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>400</td>
<td>500</td>
<td>600</td>
<td>700</td>
</tr>
<tr>
<td>Total trash deposited</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Total trash picked up</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>140</td>
</tr>
<tr>
<td>Trash left</td>
<td>80</td>
<td>120</td>
<td>160</td>
<td>200</td>
<td>200</td>
<td>180</td>
<td>140</td>
</tr>
</tbody>
</table>

Sum of an arithmetic sequence is

\[ S_n = \frac{(\text{first term} + \text{last term}) \cdot \text{number of terms}}{2} \]

- Total daily trash deposited: \( u(n) = 100n \)
- Daily picked up: \( 20n \)
  - Total trash picked up \( v(n) = (20 + 20n)n/2 = 10n + 10n^2 \)
  - Trash left: \( w(n) = u(n) - v(n) = 100n - (10n + 10n^2) = -10n^2 + 90n \)

Extensions!

1. What if someone was dumping 200 pounds of garbage each day instead of 100 pounds? Will the group be able to clean the park? How long will it take?
2. What if someone dumped 500 pounds of garbage a day instead of 100 pounds?
3. *Look at this in functional notation and explain why this happens.

Follow up with student's project. They research and find data about a particular garbage problem they find interesting – perhaps the Pacific garbage dump, your county's garbage problem, the amount of tires, mattresses, clothes, straws, phones, computers, etc. being trashed every day/year. They should then create a model to help solve or at least lessen the problem and use mathematics to determine/predict the effect of that model.
Extensions!

4. What if the garbage dumpers started with 100 pounds of garbage, but instead of dumping the same amount each day, they dumped 10% more garbage than they had dumped the day before? Will Ann ever be able to clean up the park? If so, how many days will it take?

5. What if the garbage dumpers started with 100 pounds of garbage, but dumped only 5% more garbage than the day before? Will Ann ever be able to clean up the park? If so, how many days will it take?

6. *Look at the functional notation here and explain why these results are different.

**Follow up with student's project. They research and find data about a particular garbage problem they find interesting – perhaps the Pacific garbage dump, your county's garbage problem, the amount of tires, mattresses, clothes, straws, phones, computers, etc. being trashed every day, year. They should then create a model to help solve or at least lessen the problem.
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Make a table or chart...

- Use named lists.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Trash</td>
<td>100</td>
<td>180</td>
<td>240</td>
<td>280</td>
<td>300</td>
<td>300</td>
<td>280</td>
</tr>
<tr>
<td>Picked up</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>140</td>
</tr>
<tr>
<td>Trash left</td>
<td>80</td>
<td>140</td>
<td>180</td>
<td>200</td>
<td>200</td>
<td>180</td>
<td>140</td>
</tr>
</tbody>
</table>

- Finish the table.
- Look for patterns first.
- Initial trash – what is happening?
- Picked up – what is happening?
- Insert actual calculator table.

Let's look at a hypothetical problem...

Ann went to the park one day to find someone had dumped 100 pounds of garbage. Frustrated, Ann picked up 20 pounds, but that’s all she had time to get.

- The next day someone dumped another 100 pounds of garbage on top of the 80 pounds from the day before, but Ann brought a friend to help her pick up garbage. So, by the end of day 2, although there was 180 pounds of garbage, they were able to pick up 40 pounds of it, so they left 140 pounds for the next day.

- At the beginning of day 3, there were 240 pounds of garbage, but Ann brought 2 friends, so they were able to clean up 60 pounds.

- If, each night, someone continues to dump 100 more pounds of garbage, but Ann continues to bring one additional friend each day and each person can clean up 20 pounds of garbage, will they ever be able to clean up the park? If so, how many days will it take? Will they be able to keep the park clean?