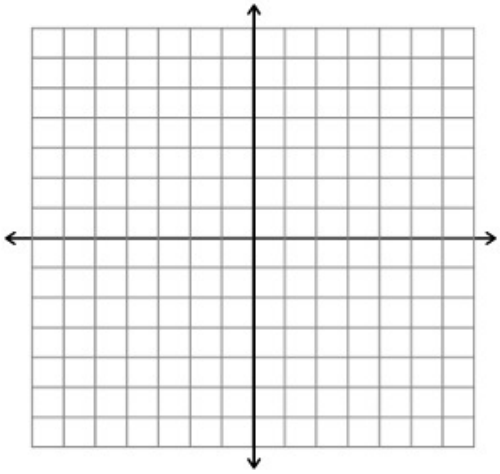
Growth Investigation: Linear vs. Exponential

1. On your calculator, graph these functions:

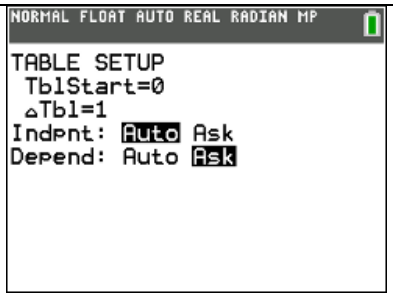
Y1= 2x

Y2= 2^x

What do you observe?



2. Press [2nd] [window] to access [tablset].
Set the Independent variable to AUTO and the Dependent variable to ASK



Look at the Table for these functions. Move down the column for each, pressing [enter] to find the value.

How does the Y-value change for **Y1=** 2x?

How does the Y-value change for **Y2=** 2^x?
At what value(s) of x does Y1 = Y2?

At what point(s) do the graphs intersect?

X	**Y1**=2x	**Y2**=2^x

3. Some values for the functions f and g are shown in the table. One of the functions is linear. The other is exponential. Use the patterns to complete the remaining entries in the table.

How are f and g similar to each other?

How does the Y-value change for each?

Which function increases faster?

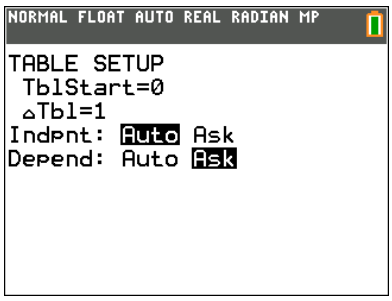
X	**f**	**g**
0	2	2
1	12	20
2	22	200
3	32	
4		
5		20000
6	62	
7		

4. Which scenario grows faster?

A. You start with $150 and save $2 each week.	B. You start with $5 and each week and double the amount you saved the previous week.

Write an equation for each.
Enter them in [y=].

In [tablset], set the Independent variable to ASK and the Dependent variable to AUTO. This time, as you enter the X-value, the Y-values will fill in.



How much money do you have after 1 week in each scenario?

When does scenario B catch up with scenario A?

5. Which population grows faster?

A. You have a population of 100 guppies, which increases by 20 guppies each year.	B. You have a population of 65 frogs, which increases by 20% each year.

Write an equation for each.

How many of each will there be in 5 years? (Round to whole numbers).

After how many years do the frogs first overtake the guppies? Report to the nearest year.

6. Which would you rather have?

Job A. Salary of $25,000 with an 8% raise each year.	Job B. Salary of $25000 with a $3000 raise each year.

Write an equation for each.

When will Job A have a higher salary than Job B? Report to the nearest year.

Teacher Notes

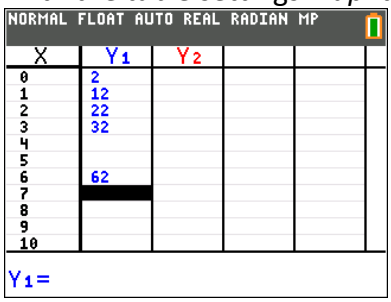
Answers to questions and suggested teaching strategies follow.

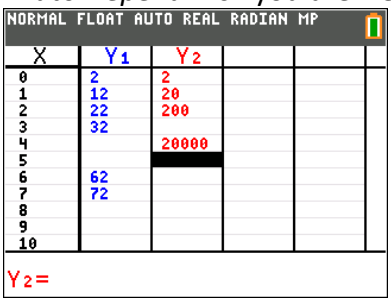
1. Students may observe which graph is linear and which is exponential. They appear to intersect.
2. Y1 climbs by 2. Y2 doubles. The graphs intersect at x = 1 and x =2, i.e., at points (1, 2) and (2, 4).
3. The teacher enters the following in the display calculator, but does not show students (yet).

   Y1= 2+10x

   Y2= $2 \bullet 10^x$

   With the table settings *Indpnt: Auto Depend: Ask* you are free to ask for any output.





   Y1 = f and Y2 = g are similar in that they have the same initial value of *y* = 2 at x =0.

   For f, the difference of consecutive outputs is 10. Alternatively, the next value is 10 more than the previous.

   For g, the ratio of consecutive outputs is 10. Alternatively, the next value grows by a factor of 10.

   Challenge students to find the equation for each.

   Ask students to observe the role that 2 and 10 play in each formula.

On the second page of the activity we use tables to find intersection points and relate the functions to a real world context. With the table settings *Indpnt: Ask Depend: Auto* you are free to ask for any input. This facilitates "table walking," i.e., producing the next pair of outputs one row at a time. Using this setting might be easier to find intersection points.

Note that you are also free to produce nonconsecutive values and can delete any row you wish.

4. B grows faster. **Y1=** 150+2x and **Y2=** $2 \bullet 2^x$

   After 1 week, scenario A has $152 and B has $10.

   At week 5 scenario B catches up with A.

X	Y1	Y2
0	150	5
1	152	10
2	154	20
3	156	40
4	158	80
5	160	160

5. B grows faster.

   Y1= 100+20x and **Y2=** $65 \bullet 1.20^x$. In 5 years, we have 200 guppies and 162 frogs.

   Frogs first overtake guppies between years 7 and 8.

X	Y1	Y2
0	100	65
1	120	78
2	140	93.6
3	160	112.32
4	180	134.78
5	200	161.74
6		
7		
8		

X	Y1	Y2
0	100	65
1	120	78
2	140	93.6
3	160	112.32
4	180	134.78
5	200	161.74
6	220	194.09
7	240	232.91
8	260	279.49

X	Y1	Y2
0	100	65
1	120	78
2	140	93.6
3	160	112.32
4	180	134.78
5	200	161.74
6	220	194.09
7	240	232.91
7.1	242	237.19
7.2	244	241.56
7.3	246	246

   Extension: When do frogs catch up with guppies to the *nearest 0.1 year*?

   Answer: *x* = 7.3 years.

6. After 11 years Job A pays better.

   Y1= $25000 \bullet 1.08^x$ and **Y2=** 25000+3000x

   Extension: How many years, *to the nearest 0.1 year*, does it take for your salary for Job A to surpass Job B?

   Answer: *x* = 10.8 years.

X	Y1	Y2
0	25000	25000
1	27000	28000
5	36733	40000
10	53973	55000
11	58291	58000
10.5	56091	56500
10.6	56524	56800
10.7	56961	57100
10.8	57401	57400

Strategies to Help Make It Stick

Maze Task

One way to help elicit if students can recognize if a table represents a linear function or an exponential function is to use a "maze technique" shown below. Students circle one of the words in each bracket and complete the blank to make the sentence correct. It may be helpful to clarify that "consecutive outputs in a table" means the inputs are 1 apart ($\Delta x=1$).

Advantages of the template in the box below:

- It can help students see the similar structure between the functions.
- It helps them reflect and generate the connection between the behavior in the table and the parameters in the formula.
- It is easy for a teacher to calibrate if the student is correct at a glance.

The linear formula is written $\boldsymbol{y = b + mx}$ intentionally to compare the parameters in the order of the appearance (initial value followed by how it changes), even though the formula $\boldsymbol{y = mx + b}$ is more traditional. This assessment could also follow the reading of a piece of relevant text.

For a **linear** function $\boldsymbol{y = b + mx}$, the $\left\{\begin{matrix}\textit{sum}\\ \textit{difference}\\ \textit{product}\\ \textit{ratio}\end{matrix}\right\}$ of consecutive outputs in a table is constant and is

equal to the value of $\left\{\begin{matrix}b\\ m\end{matrix}\right\}$. This is the $\left\{\begin{matrix}\textit{slope}\\ \textit{growth or decay factor}\end{matrix}\right\}$ of the linear function.

The value of $\left\{\begin{matrix}b\\ m\end{matrix}\right\}$ represents the *y*-intercept or initial value at *x* = ___.

For an **exponential** function $\mathbf{y = a \bullet b^x}$, the $\left\{\begin{matrix}\textit{sum}\\ \textit{difference}\\ \textit{product}\\ \textit{ratio}\end{matrix}\right\}$ of consecutive outputs in a table is constant and is

equal to the value of $\left\{\begin{matrix}a\\ b\end{matrix}\right\}$. This is the $\left\{\begin{matrix}\textit{slope}\\ \textit{growth or decay factor}\end{matrix}\right\}$ of the exponential function.

The value of $\left\{\begin{matrix}a\\ b\end{matrix}\right\}$ represents the *y*-intercept or initial value at *x* = ___.

Reversals

By reversing the directions you can create open questions which can help deepen students' understanding.

1. Give an example of a table and formula of a linear function and an exponential function which have the same initial value. Construct a real works scenario which would fit the mathematical model.
2. Give an example of a table and formulas of a linear and exponential function which have the same initial value and also intersect at the value *x* = 5.