

Activity Overview

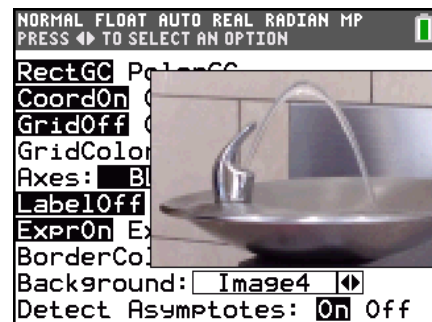
In this activity, students determine the model for the path of a parabolic stream of water analytically as well as by using the Quick Plot – FitEq feature.

Topic: Quadratic Functions

- Use the symmetry of the graph of a quadratic function to find its equation.
- Explore how the equation of the model varies with a choice of axes and how the values of a , b , and c in the equation $y = ax^2 + bx + c$ relate to the graph of the parabola.

Teacher Preparation and Notes

- Prior to this activity, students should be able to:
 - report the x -coordinate of the vertex of a parabola and its axis of symmetry if given its zeros by looking for the value of x midway between.
 - find the factored form of a quadratic function if given its zeros and another point on the graph.
 - expand the factored form of a quadratic function to write it in standard form $y = ax^2 + bx + c$



Compatible Devices:

- TI-84 Plus C Silver Edition
- TI-84 Plus CE

Click [HERE](#) for Graphing Calculator Tutorials.

Problem 1 – Modeling the Stream of a Water Fountain: Analytical Solution

Do not give students the window settings to produce screen shown just yet. Instead, ask students to estimate points on the graph of a parabola through the origin using only the photo on the handout. This will provide an opportunity to use analytical methods to produce a model, as shown below, which can be confirmed (and improved) with features of the calculator in Problem 2.



1. If each tick mark represents one unit, what is a reasonable estimate for the other zero, to the nearest integer? **Ans:** -6
2. Lauren now has both zeros of her parabola. Using the image above, what other points might be on the graph? Complete the table to the right with reasonable estimates.
Ans: (The table may give one possibility. The y-coordinate of the vertex should be close to 3.5 or 3.6.)
3. Using what you know about the symmetry of a parabola, report the equation of its axis of symmetry. **Ans:** $x = -3$

x	y
-6	0
-5	2
-4	3
-3	3.5 or 3.6
-2	3
-1	2
0	0

4. Given the two zeros of the parabola and one other point, you can use algebra to find its equation. Choose another point from the table to create an appropriate model of the parabolic stream of water.

Ans: Using the zeros 0 and -6, students could use the factored form $y = ax(x+6)$ and substitute a point in the table to find a . For example, if they substitute $x = -1$ and $y = 2$, then

$$2 = a(-1)(-1 + 6)$$

$$2 = a(-5)$$

$$a = -2/5 = -0.4 \text{ so } y = -0.4x(x+6).$$

They will have the same equation if they substituted the point $(-5, 2)$, $(-4, 3)$, $(-2, 3)$ or $(-3, 3.6)$

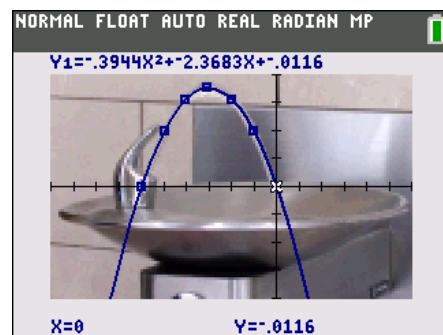
Problem 2 – Using QuickPlot&Fit-EQ for Lauren's Choice of Axes

Students set up the viewing window to position the axes so that the parabola passes through the origin and use **QuickPlot&Fit-EQ** to drop points on the image and confirm their analytical solution.

- The value of c in the equation of the model should be near 0. Why?

Ans: In the equation $y = ax^2 + bx + c$, the y -intercept of the graph will be $(0, c)$. Since the axes were chosen so that the graph passes through the origin, c should be near 0.

The figure to the right confirms c is the value of y when $x = 0$. (Press TRACE, press the down arrow key to be on the parabola, and press the number 0.)



- How do the values of a and b in the **QuickPlot&Fit-EQ** regression equation compare with those of your model found analytically in Problem 1?

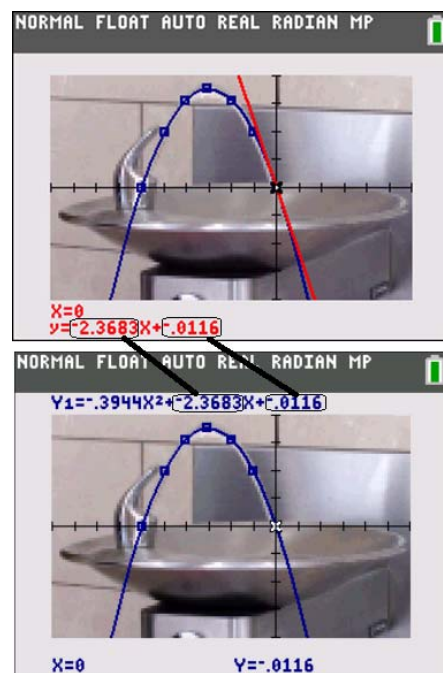
Ans: Students should assess how well their equation matches the analytical fit from Problem 1 by converting its factored form, i.e., $y = -0.4x(x+6)$, to standard (or expanded) form $y = -0.4x^2 + 2.4$. The values $a = -0.4$ and $b = 2.4$ should be similar to the **QuickPlot&Fit-EQ** regression equation.

Calculus Extension - Draw a tangent line at the origin.

A class discussion may develop about the meaning of the value of b in the equation $y = ax^2 + bx + c$, namely the slope of the curve at the origin. You can foreshadow this concept to precalculus students without finding the derivative.

The first derivative of $y = ax^2 + bx + c$ is $y' = 2ax + b$ so the slope of tangent line at the origin will be identical to the value of b in the equation from **QuickPlot&Fit-EQ**.

In general, the parabola with formula $y = ax^2 + bx + c$ will have the same y -intercept as the tangent line at the origin, i.e., $(0, c)$.



Problem 3 – Using QuickPlot&Fit-EQ for Warren’s Choice of Axes

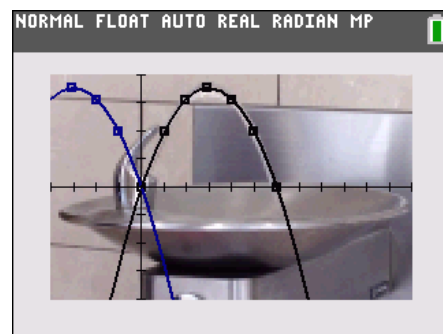
- How would his value of a in his quadratic equation compare with the value of a found in Lauren’s model? Why? **Ans:** They should be the same since the parabolas should have the same shape.
- How would his value of b compare with the value of b found in Lauren’s model? Why? **Ans:** They should be opposite in value due to symmetry.

Students graph both models in the same viewing window.

How are these graphs related?

Ans: The graphs are horizontal reflections of each other about the y -axis.

You may want to discuss how their equations compare, i.e., they are of the form $y = ax^2 + bx$ and $y = ax^2 - bx$ or, if 0 and r are zeros, $y = ax(x + r)$ and $y = ax(x - r)$.



Extension:

- What viewing window might position the y -axis on the vertex?

Ans: There are many possible answers.

If students modify the window in Problem 3 ($X_{\min} = -4$, $X_{\max} = 12$, $Y_{\min} = -4$, $Y_{\max} = 4$) they could subtract 3 units from X_{\min} and X_{\max} , i.e. $X_{\min} = -7$, $X_{\max} = 9$, $Y_{\min} = -4$, $Y_{\max} = 4$

- How would this affect the equation of the model?

Ans: The value of a would be the same as in all of the other models, since the parabola has kept its shape

However, the value of b would be 0 and the y -intercept would be the vertex. Its zeros would be -3 and 3 .

One possible model would be

$y = -0.4(x - 3)(x + 3)$ which, when expanded becomes $y = -0.4(x^2 - 9) = -0.4x^2 + 3.6$.

