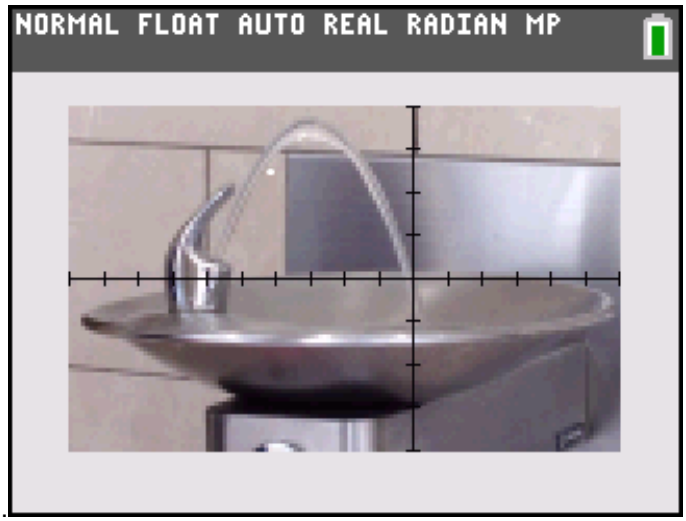
Problem 1 – Modeling the Stream of a Water Fountain: Analytical Solution

Lauren wants to overlay the graph of a parabola onto the photo of the drinking fountain. She has many ways she can position the axes. If she wants the graph to pass through the origin, one possibility is shown to the right.



1. If each tick mark represents one unit, what is a reasonable estimate for the other zero, to the nearest integer?

2. Lauren now has both zeros of her parabola. Using the image above, what other points might be on the graph? Complete the table to the right with reasonable estimates.

x	*y*
−6	
−5	
−4	
−3	
−2	
−1	
0	

3. Using what you know about the symmetry of a parabola, report the equation of its axis of symmetry.

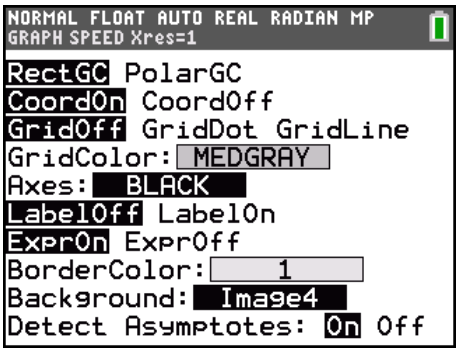
4. Given the two zeros of the parabola and one other point, you can use algebra to find its equation. Choose another point from the table to create an appropriate model of the parabolic stream of water.

   y = _____

Problem 2–Using QuickPlot&Fit-EQ for Lauren's Choice of Axes

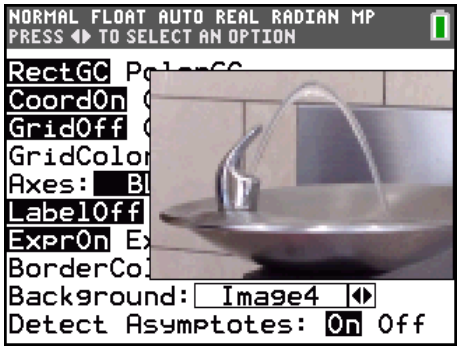
Press [2nd][FORMAT] to access the Graph Format menu.

Press the down arrow key to reach the Background.



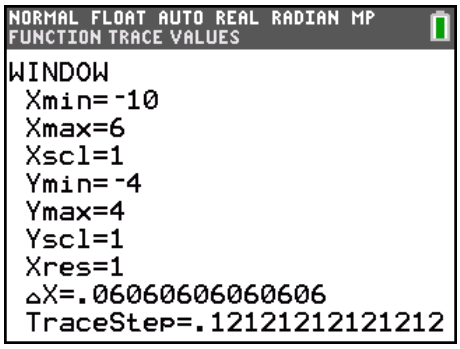
Press the left or right arrow keys to view the options. Image variables are shown, followed by choices for solid color backgrounds.

Press the up or down arrow keys once you have made your selection.



Press [WINDOW] and enter the values shown.

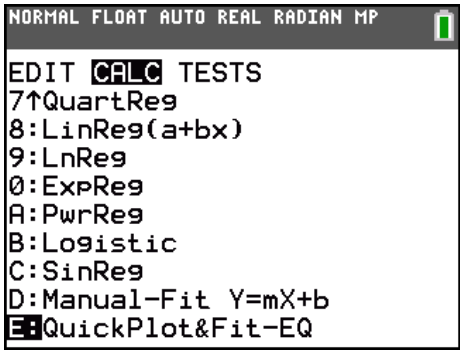
Press [GRAPH] to see the background and axes displayed.



Press [STAT].

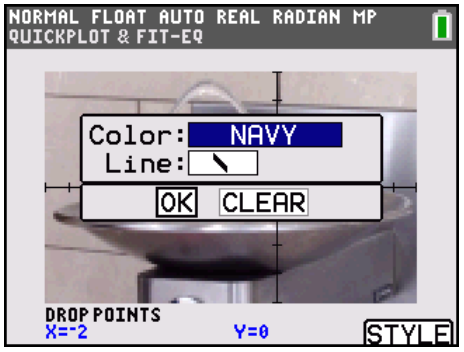
Press the right arrow to reach the **CALC** menu.

Scroll down to **E:QuickPlot&Fit-EQ**. Press [ENTER].



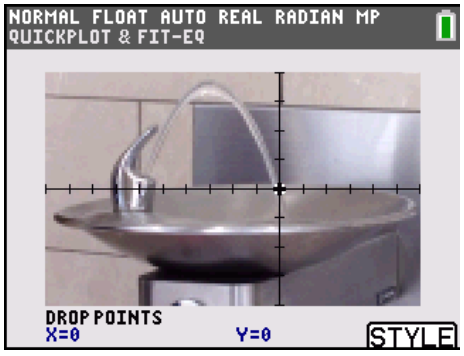
Press the **STYLE** soft key. Use the left and right arrow keys to change the color of the drop points and the equation. (The *x*- and *y*-coordinates will also change to the selected color.) You can also change the line style.

Highlight **OK** and press [ENTER].



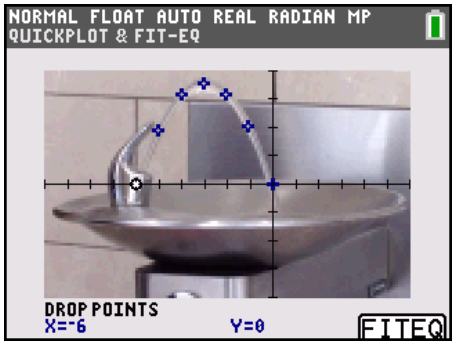
Move the cursor to the point (0, 0).
Press enter to drop the point.

Continue moving the cursor and dropping points on the graph. The screen coordinates that will be displayed are based on your values for Xmin, Xmax, Ymin, and Ymax. Do not feel you need to restrict yourself to use the values that you wrote in the table in Problem 1.

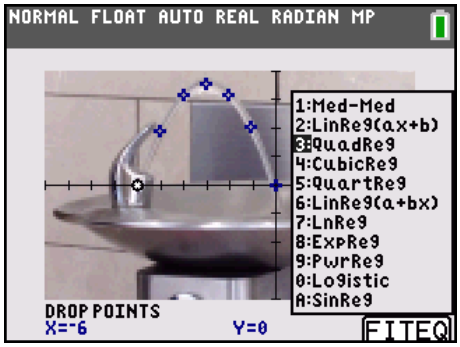


Note: If you make an error, press 2nd [QUIT].
To clear the screen, press 2nd [DRAW] **1:ClrDraw** ENTER.

When you have dropped the desired number of points, press the **FITEQ** soft key.

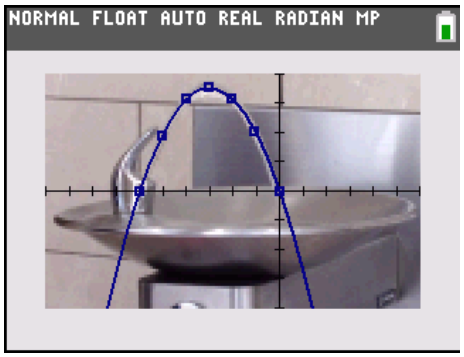


Choose the regression model **3:QuadReg**.

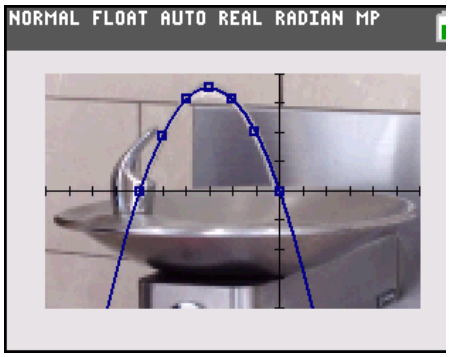


The equation of the regression model $y = ax^2 + bx + c$ is displayed and graphed. Follow the prompts to store the data in L1 and L2 and the equation in Y1.

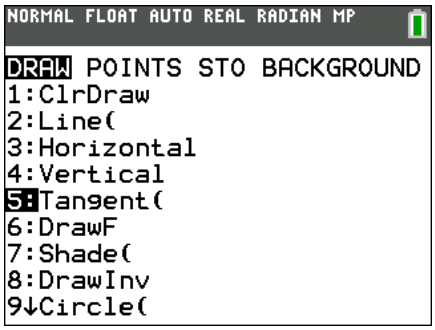
1. The value of *c* in the equation of the model should be near 0. Why?
2. How do the values of *a* and *b* in the **QuickPlot&Fit-EQ** quadratic regression equation compare with those of your model found analytically in Problem 1?



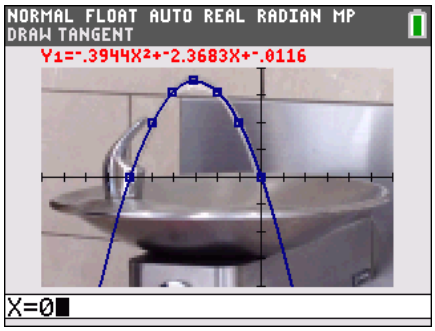
Calculus Extension - Draw a tangent line at the origin.



1. First press GRAPH, which will cue the TI-84 to prompt you for arguments from the Graph Screen, followed by 2nd [DRAW].

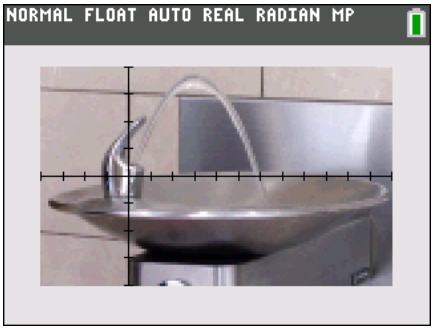


2. While the left and right arrow keys can be used to locate the desired point of tangency on the curve, you can also just type the x-coordinate. Press the number 0 to ask for a tangent line at the origin. Press ENTER.



3. The tangent line is drawn and its equation at the origin is displayed. Compare the equation of the tangent line and the equation of the parabola. What do you notice?
Why is this the case?

Problem 3 – Using QuickPlot&Fit-EQ for Warren's Choice of Axes



Warren models the parabolic stream using the axes shown to the right. He uses the same values as Lauren for Ymin and Ymax, but uses Xmin = −4, Xmax = 12. Suppose he too uses integer values for the zeros of his parabola. Predict the following:

1. How would his value of *a* in his quadratic equation compare with the value of *a* found in Lauren's model? Why?
2. How would his value of *b* compare with the value of *b* found in Lauren's model? Why?
3. Find an equation for Warren's model using **QuickPlot&Fit-EQ**. Store the data in L_3 and L_4 and the equation in Y_2. Graph the equations for Lauren's and Warren's model in the same viewing window (Xmin = −4, Xmax = 12, Ymin: = −4, Ymax = 4). How are these graphs related?

Extension

1. What viewing window might position the *y*-axis on the vertex?
2. How would this affect the equation of the model?