Technology and the AP Calculus Exam

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Topics

- The four graphing calculator capabilities.
  Remarks.
- For Each Calculator Function:
  AP Calculus Free-Response Question
  Calculator Solution.

Calculator Use on the Exams

- Both multiple-choice and free-response sections include problems that require the use of a graphing calculator.
- A graphing calculator is expected to have these capabilities.
  (1) Plot the graph of a function within an arbitrary viewing window.
  (2) Find the zeros of a functions (solve equations numerically).
  (3) Numerically calculate the derivative of a function.
  (4) Numerically calculate the value of a definite integral.

Remarks

- A student may not need to use a graphing calculator to solve a *calculator active question*.
- The four *capabilities* are the only calculator functions a student can use to present a solution.
- A student can use any other available graphing calculator function.
- Great care is taken in writing the questions to ensure that students using an advanced machine do not have an advantage.
- Calculator issues:
  (1) Increase the number of acceptable capabilities.
  (2) Equity among students.
  (3) Number of calculator active questions.

2002 Form B AB-3

A particle moves along the x-axis so that its velocity v at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

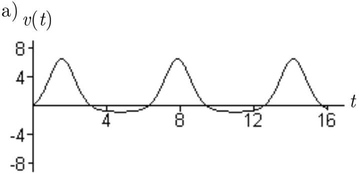
(a) On the axes provided, sketch the graph of $v(t)$ for $0 \le t \le 16$.

(b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

(d) Is there any time t, $0 < t \le 16$, at which the particle returns to the origin? Justify your answer.

(a)



1 : graph

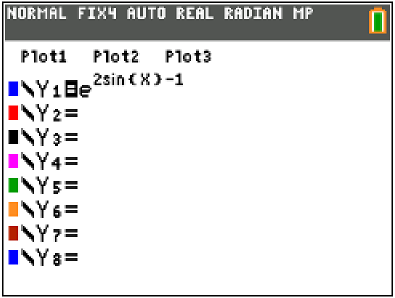
three "humps"

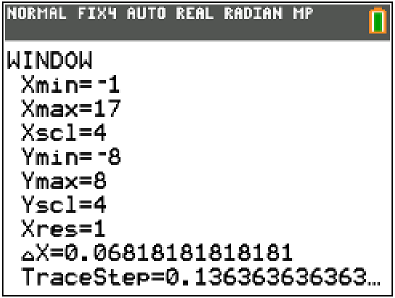
periodic behavior

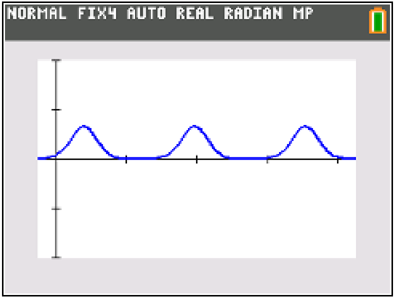
starts at origin

reasonable relative max and min values

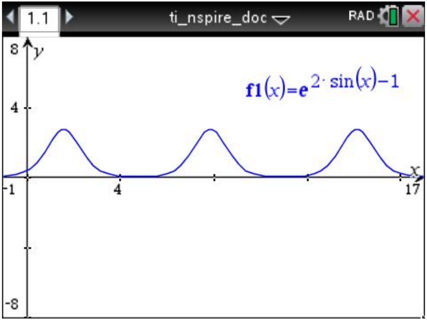
Calculator Solution: TI-84 Plus CE







Calculator Solution: TI-nspire



2005 AB-4

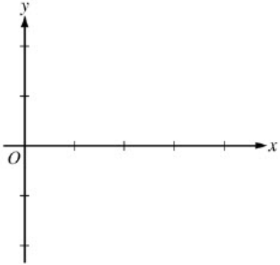
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

   (a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

   (b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

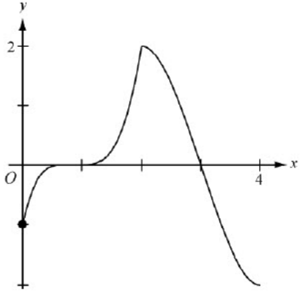
   (Note: Use the axes provided in the pink test booklet.)



2005 AB-4

Part (b): Solution

(b)



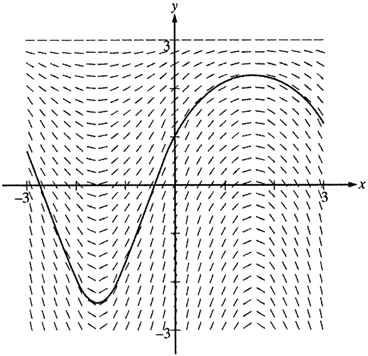
$$2:\begin{cases}1: \text{points at } x = 0, 1, 2, 3 \\ \quad\ \text{and behavior at } (2, 2) \\ 1: \text{appropriate increasing/decreasing} \\ \quad\ \text{and concavity behavior}\end{cases}$$

2014 AB-6

Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.

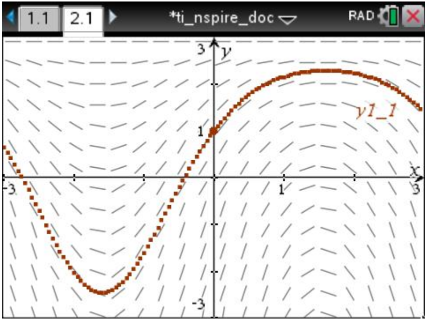
(a)



1 : solution curve

2014 AB-6

Part (a): Calculator Solution: TI-nspire



Note: TI-84 programs to construct slope fields.

2015 AB-1 / BC-1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

2015 AB-1 / BC-1

Part (c): Solution

(c) The amount of water in the pipe at time t, $0 \le t \le 8$, is

$$30 + \int_0^t [R(x) - D(x)]\,dx.$$

$$R(t) - D(t) = 0 \Rightarrow t = 0,\ 3.271658$$

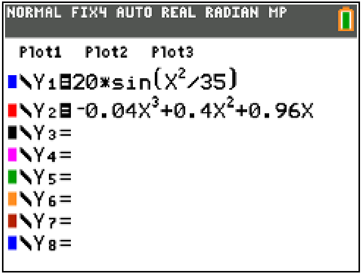
t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

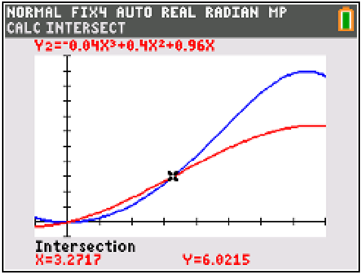
The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

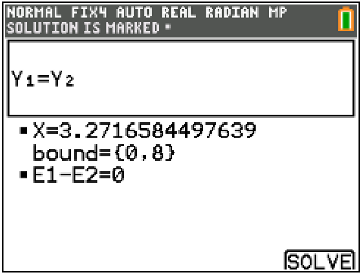
$$3 : \begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

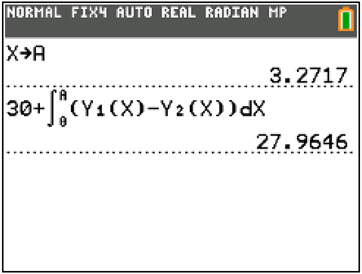
2015 AB-1 / BC-1

Part (c): Calculator Solution, TI-84 Plus CE





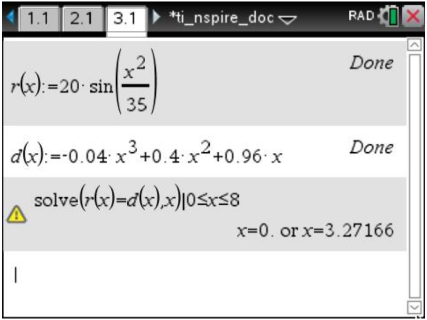




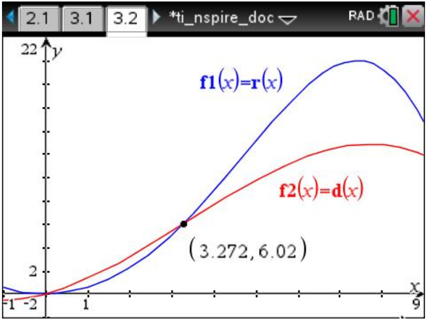
2015 AB-1 / BC-1

Part (c): Calculator Solution, TI-nspire

Home Screen

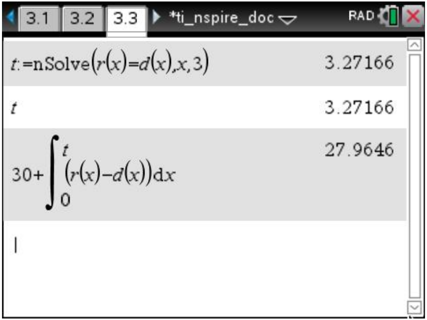


Graph Screen



2015 AB-1 / BC-1

Part (c): Calculator Solution, TI-nspire, Continued



2015 BC-2, Part (b)

At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

(c) Find the time at which the speed of the particle is 3.

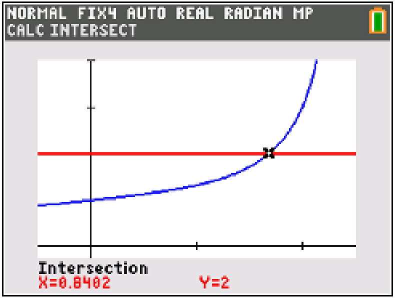
(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

(b) $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{e^{0.5t}}{\cos\left(t^2\right)}$ $\dfrac{e^{0.5t}}{\cos\left(t^2\right)} = 2$ $t = 0.840$	$2 : \begin{cases} 1 : \text{slope in terms of } t \\ 1 : \text{answer} \end{cases}$

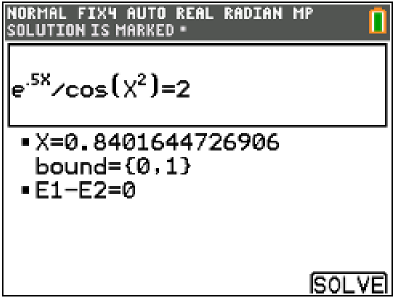
2015 BC-2, Part (b)

Calculator Solution: TI-84 Plus CE

Graph Screen



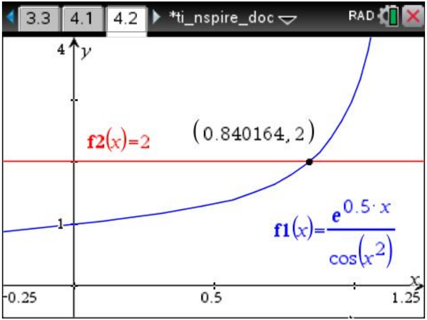
Home Screen



2015 BC-2, Part (b)

Calculator Solution: TI-nspire

Graph Screen



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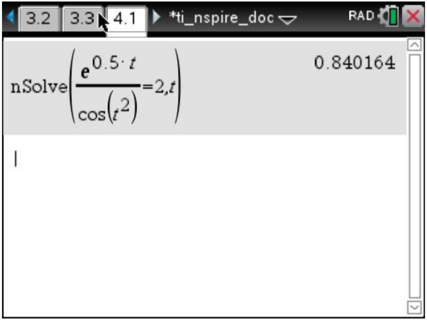


2014 AB-1 / BC-1, Parts (b) and (d)

Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

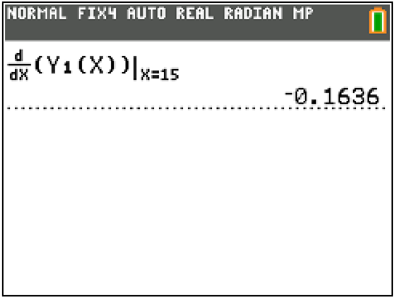
(d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

(b) $A'(15) = -0.164$ (or -0.163) The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t = 15$ days.	$2 : \begin{cases} 1 : A'(15) \\ 1 : \text{interpretation} \end{cases}$
(d) $L(t) = A(30) + A'(30) \cdot (t - 30)$ $A'(30) = -0.055976$ $A(30) = 0.782928$ $L(t) = 0.5 \Rightarrow t = 35.054$	$4 : \begin{cases} 2 : \text{expression for } L(t) \\ 1 : L(t) = 0.5 \\ 1 : \text{answer} \end{cases}$

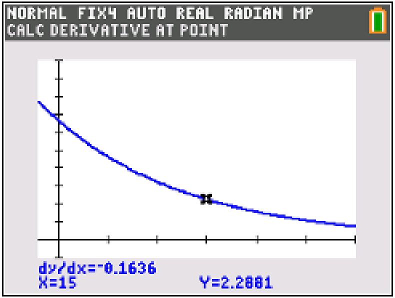
2014 AB-1 / BC-1, Part (b)

Calculator Solution: TI-84 Plus CE

Home Screen



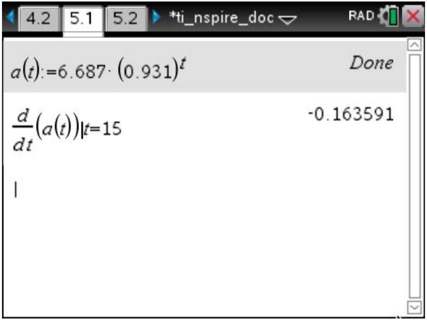
Graph Screen



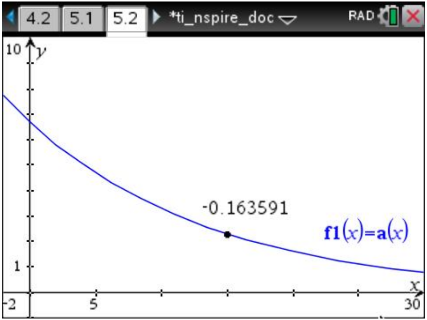
2014 AB-1 / BC-1, Part (b)

Calculator Solution: TI-nspire

Home Screen



Graph Screen



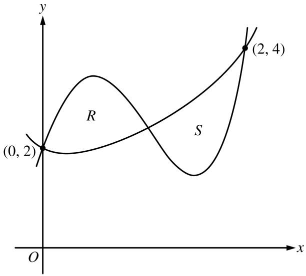
2015 AB-2

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

(a) Find the sum of the areas of regions R and S.

(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.



2015 AB-2

Solution

(a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points $(0, 2)$, $(2,4)$, and $(A, B) = (1.032832, 2.401108)$.

$$\text{Area} = \int_0^A [g(x) - f(x)]\,dx + \int_A^2 [f(x) - g(x)]\,dx$$
$$= 0.997427 + 1.006919 = 2.004$$

$4 : \begin{cases} 1 : \text{limits} \\ 2 : \text{integrands} \\ 1 : \text{answer} \end{cases}$

(b) $\text{Volume} = \int_A^2 [f(x) - g(x)]^2\,dx = 1.283$

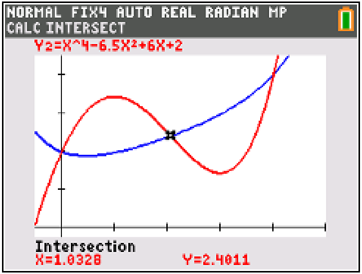
$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

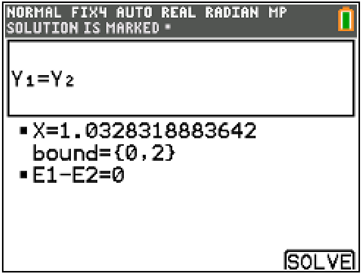
(c) $h(x) = f(x) - g(x)$
$h'(x) = f'(x) - g'(x)$
$h'(1.8) = f'(1.8) - g'(1.8) = -3.812 \text{ (or } -3.811)$

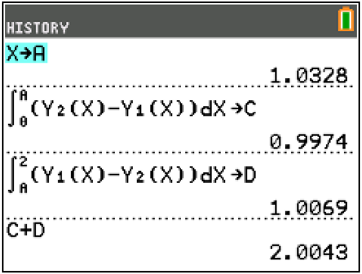
$2 : \begin{cases} 1 : \text{considers } h' \\ 1 : \text{answer} \end{cases}$

2015 AB-2

Explore on the Graph Screen

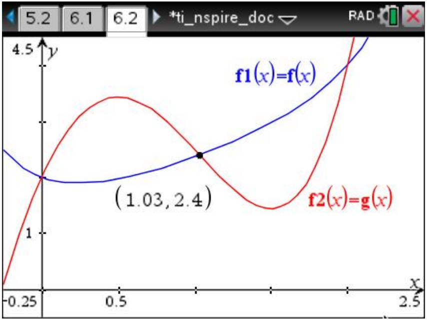






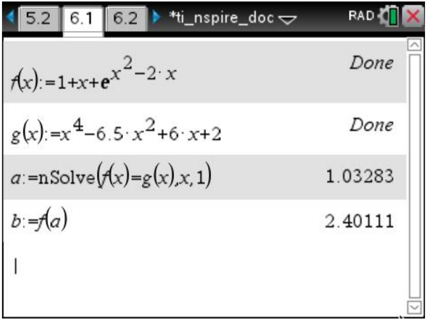
2015 AB-2

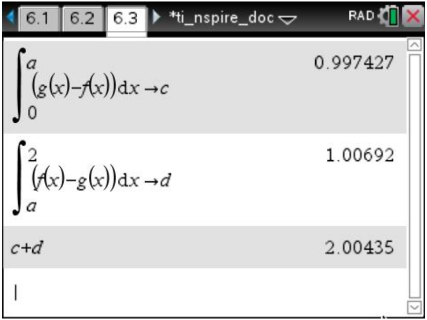
Explore on the Graph Screen



2015 AB-2

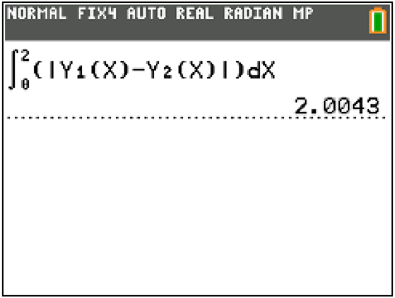
Confirm on the Home Screen

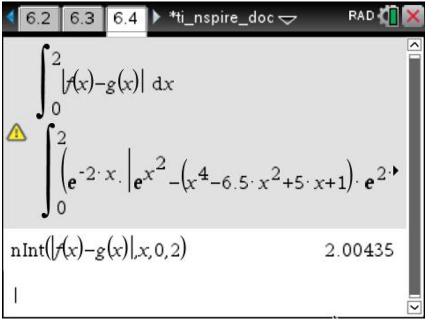




2015 AB-2

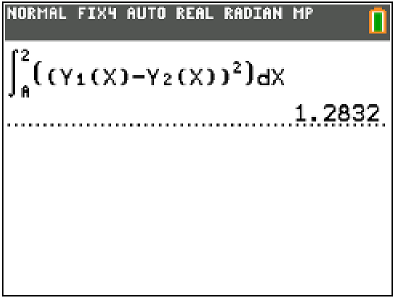
Part (a): Alternate Solution

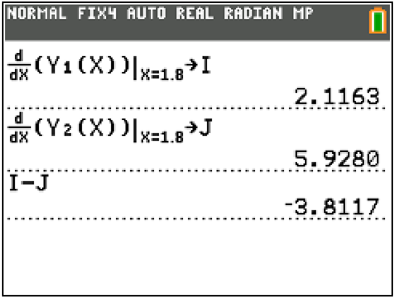




2015 AB-2

Parts (b) and (c): Calculator Solution





2015 AB-2

Parts (b) and (c): Calculator Solution