DataQuest

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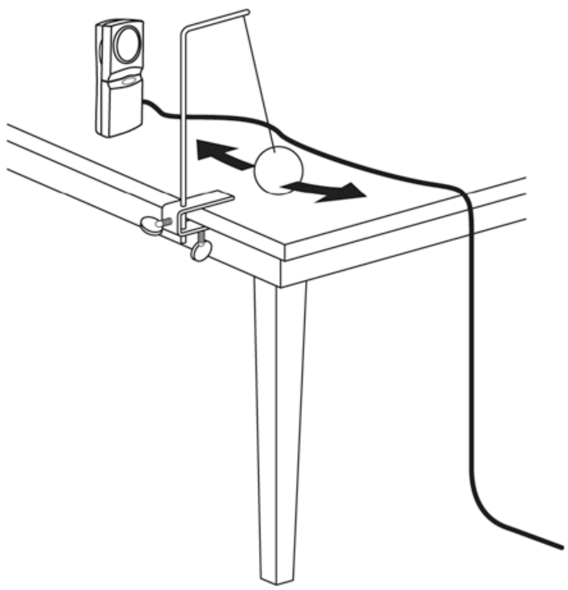
Tic, Toc: Pendulum Motion

Pendulum motion has long fascinated people. Galileo studied pendulum motion by watching a swinging chandelier and timing it with his pulse. In 1851 Jean Foucault demonstrated that the earth rotates by using a long pendulum which swung in the same plane while the earth rotated beneath it.

As long as the swing is not too wide, the pendulum approximates simple harmonic motion and produces a sinusoidal pattern. In this activity, you will use a Motion Detector to plot the position versus time graph for a simple pendulum. You will time the motion to calculate the period, and use a ruler to measure the maximum displacement. You will then use the data to model the motion with the cosine function $y = A\cos(B(x - C)) + D$ to mimic the position versus time graph.



OBJECTIVES

- Record the horizontal position versus time for a swinging pendulum.
- Determine the period of the pendulum motion.
- Model the position data using a cosine function.

MATERIALS

TI-Nspire handheld **or** computer and TI-Nspire software	pendulum bob
CBR 2 **or** Go! Motion **or** Motion Detector and data-collection interface	string
	meter stick
	stopwatch

PROCEDURE

1. Hang the pendulum bob from about 80 cm of string from a rigid support. Arrange the support so the bottom of the pendulum bob clears the table by several centimeters. Position the Motion Detector about 1 m away from the bob, pointing at the bob. Elevate the detector slightly so that it does not respond to the table.

2. If your Motion Detector has a switch, set it to Normal. Connect the Motion Detector to the data-collection interface. Connect the interface to the TI-Nspire handheld or computer. (If you are using a CBR 2 or Go! Motion, you do not need a data-collection interface.)

3. Choose New Experiment from the Experiment menu. For this experiment, the default data-collection parameters for a Motion Detector will be used (Rate: 20 samples per second; Duration: 5 seconds). The number of points collected should be 101.

4. Click the Graph View tab ( ). Choose Show Graph ► Graph 1 from the Graph menu. Only the position *vs*. time graph will be displayed.

5. Measure the distance from the bob to the Motion Detector; this distance must be at least 50 cm. Record this distance *D*, in meters, in your data table.

6. Place the meter stick under the bob, along the line between the detector and the bob. Arrange the stick so that the zero point is under the bob when it is hanging still. Determine how far you will pull back the bob before releasing it. This distance should be less than 20 cm. Record this value, in meters, as the amplitude *A* in your data table.

7. Use the stopwatch to measure the period of the pendulum. The period is the time taken by the pendulum to complete one back and forth cycle. Use the amplitude of motion you determined in the previous step. Measure the time for *ten* complete cycles, and record this time in your data table.

   Take care to count carefully: One way to do this is to start the stopwatch when the bob is farthest from the Motion Detector, and count one cycle when it returns to that spot. Keep the stopwatch running until ten cycles are completed.

8. Practice swinging the ball by pulling it back the distance you recorded above, and then releasing it so that the ball swings in a line directly away from the Motion Detector.

9. With the ball swinging properly, start data collection ( ). Data collection will run for five seconds.

10. When data collection is complete, a graph of position versus time will be displayed. Examine the graph, which should be sinusoidal. Check with your teacher if you are not sure whether you need to repeat the data collection. To repeat data collection, repeat Step 9.

DATA TABLE

A (m)	
B	
C (s)	
D (m)	
Time for ten cycles (s)	
Period (s)	

ANALYSIS

1. Be sure your TI-TI-Nspire software is set to perform angle calculations in Radians.

2. As part of your analysis, you will compare the Motion Detector data to the cosine model of $y = A\cos(B(x - C)) + D$. Your setup measurements will allow you to determine the parameters *A*, *B*, and *D*. You can determine *C* from your graph of the Motion Detector data.
   a. Click any data point and use ► and ◄ to examine the data points on the graph.
   b. Since the cosine function starts at a maximum value when its argument is zero, you can use the location of a maximum to determine the value of *C*, which represents the horizontal shift of the data. Trace across your data to any maximum and read the time (*x*) value. Record this value as *C* in your data table.

3. During the procedure, you measured the time for ten complete cycles of the pendulum. Use this value to find the period of the motion, which is the time for one complete cycle. Enter this value in your data table.

4. The sinusoidal model has a parameter *B* (called the angular frequency) that represents the number of cycles the function makes during the natural period of the cosine function. Find *B* by taking 2π (the natural period of the cosine function) divided by the period of the pendulum (the time for one cycle). Record the value in your data table.

5. Display a graph of the Motion Detector data and the model equation. After entering the model equation, you'll enter the values of the four parameters found in your data table.
   a. Choose Model from the Analyze menu.
   b. Enter **A*cos(B*(x–C))+D** as the equation for your model. Select OK.
   c. In the A field, enter the amplitude of your bob motion.
   d. In the B field, enter the angular frequency.
   e. In the C field, enter the horizontal time shift value.
   f. In the D field, enter the value for the offset of the cosine function from the distance axis.
   g. Select OK to view the model on your graph and answer Analysis Questions 1–4.

ANALYSIS QUESTIONS

1. How well does your model equation fit your data? If your fit is acceptable, write the model equation below, and suggest explanations for any discrepancies. If the fit of the model is not acceptable, deduce which of your parameters is producing the problem. Make changes as necessary to the parameters, and discuss why the changes were necessary. Write out the equation that produced a good fit. When you're done, select OK to close the curve fit dialog box.

2. How would the parameters A, B, C, and D change if you were to use the sine function $y = A \sin(B(x–C))+D$ instead of the cosine function? Predict the values below and explain your reasoning for each.

3. Test your predictions by storing any changed values in the four parameters using the same method you used above. Also using the same method as above, change the model equation to a sine function. Redisplay the graph to compare the data and sine model. How well does the sine model fit the data? Explain any discrepancies.

4. Give a physical interpretation of each of the parameters A, B, C, and D from the model $y = A \cos(B(x–C))+D$ in terms of the pendulum.

CALCULUS EXTENSION

Once you have an equation for the position versus time graph of the pendulum motion, take the derivative of the equation. This represents the velocity of the pendulum at any time t. How does the velocity versus time graph compare with the position versus time graph? When during the pendulum motion is the velocity zero? When is the velocity a maximum?

The derivative of velocity is *acceleration*. Take the second derivative of the position equation. Describe the position and velocity when the acceleration is a maximum. Do the same when the acceleration is zero.

Give a general description of the pendulum's position, velocity, and acceleration when the bob is passing through the at-rest position and when it is farthest from the detector.