



Math Objectives

- Students will model a basketball's flight through the air using a quadratic function.
- Students will analyze their quadratic function and interpret the parameters of the function in relation to the flight path of the basketball.
- Students will reason abstractly and quantitatively, model with mathematics, and attend to precision.

Vocabulary

- parent quadratic function
- maximum value of a quadratic function
- zero of a quadratic function

About the Lesson




- In this lesson, students use a quadratic function to model the flight path of a basketball. Students will interpret the parameters of the quadratic model to answer questions related to the path of the basketball.
- As a result, students will:
 - Model data with quadratic functions.
 - Interpret the vertex of a quadratic function in a real-world context.
 - Determine a zero of a quadratic equation.
 - Use a mathematical model to make predictions.

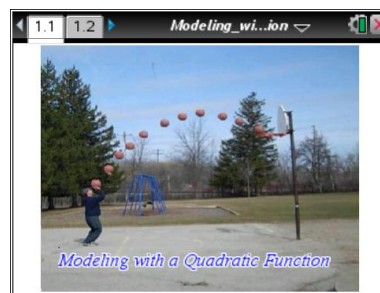


TI-Nspire™ Navigator™

- Send out the *Modeling_with_a_Quadratic_Function.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies :  TI-Nspire™ CX Handhelds,  TI-Nspire™ Apps for iPad®,  TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>

Lesson Files:

Student Activity:

- Modeling_with_a_Quadratic_Function_Student.pdf
- Modeling_with_a_Quadratic_Function_Student.doc


TI-Nspire document

- Modeling_with_a_Quadratic_Function.tns



Discussion Points and Possible Answers



Tech Tip: If students experience difficulty dilating or translating the parabola, check to make sure that they have moved the cursor near the parabola until the cursor becomes a dilation symbol (\sphericalangle) or a translation symbol (\rightarrow). They should then press **ctrl**  to grab the parabola and use the Touchpad to dilate or translate the parabola.



Tech Tip: If students experience difficulty dilating or translating the parabola, point out that selecting the graph near the vertex will translate it while selecting the graph elsewhere will dilate it.

Move to page 1.2.

Note: Two tick marks represent one meter.

1. Graph the parent quadratic function, $f(x) = x^2$. Transform the parent function so that it matches the path of the basketball. What is the equation of the quadratic function that matches the path of the basketball?

Sample Answer: Student models will vary slightly. A possible equation is

$$f(x) = -0.093(x - 7.17)^2 + 7.98.$$


2. Attending to precision is important in mathematics. At the distance this picture was taken and with the scale provided, is it reasonable to get results that are correct to the nearest millimeter, centimeter, decimeter, or meter? In other words, how many decimal places should you keep?

Answer: 1 decimal place of precision is possible. Answers should be given to the nearest decimeter. This is true since the marking shows 0.5 meter, so estimating to the nearest 0.1 meter is possible.



3. In this activity, the horizontal distance traveled by the basketball is the independent variable. What is the dependent variable?

Answer: The dependent variable represents the vertical height of the basketball.



Tech Tip: To determine the maximum height, students can use the **Trace** command by pressing **Menu > Trace > Graph Trace** and moving the cursor to the maximum value on the parabola. Alternately, students can select **Menu > Analyze Graph > Maximum**. They must set the lower bound by moving the cursor to the left of the maximum and pressing **enter**, or enter a value and press **enter**. They must then set the upper bound by moving to the right of the maximum and pressing **enter** or , or enter a value. As they move the cursor, the coordinates of the maximum will be “ghosted” until the upper bound is set.



Tech Tip: To determine the maximum height, students can use the **Trace** command by selecting  **> Trace > Graph Trace** and then selecting the maximum value on the parabola. Alternately, students can select  **> Tools > Analyze Graph > Maximum**. Students must select a location to the left of the maximum, then a location to the right of the maximum. The coordinates of the maximum will be “ghosted” until the upper bound is set.

4. Determine the maximum height of the basketball in meters. Explain your reasoning.

Note: Two tick marks on the axes represent 1 meter.

Sample Answer: The maximum height of the basketball is 3.99 m. The y-coordinate of the vertex is 7.98. Using the scale provided, this value is divided by two to determine the height of the ball.

Teacher Tip: Students can justify their reasoning by using strategies such as reading the value from the graph or using the **Trace** command or the **Maximum** command.

5. Visualize a point on the ground directly beneath the ball when it reaches its maximum height. How far is this point from the person shooting the basketball? Explain your reasoning.

Sample Answer: The point directly beneath the ball when it reaches its maximum height is 3.59 m from the person on the left. The x-coordinate of the vertex is 7.17. Using the scale provided, divide this number by 2 to determine the point at which the maximum occurs.

Teacher Tip: Students can also justify their reasoning by using other strategies, such as reading the value from the graph or using the **Trace** command or the **Maximum** command.



6. How high is the ball when it is a horizontal distance of 2 m from the person shooting the basketball? Explain your reasoning.

Sample Answer: The ball is 3.53 m high at a horizontal distance of 2 m from the person on the left. Use $x = 4$ since 4 tick marks on the graph represents 2 m. The function value at $x = 4$ is 7.05. Divide this value by 2 because of the scale.



Tech Tip: Students can answer question 5 by evaluating the function for an x -value of 4. They can use Trace and type 4 or they can either insert a Calculator page and type **f1(4)** or use the **Scratchpad**. If they use the Scratchpad, they will need to record their equation on a piece of paper and then type the function in and evaluate the function at $x = 4$.



Tech Tip: Students can answer question 5 by evaluating the function for an x -value of 4. They can insert a Calculator page and type **f1(4)** or Trace on the graph to $x = 4$.

7. If the ball follows the path modeled by your quadratic function, and the basket is not there, how far will the ball land from the person on the left? Explain your reasoning.


Sample Answer: The ball will land 8.2 m from the person on the left. The x -intercept to the right of the vertex is (16.4, 0). To determine this, place a point on the function and then drag it toward the x -axis. This is the point where the ball hits the ground. Take this value and divide by 2 because of the scale.

Teacher Tip: Students can also justify their reasoning by using other strategies, such as reading the value from the graph, using the **Trace** or **Zero** commands in a Graphs page, or using the **nSolve** command in a Calculator page.



Tech Tip: To use the **Zero** command, select **Menu > Analyze Graph > Zero**. Select the region.



Tech Tip: To use the **Zero** command, select  **> Analyze Graph > Zero**. Use the touchpad to select the lower and upper bound of the zero or enter values for the lower and upper.



Tech Tip: If a student uses the **nSolve** command after adding a calculator page, they will get the zero on the left, which is the one closest to the origin. They will need to specify the constraints to solve for the one on the right. The command could be **nSolve (f1(x) = 0, x, 0, 20)** or use the vertical bar for 'such that' found in **[ctrl] [=]** to limit the domain and type **nSolve (f1(x) = 0, x) | 0 < x < 20**.

Extensions

1. How far, horizontally, from the person shooting the basketball is the ball when the ball reaches a height of 3.7 m?

Teacher Tip: This height is interesting because, the ball is at that height two times.

2. How far is the person standing from the basket? Explain your reasoning.
3. The equation $y = -0.1(x - 8)^2 + 9$ describes the path of another basketball. The person throwing the ball is positioned on the y -axis. How is the path of this ball similar, and how is it different from the path of the ball in the activity? (Assume the scale is the same.)
4. What assumptions have been made when creating this model?

Teacher Tip: Do students know the height of a basketball goal in meters? A quick Google search shows that the height is 10 feet, or about 3 m. In the model for the image, the rim is about 3.3 m from the x -axis and the bottom of the basketball pole is at about 0.3 m from the x -axis. Have students discuss how the model can be improved. Basketballs travel a nearly parabolic trajectory. Although the air resistance is small, causing it to slow down at about 1 m/s each second tangent to its path, the path is not perfectly parabolic.